Propulsion Performance Measures

(Supplementary Notes)

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1 Introduction

The purpose of these notes in to outline the basic performance parameters used to evaluate propulsive devices. These include thrust, propulsive and thermal efficiency as well as thrust specific fuel consumption (TSFC) and specific impulse. These parameter will be discussed in the context of an air breathing propulsive device such as a turbojet as well as rockets.

1.1 Basic concepts

braces Thrust of course is a measure of the propulsive force generated by a jet engine or rocket. The thrust equation will be developed in class and below for both types of devices. Here we will take up the discussion of the term efficiency. What is an efficiency? Very simply put it is the ratio a desired output of the system to the required input to achieve the desired output or "what you want divided by what it cost to get it". Of course we could be discussing the actual "cost" in some currency, but in general we will be discussing some energy input. Let's look at some examples you should have learned from thermodynamics. When looking at these expressions below ask yourself what the desired output is from each process or device.

Combustion Efficiency

$$\eta_{combustion} = \frac{\text{Amount of heat released}}{\text{Fuel Heating Value}} = \frac{Q}{HV}, \tag{1}$$

Electric Motor

$$\eta_{motor} = \frac{\text{Mechanical power output}}{\text{Electrical power input}} = \frac{P_{shaft}}{P_{elec}}, \tag{2}$$

Electric Generator

$$\eta_{generator} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{P_{elec}}{P_{shaft}}, \tag{3}$$

Pump

$$\eta_{pump} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{P_{pump}}{P_{shaft}}, \tag{4}$$

If we want to know the efficiency of a electric motor and pump combination it is simply the product of the two efficiencies.

$$\eta_{pump+motor} = \frac{P_{shaft}}{P_{elec}}\frac{P_{pump}}{P_{shaft}} = \frac{P_{pump}}{P_{elec}}, \tag{5}$$

See your thermodynamics book for more examples. You should make sure you can write out the efficiency of the combination of a turbine and a generator.

1.2 Thrust

In class we derived the thrust equation from first principles using the Reynolds Transport Theorem. This resulted in a form of the net thrust equation for the air breathing device and the rocket.

$$T = \dot{m}_e V_e - \dot{m}_i V_i + (P_e - P_{atm})A_e \tag{6}$$

To defining the effective or "equivalent" exhaust velocity we rearrange the net thrust equation.

$$T = \dot{m}_e(V_e + \frac{(P_e - P_{atm})A_e}{\dot{m}_e}) - \dot{m}_i V_i \tag{7}$$

The term in brackets is usuall labeled "C" or in the case of the text "u_{eq}".

$$C = V_e + \frac{(P_e - P_{atm})A_e}{\dot{m}_e} \tag{8}$$

The thrust equation can then be written in the following form:

$$T = \dot{m}_e C - \dot{m}_i V_i = \text{gross thrust} - \text{ram drag} \tag{9}$$

For the rocket this simplifies to

$$T = \dot{m}_e V_e + (P_e - P_{atm})A_e = \dot{m}_e C \tag{10}$$

2 The Efficiencies

braces In the section the thermal an propulsive efficiencies will be defined and discussed. Understanding these basic performance measures in critical to understanding chemical as well as other propulsion systems.

2.1 Thermal Efficiency

In thermodyanamics you learned the cycle thermal efficiency was a ratio of the "net work" divided by the "heat in". In the context of a chemical system the heat in is simply the energy consumed during combustion of a fuel. This is usually defined as the heat release (Q_R) or heating value (HV) For non-chemical systems it could simply be a measure of the heat transfer from a heat exchanger heated by a nuclear reactor or a solar concentrator. In general this could be considered the available energy or power. Determining the net work in the context of a propulsive system we are interested in the change in energy of or the work done on the working fluid. In general this could be called the thermal power and is manifested in the change in the kinetic energy of the working fluid. As an example, for the air breathing system assuming one-dimensional flow and chemical propulsion we have:

Thermal Efficiency (Air Breathing System)

$$\eta_{th} = \frac{\text{Rate of kinetic energy change of the working fluid}}{\text{Rate of energy consumed by combustion}} = \frac{\dot{m}(C^2/2 - V_i^2/2)}{\dot{m}_f Q_R} \tag{11}$$

Thermal (Internal) Efficiency (Rocket)

$$\eta_{th} = \frac{\frac{1}{2}\dot{m}C^2}{\dot{m}Q_R} = \frac{C^2}{2Q_R} \tag{12}$$

Note that for the rocket the change in kinetic energy of the jet starts nearly from rest.

2.2 Propulsive Efficiency

It is very important for a thermal system to change the total energy in the system to generate useful work or power. However for propulsion systems the useful effort is usually measured in terms of thrust and useful power in terms of thrust multiplied by flight velocity TV_i. We define the propulsive efficiency as the ratio of this useful power to the power in the jet or the rate of change in the kinetic energy of the working fluid described above.

Propulsive Efficiency (general definition works for both air breathers and rockets)

$$\eta_p = \frac{\text{Useful or thrust power}}{\text{Thrust power} + \text{Rate of residual kinetic energy in the jet}} = \frac{TV_i}{TV_i + \dot{m}(C - V_i)^2} \tag{13}$$

Propulsive Efficiency (works best for air breather applications)

$$\eta_p = \frac{\text{Useful or thrust power}}{\text{Rate of kinetic energy change of the working fluid}} = \frac{TV_i}{\dot{m}(C^2/2 - V_i^2/2)} \tag{14}$$

2.2.1 Air Breathing Systems

For the air breather we substitute the thrust equation $T = \dot{m}_e C - \dot{m}_i V_i$ into equation (13) or (14) and we obtain the following and assuming the fuel air ratio is very small such that the inlet and exit mass flows are equal:

$$\eta_p = \frac{2V_i}{C + V_i} \tag{15}$$

2.2.2 Rockets

For the rocket we substitute the thrust equation $T = \dot{m}C$ into equation (13).

$$\eta_p = \frac{2CV_i}{C^2 + V_i^2} \tag{16}$$