Counting

In computer science, we want to count things all the time. How many steps does this take? How much memory will it use? How many different answers can I get? All these questions require that we be able to count.

1. The basic rule is the product rule. If there are m ways of doing one task, and n ways of doing another task, then there are mn ways of doing both tasks, one after the other.

   (a) I'm tired of always typing long alpha codes. If I were to propose replacing the alpha code with a 3 character long code, with onde digit, and two upper case letters, how many midshipmen could I represent with this system? $10 \times 26 \times 26 = 6760$.

   (b) If I have a Snickers, a Mountain Dew, a Strawberry Pop-Tart, and a class of 20 students, and I will only give one item to any single student. How many ways can I distribute my junk food? $20 \times 19 \times 18 = 6840$.

   (c) If I have a set S of size $|S|$. How many subsets can I make of S? If I go through each element of the set in turn, I can either add that element to the subset or not. Thus there are 2 choices for each of the $|S|$ elements in the set, or there are $2^{|S|}$ subsets of S.

   (d)

$$\begin{aligned}
&\text{for}(i = 0; i < 50; i++)\\
&\quad\text{for}(j = 0; j < 25; j++)\\
&\qquad\text{for}(k = 0; k < 10; k++)\\
&\qquad\quad\text{cout} << \text{"Yao Ming!"};
\end{aligned}$$

   How many times is this sage knowlege printed? 12500.

2. The sum rule. If there are m ways of doing one task, and n ways of doing another task, and only one task can be done, then there are $m + n$ ways of doing one of these tasks.

   (a) If there are 22 mids in one of my classes, and 19 in the other, there are 41 ways I can pick a single midshipman from one of my classes.

3. This is a good problem that combines the sum and product rule, and also shows you an importatnt trick. Imagine a computer system that requires a password at least 6, but no more than 8 characters long, where a character can be an uppercase letter, lowercase letter or a digit, and there must be at least 1 digit.

   (a) First, lets calculate how many 6 character passwords there are. We can easily find the number of passwords if there wasn't the 1 digit requirement: $(26+26+10)^6 = 56,800,235,584$, but how do we include the restriction? All we have to do is subtract away the number of passwords without *any* digits to be left with the number with at least 1 digit:$(26+26+10)^6 - (26+26)^6 = 37,029,625,920$

   (b) Now we just reapeat this for 7 & 8character passwords: $(26+26+10)^7 - (26+26)^7 = 2.49354 \times 10^{12}$ and $(26 + 26 + 10)^8 - (26 + 26)^8 = 1.6488 \times 10^{14}$

   (c) Finally, since we can pick either a 6 character or a 7 character or an 8 character password, we add them up: $37029625920 + 2.49354 \times 10^{12} + 1.6488 \times 10^{14} = 1.67411 \times 10^{14}$

   (d) note that the shorter passwords don't add much, do they?

4. Inclusion-exclusion principle. Sometimes we want to use the sum rule but there s some overlap. For example, when I was picking a midshipman from one of my classes before, I assumed there was no overlap in students between the two classes. What if there are 22 mids in one of my classes, and 19 in the other, but all 19 are also in the first class? How many ways can I pick a unique mid from one of my classes now? 22, right. The basis of the exclusion principle is that when you're tempted to use the sum rule for m and n different ways, but there is overlap of o things, then the number of ways to do one of these tasks is $m + n - o$.

   (a) If I have 10 students who are in both of my classes, then there are $22 + 19 - 10$ different ways to pick a uniqie student from one of my classes.

   (b) How many 8-bit numbers start with 1, or end in 00? Well, there are 2^7 numbers that start with 1, and 2^6 numbers that end with 00, but we can't just add them to gether because there are some that start with 1 AND end with 00. So we subtract out those double counted numbers to get: $128 + 64 - 32 = 160$

5. Tree diagrams. Sometimes drawing a picture of the situation is what's called for. We draw what we'll call a tree, branching with each possible choice. How many 4 bit sequences have no double 1s (sub-sequences like 11). We draw the tree, and then count.

6. Pigeon Hole Principle: if there are $k + 1$ or more pigeons and only k pigeon holes, then at least one hole will have more than one pigeon.

   (a) well duh.

   (b) How many people do you need to guarantee that two have the same Birthday? 367.

   (c) OK, but how about this one: Show that for any ingeger, there is a multiple of that number that is made up of only 1 and 0 (it's still a deciman number, it just doesn't have any of the other digits).

   Let n be a positive integer. There are $n + 1$ integers of the form: $1, 11, 111, ..., 1...1$ where the last integer has $n + 1$ 1s. Take each of the integers and $\mod$ it by n, leaving $n + 1$ results. Of these results, at least 2 must be the same, by the pigeon hole principle. If you take any 2 integers that yeild the same result when modded by some n, then the difference between the 2 integers is divisible by n (try it!). But the difference of two integers made up entirely of 1s is made up entirely of 0s and 1s. $\square$