

Map Making

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We've been talking about navigation using maps, but where do these maps come from?

- Hand Crafted by people. Very accurate, but work intensive. Robot limited to navigating in areas already mapped.
- Robot Crafted. Robot explores and builds a map automatically. often less accurate. Why?

How do we represent the map?

- Divide world into a grid, just like graph paper.
- Mark some cells as occupied, and some as unoccupied.
- This is the map.
- Called occupancy grid.
- If we need to we can idealize blobs into polygons for those path planning algorithms.

Question: If a sensor tells us that some particular cell is occupied (or not occupied), what should we do?

- Mark the cell as occupied (or unoccupied)?
- What if the sensor is wrong?
- How do we incorporate this uncertainty into our model?
- Probability

Probability is a measure of uncertainty that you've all covered before. Just as a basic reminder:

- $P(H=a)$ is the probability of some variable H having value a .
- e.g. $P(\text{CoinFlip} = \text{heads})$, $P(\text{CellOccupied} = \text{true})$ This is called a hypothesis.
- The hypothesis has some number p $0 \leq p \leq 1$: $P(\text{CoinFlip} = \text{heads}) = 0.5$, $P(\text{CellOccupied} = \text{true}) = 0.33$.
- The sum of the p 's across all values of the variable must be 1.: $P(\text{CoinFlip} = \text{heads}) = 0.5 + P(\text{CoinFlip} = \text{tails}) = 0.5 = 1$ or $P(\text{CellOccupied} = \text{true}) = 0.33 + P(\text{CellOccupied} = \text{false}) = 0.67 = 1$.
- If the values are true or false, we often use a shorthand: $P(\text{CellOccupied} = \text{true})$ becomes $P(\text{CellOccupied})$ and $P(\text{CellOccupied} = \text{false})$ becomes $P(\neg\text{CellOccupied})$
- the set of p 's for all values that H can be is called the probability distribution.
- $P(A \wedge B) = P(A)P(B)$
- $P(A \vee B) = P(A) + P(B)$
- $P(A) + P(\neg A) = 1$.

Some of what I said above applies only to cases where two variables are independent.

- Variables are independent when the value of one does not affect the value of another.
- When flipping 2 different coins, the value of one does not affect the value of the other.
- But lets take a different example:
 - Imagine we live in a town where it rains 50% of the time.
 - $P(\text{Raining}) = 0.5$
 - Most of the time it rains, the sidewalk gets wet. Sometimes it doesn't because the tent for the local art fair keeps it dry.
 - $P(\text{Wet}) = 0.49$
 - If we use the rule $P(A \wedge B) = P(A)P(B)$, then the $P(\text{Raining} \wedge \text{Wet}) = 0.245$.
 - That says that $\frac{1}{4}$ of the time, it will be raining and the sidewalks will be wet.
 - But does that make sense? You expect that $\frac{1}{2}$ of the time it will be raining, and nearly 100% of the time it is raining, the sidewalks will be wet.
 - It doesn't make sense, so instead we use a Rule Based on Conditional Probability.
- Conditional Probability is a way of expressing the relationship between two variables, expressed as: $P(A = x|B = y)$ The probability of A being x, given that B is y, or more succinctly: $P(A|B)$, the probability of A given B.
- Properties of conditional probability:
 - $P(A|B) = P(A)$ if A and B are independent.
 - $P(A \wedge B) = P(A)P(B|A)$ Note that the independent case derives from these two.
 - $P(\text{Raining} \wedge \text{Wet}) = P(\text{Raining})P(\text{Wet}|\text{Raining})$
 - $P(\text{Raining} \wedge \text{Wet}) = 0.5 \times 0.99 = 0.495$
- If $P(A \wedge B) = P(A)P(B|A)$, then $P(B \wedge A) = P(B)P(A|B)$
- $P(A \wedge B) = P(B \wedge A)$
- $\therefore P(A)P(B|A) = P(B)P(A|B)$
- If we solve for $P(A|B)$ we get: $P(A|B) = \frac{P(A)P(B|A)}{P(B)}$
- If that is true, then $P(\neg A|B) = \frac{P(\neg A)P(B|\neg A)}{P(B)}$ too. These are known as Bayes' Rule.
- Adding those two equations together, we get:

$$P(A|B) + P(\neg A|B) = \frac{P(A)P(B|A)}{P(B)} + \frac{P(\neg A)P(B|\neg A)}{P(B)}.$$
- We also know that $P(A|B) + P(\neg A|B) = 1$
- and substituting in $P(A|B) + P(\neg A|B) = 1$, we get:

$$1 = \frac{P(A)P(B|A)}{P(B)} + \frac{P(\neg A)P(B|\neg A)}{P(B)}.$$
- Solve for P(B) to get:

$$P(B) = P(A)P(B|A) + P(\neg A)P(B|\neg A)$$
- Substitute that back into the original equation to get:

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\neg A)P(B|\neg A)}$$

So what does this have to do with mapping? Well, if A is the hypothesis that some cell is occupied, and B is a sensor reading, then we can calculate the probability that some cell is occupied given some sensor reading: $P(O|s)$.

$$P(O|s) = \frac{P(O)P(s|O)}{P(O)P(s|O) + P(-O)P(s|-O)}$$

If we know all the things on the right hand side, then we can easily calculate the left.

- $P(O)$ is the prior probability. That is the probability that some cell is occupied, without any other knowledge. At the outset, this is set to some reasonable value. Could be 0.5, but could be a better guess based on what you know about the environment.
- $P(s|O)$ is the probability that a sensor returns the value, given that the cell is occupied. This is a property of the sensor (called the sensor model): how likely is it that, when an object is in a location, the sensor will sense it? We get this number by testing the sensor thoroughly.
- $P(s|-O)$ is also part of the sensor model and is generated by testing.
- $P(-O)$ is just $1 - P(O)$.

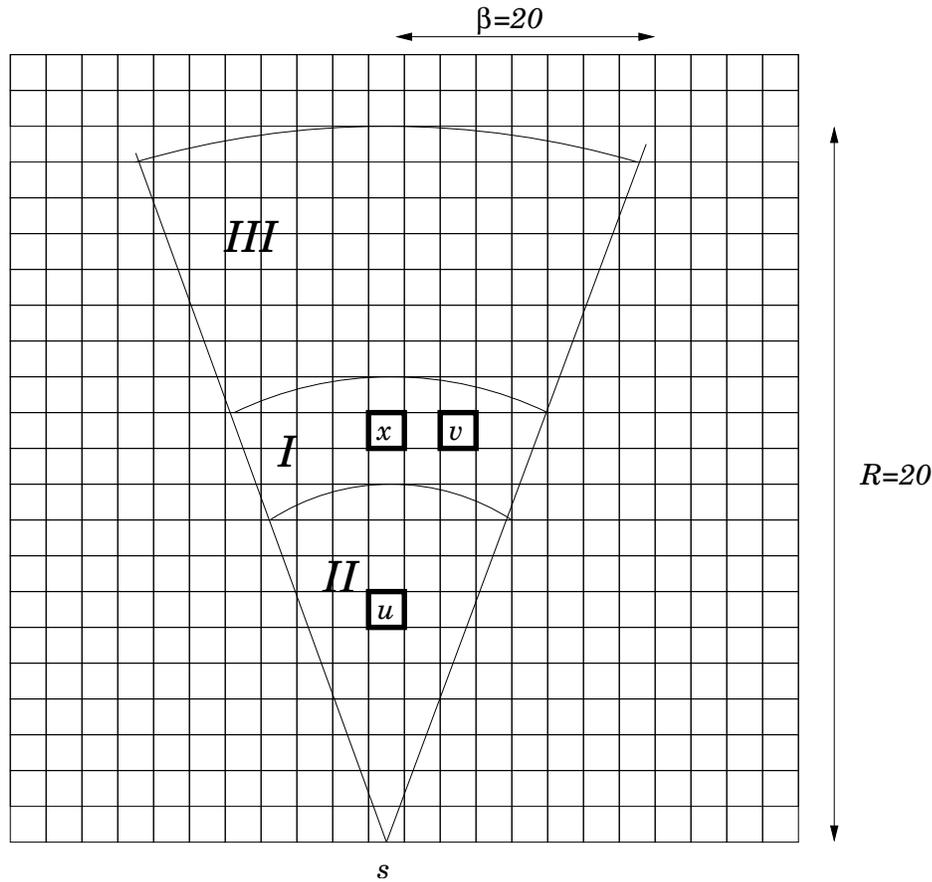
OK, so what's the algorithm?

1. Initialize all the cells of the map to a reasonable prior
2. Repeat:
 - (a) Take a sensor scan.
 - (b) For each cell on the map in sensor range, update the probability in that cell using the sensor model and the current value in the cell as the prior.
 - (c) Move to somewhere else.

Note that once a pass through the loop is done, the values of the cells are replaced. These numbers become the new priors in the next pass through the loop.

Question: What might a sensor model look like? The following is a typical model for a Polaroid sonar.
Facts:

- The sonar can only detect object within a the cone seen here:



- If the sonar returns a value x , there is an area (labeled Region I above) where an object is pretty likely to be. Region II is an area where it's unlikely an object is, and Region III is an area inside the sensor range, but we don't know anything about because there is probably an object in front of it.
- When considering a particular cell with the sensor model, we'll say that r is the distance from the sensor to the cell, R is the distance from the sensor to the most distant cell it can sense. α is the angle to the cell, and β is the angle to the most left (or right) cell that the sensor can detect.
- For every cell in Region I, $P(s = x|O = true) = \frac{R-r + \frac{\beta-\alpha}{\beta}}{2} Max_o$
- For cells in I, we say $P(s|-O) = 1 - P(s|O)$.
- For every cell in Region II, $P(s|-O) = \frac{R-r + \frac{\beta-\alpha}{\beta}}{2} Max_o$
- For cells in II we say $P(s|O) = 1 - P(s|-O)$.
- We do not update cells in region III.
- Max_o is just a constant, usually less than but close to 1, which we use to express the idea that we never completely believe our sensors, so we make sure $P(s|O)$ is never actually 1.

And that is enough to use Bayesian updating for mapping. So what about an example?

- Look at two of the cells we need to update in the figure, the ones labeled u, v .
- Assume we get the first sonar scan that returns a distance 12.
- $P(s = 12|-O_u) = \frac{\frac{20-7}{20} + \frac{20-0}{20}}{2} 0.98 = 0.8085$

- $\therefore P(s = 6|O_u) = 0.1915$
- $P(s = 12|O_v) = \frac{\frac{20-12}{20} + \frac{20-10}{20}}{2} \cdot 0.98 = 0.441$
- $\therefore P(s = 12|\neg O_v) = 0.559$
- $P(O_u|s = 12) = \frac{0.5 \times 0.1915}{0.5 \times 0.1915 + 0.5 \times 0.8085} = 0.1915$
- $P(O_v|s = 12) = \frac{0.5 \times 0.441}{0.5 \times 0.441 + 0.5 \times 0.559} = 0.441$
- 2 questions:
 1. Why was $P(O|s) = P(s|O)$? Because the priors were 0.5.
 2. Why the the value for O_v go down? Because the sensor is unreliable at that distance.
- What If the robot was teleported 6 cells forward and facing v , took another reading and got back 6. What is the value of $P(O_v|s = 6)$?
- $P(s = 6|O_v) = \frac{\frac{20-6}{20} + \frac{20-020}{20}}{2} \cdot 0.98 = 0.83$
- $P(O_v|s = 6) = \frac{0.441 \times 0.83}{0.441 \times 0.83 + 0.559 \times 0.17} = 0.793887997$