# IC220
# Slide Set #7: Digital Logic
# (more Appendix B)

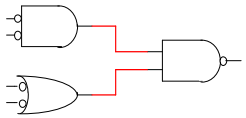


## DeMorgan's Law and Bubble Pushing

$$\overline{A+B} = \overline{A} \bullet \overline{B} \qquad \overline{A \bullet B} = \overline{A} + \overline{B}$$



## Bubble Pushing Example





## Representing Combinational Logic

Truth Table

Boolean Formula

Circuit

For combinational logic, these three:
- are equivalently _____
- straight-forward to _____
- have no _____

## 2-Level Logic

- **Represent _____ logic function(s)**
  - **Utilizing just two types of gates**

    (assuming we get NOT for free)
  - **Two forms**
    - Sum of products
    - Product of sums
  - **Relationship with truth table**
    - Generate a gate level implementation of any set of logic functions
    - Allows for simple reduction/minimization



## Example

**EX: B-11 to B-14**

- **Show the sum of products for the following truth table.**
- **Strategy: _____ all the products where the output is _____**

| A | B | C | z |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- **z =**

- **Is this optimal?**



## Reduction/Minimization

- **Reduction is important to reduce the size of the circuit that performs the function. This, in turn, reduces the cost of, and delay through, the circuit.**

- **What?**
  - **Less power consumption**
  - **Less heat**
  - **Less space**
  - **Less time to propagate a signal through the circuit**
  - **Less points of possible failure**

- **It makes good engineering and economic sense!**



## Minimization by Hand

- **Sum of Products:**

$$z = (\overline{A} \bullet \overline{B} \bullet C) + (A \bullet \overline{B} \bullet \overline{C}) + (A \bullet \overline{B} \bullet C) + (A \bullet B \bullet C)$$

**Truth Table:**

| A | B | C | z |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- **Okay to duplicate terms while minimizing**

## Karnaugh Maps (k-Maps)

- A graphical (pictorial) method used to minimize Boolean expressions.
- Don't require the use of Boolean algebra theorems and equation manipulations.
- A special version of a truth table.
- Works with two to four input variables
  (gets more and more difficult with more variables)
- Groupings must be _____
- Final result is in _____ form



## Karnaugh Maps (k-Maps) Example #1

- Lets create a k-map table
  - Borders represent all possible conditions
  - NOT in counting order
  - Be consistent
- -What are the values for the map?
  - The values of ___
- To reduce, circle our powers of 2!

| A | B | C | z |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

| | $\overline{B}\,\overline{C}$ | $\overline{B}C$ | $BC$ | $B\overline{C}$ |
|---|---|---|---|---|
| $A$ | | | | |
| $\overline{A}$ | | | | |

- Result:



## K-Maps Example #2

- Suppose we already have this k-Map. Minimize the function.

| | $\overline{C}\,\overline{D}$ | $\overline{C}D$ | $CD$ | $C\overline{D}$ |
|---|---|---|---|---|
| $\overline{A}\,\overline{B}$ | 1 | 0 | 0 | 0 |
| $\overline{A}B$ | 0 | 0 | 0 | 1 |
| $AB$ | 0 | 1 | 1 | 0 |
| $A\overline{B}$ | 1 | 1 | 1 | 1 |

- Every "1" must be _____ by at least one term
- Larger blocks in k-Map produce smaller product terms



## Truth Table and Logical Circuit Example

- How does a truth table and subsequent sum of products equation create a logic circuit?
- From the earlier example:

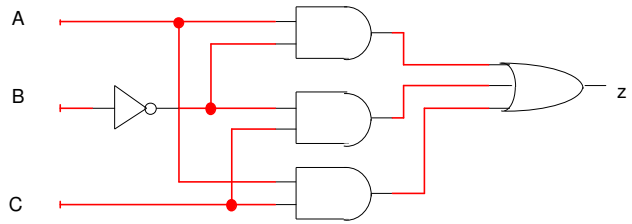
  z = $\overline{B} \bullet C + A \bullet \overline{B} + A \bullet C$

- Lets build the logical circuit:
  - Which gates do we need?
  - How many inputs do we have?
  - How do we connect the circuit?

## Example Circuit

EX: B-21 to B-24

$z = \overline{B} \bullet C + A \bullet \overline{B} + A \bullet C$






## Don't Cares

- Sometimes don't care about the output.

| | $\overline{C}\,\overline{D}$ | $\overline{C}D$ | $CD$ | $C\overline{D}$ |
|---|---|---|---|---|
| $\overline{A}\,\overline{B}$ | $X$ | 0 | 0 | 0 |
| $\overline{A}B$ | 1 | 0 | 0 | $X$ |
| $AB$ | 1 | 1 | 1 | 1 |
| $A\overline{B}$ | 1 | 0 | 0 | 0 |

- Each X can be either a 0 or 1 (helps with minimization)
- But in actual circuit, each X will have some specific value



## General Skills

- Make sure you can populate a K-Map from a truth table
- Make sure you can populate a truth table from a K-Map
- Given a circuit, know how to construct a truth table
- Given a truth table, know how to produce a sum-of-products, and how to draw a circuit
- Be able to understand minimization and use it
- Know DeMorgan's Law and other Boolean laws