
IC220
Slide Set #7: Digital Logic
(more Appendix C)

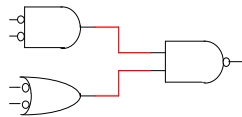
1

DeMorgan's Law and Bubble Pushing

$$\overline{A + B} = \overline{A} \cdot \overline{B} \qquad \overline{A \cdot B} = \overline{A} + \overline{B}$$

2

Bubble Pushing Example



3

Representing Combinational Logic

Truth Table

Boolean Formula

Circuit

For combinational logic, these three:

- are equivalently _____
- straight-forward to _____
- have no _____

4

2-Level Logic

- Represent _____ logic function(s)
 - Utilizing just two types of gates

- (assuming we get NOT for free)
- **Two forms**
 - Sum of products
 - Product of sums
- **Relationship with truth table**
 - Generate a gate level implementation of any set of logic functions
 - Allows for simple reduction/minimization

5

Reduction/Minimization

- Reduction is important to reduce the size of the circuit that performs the function. This, in turn, reduces the cost of, and delay through, the circuit.

- What?
 - Less power consumption
 - Less heat
 - Less space
 - Less time to propagate a signal through the circuit
 - Less points of possible failure

- It makes good engineering and economic sense!

7

Example

EX: B-11 to B-14

- Show the sum of products for the following truth table.
- Strategy: _____ all the products where the output is _____

A	B	C	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

- z =

- Is this optimal?

6

Minimization by Hand

- Sum of Products: Truth Table:

$$z = (\bar{A} \cdot \bar{B} \cdot C) + (A \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (A \cdot B \cdot C)$$

A	B	C	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

- Okay to duplicate terms while minimizing

8

Karnaugh Maps (k-Maps)

- A graphical (pictorial) method used to minimize Boolean expressions.
- Don't require the use of Boolean algebra theorems and equation manipulations.
- A special version of a truth table.
- Works with two to four input variables (gets more and more difficult with more variables)
- Groupings must be _____
- Final result is in _____ form

9

Karnaugh Maps (k-Maps) Example #1

- Lets create a k-map table
 - Borders represent all possible conditions
 - **NOT** in counting order
 - Be consistent
- -What are the values for the map?
 - The values of _____
- To reduce, circle our powers of 2!

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
A				
\overline{A}				

- Result:

10

K-Maps Example #2

- Suppose we already have this k-Map. Minimize the function.

	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	1	0	0	0
$\overline{A}B$	0	0	0	1
AB	0	1	1	0
$A\overline{B}$	1	1	1	1

- Every "1" must be _____ by at least one term
- Larger blocks in k-Map produce smaller product terms

11

Truth Table and Logical Circuit Example

- How does a truth table and subsequent sum of products equation create a logic circuit?
- From the earlier example:

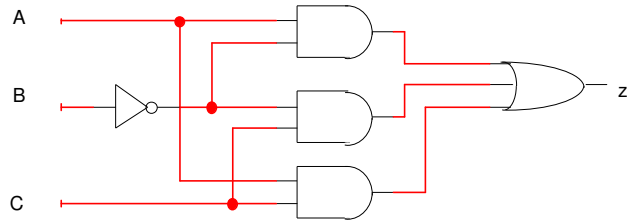
$$Z = \overline{B} \cdot C + A \cdot \overline{B} + A \cdot C$$
- Lets build the logical circuit:
 - Which gates do we need?
 - How many inputs do we have?
 - How do we connect the circuit?

12

Example Circuit

EX: B-21 to B-24

$$z = \bar{B} \cdot C + A \cdot \bar{B} + A \cdot C$$



13

Don't Cares

- Sometimes don't care about the output.

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X	0	0	0
$\bar{A}B$	1	0	0	X
AB	1	1	1	1
$A\bar{B}$	1	0	0	0

- Each X can be either a 0 or 1 (helps with minimization)
- But in actual circuit, each X will have some specific value

14

General Skills

- Make sure you can populate a K-Map from a truth table
- Make sure you can populate a truth table from a K-Map
- Given a circuit, know how to construct a truth table
- Given a truth table, know how to produce a sum-of-products, and how to draw a circuit
- Be able to understand minimization and use it
- Know DeMorgan's Law and other Boolean laws

15