

---

**IC220**  
**Slide Set #7: Digital Logic**  
**(more Appendix C)**

1

---

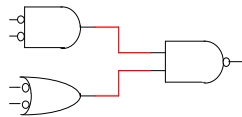
**DeMorgan's Law and Bubble Pushing**

$$\overline{A + B} = \overline{A} \cdot \overline{B} \quad \overline{A \cdot B} = \overline{A} + \overline{B}$$

2

---

**Bubble Pushing Example**



3

---

**Representing Combinational Logic**

Truth Table

Boolean Formula

Circuit

For combinational logic, these three:

- are equivalently \_\_\_\_\_
- straight-forward to \_\_\_\_\_
- have no \_\_\_\_\_

4

## 2-Level Logic

- Represent \_\_\_\_\_ logic function(s)
  - Utilizing just two types of gates
  
- (assuming we get NOT for free)
- **Two forms**
  - Sum of products
  - Product of sums
- **Relationship with truth table**
  - Generate a gate level implementation of any set of logic functions
  - Allows for simple reduction/minimization

5

## Reduction/Minimization

- Reduction is important to reduce the size of the circuit that performs the function. This, in turn, reduces the cost of, and delay through, the circuit.
  
- What?
  - Less power consumption
  - Less heat
  - Less space
  - Less time to propagate a signal through the circuit
  - Less points of possible failure
  
- It makes good engineering and economic sense!

7

## Example

EX: B-11 to B-14

- Show the sum of products for the following truth table.
- Strategy: \_\_\_\_\_ all the products where the output is \_\_\_\_\_

A	B	C	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

- z =
  
- Is this optimal?

6

## Minimization by Hand

- Sum of Products: Truth Table:

$$z = (\bar{A} \cdot \bar{B} \cdot C) + (A \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot C) + (A \cdot B \cdot C)$$

A	B	C	z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

- Okay to duplicate terms while minimizing

8

## Karnaugh Maps (k-Maps)

- A graphical (pictorial) method used to minimize Boolean expressions.
- Don't require the use of Boolean algebra theorems and equation manipulations.
- A special version of a truth table.
- Works with two to four input variables (gets more and more difficult with more variables)
- Groupings must be \_\_\_\_\_
- Final result is in \_\_\_\_\_ form

9

## Karnaugh Maps (k-Maps) Example #1

- Lets create a k-map table
  - Borders represent all possible conditions
  - **NOT** in counting order
  - Be consistent
- -What are the values for the map?
  - The values of \_\_\_\_\_
- To reduce, circle our powers of 2!

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

	$\overline{B}\overline{C}$	$\overline{B}C$	$BC$	$B\overline{C}$
A				
$\overline{A}$				

- Result:

10

## K-Maps Example #2

- Suppose we already have this k-Map. Minimize the function.

	$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
$\overline{A}\overline{B}$	1	0	0	0
$\overline{A}B$	0	0	0	1
$AB$	0	1	1	0
$A\overline{B}$	1	1	1	1

- Every "1" must be \_\_\_\_\_ by at least one term
- Larger blocks in k-Map produce smaller product terms

11

## Truth Table and Logical Circuit Example

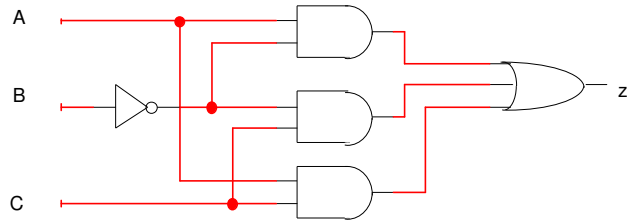
- How does a truth table and subsequent sum of products equation create a logic circuit?
- From the earlier example:
 
$$Z = \overline{B} \cdot C + A \cdot \overline{B} + A \cdot C$$
- Lets build the logical circuit:
  - Which gates do we need?
  - How many inputs do we have?
  - How do we connect the circuit?

12

## Example Circuit

EX: B-21 to B-24

$$z = \bar{B} \cdot C + A \cdot \bar{B} + A \cdot C$$



13

## Don't Cares

- Sometimes don't care about the output.

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	X	0	0	0
$\bar{A}B$	1	0	0	X
$AB$	1	1	1	1
$A\bar{B}$	1	0	0	0

- Each X can be either a 0 or 1 (helps with minimization)
- But in actual circuit, each X will have some specific value

14

## General Skills

- Make sure you can populate a K-Map from a truth table
- Make sure you can populate a truth table from a K-Map
- Given a circuit, know how to construct a truth table
- Given a truth table, know how to produce a sum-of-products, and how to draw a circuit
- Be able to understand minimization and use it
- Know DeMorgan's Law and other Boolean laws

15