SM 121 22/33/39/45/46/58

LESSON 15 LIMIT LAWS CONTINUED

(22) $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{(1+h)-1}{h(\sqrt{1+h}+1)}$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$

$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$

(33) $\lim_{x\to 0} x^2 \cos 2\pi x = 0$

$-1 \leq \cos 20\pi x \leq 1$

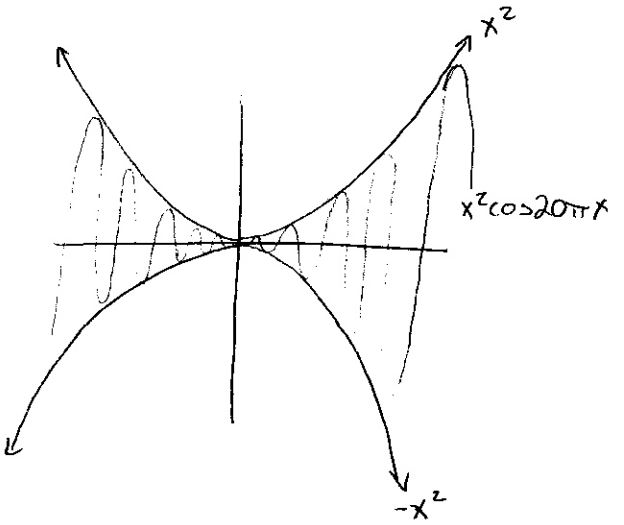
$-x^2 \leq x^2 \cos 20\pi x \leq x^2$

$\lim_{x\to 0} -x^2 \leq \lim_{x\to 0} x^2 \cos 20\pi x \leq \lim_{x\to 0} x^2$

$0 \leq \lim_{x\to 0} x^2 \cos 20\pi x \leq 0$

$\therefore$ BY THE SQUEEZE THEOREM

$\lim_{x\to 0} x^2 \cos 20\pi x = 0$



LESSON 15

(39) $\lim_{x\to 3}\left(2x+|x-3|\right) = \lim_{x\to 3} f(x) \begin{cases} 2x+(x-3) & \text{if } x \geq 0 \\ 2x-(x-3) & \text{if } x < 0 \end{cases}$

$\lim_{x\to 3^+} 2x+x-3$

$\lim_{x\to 3^+} 3x-3$

$3(3)-3$

$9-3$

$\boxed{6}$

$\lim_{x\to 3^-} 2x-x+3$

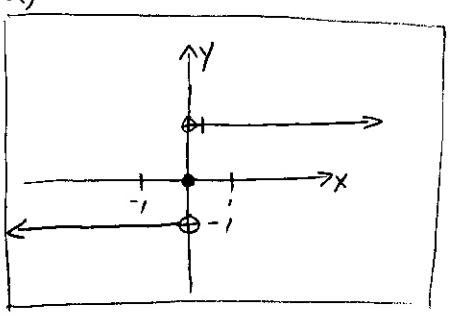
$\lim_{x\to 3^-} x+3$

$3+3$

$\boxed{6}$

$\therefore \boxed{\lim_{x\to 3}\left(2x+|x-3|\right) = 6}$

(45) a)



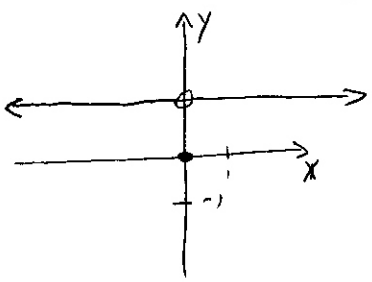
b) i) $\lim_{x\to 0^+} \operatorname{sgn} x = \boxed{1}$

ii) $\lim_{x\to 0^-} \operatorname{sgn} x = \boxed{-1}$

iii) $\lim_{x\to 0} \operatorname{sgn} x = \boxed{\text{D.N.E.}}$

iv) $\lim_{x\to 0} |\operatorname{sgn} x| = \boxed{1}$

$|\operatorname{sgn} x| = \begin{cases} \operatorname{sgn} x & \text{if } x \geq 0 \\ -(\operatorname{sgn} x) & \text{if } x < 0 \end{cases}$



LESSON 15

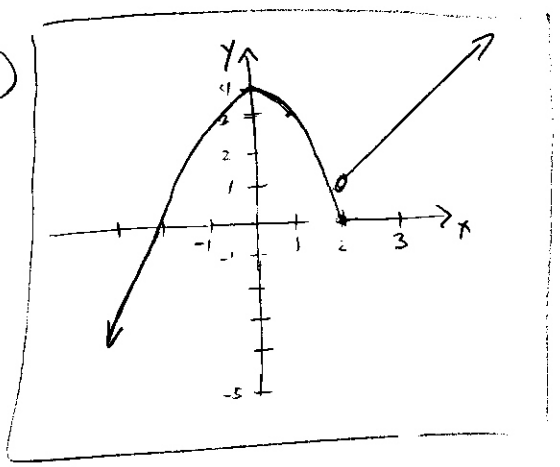
(46) $f(x) = \begin{cases} 4 - x^2 & \text{if } x \le 2 \\ x - 1 & \text{if } x > 2 \end{cases}$

a) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 4 - x^2 = 4 - 2^2 = \boxed{0}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x - 1 = 2 - 1 = \boxed{1}$

b) $\boxed{\lim_{x \to 2} f(x) \text{ DOES NOT EXIST BECAUSE } \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)}$

c)



(58) LET $f(x) = [\![x]\!]$ (sgn function)

$g(x) = -[\![x]\!]$

THEN $\lim_{x \to 3} f(x)$ AND $\lim_{x \to 3} g(x)$ DO NOT EXIST

HOWEVER $\lim_{x \to 3} (f(x) + g(x)) = \lim_{x \to 3} ([\![x]\!] - [\![x]\!])$

$= \lim_{x \to 3} (0) = 0$

$\therefore$ THE LIMIT OF $f(x) + g(x)$ CAN EXIST AT a EVEN IF $\lim_{x \to a} f(x)$ AND $\lim_{x \to a} g(x)$ DO NOT EXIST