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SECTION 2.5 CONTINUITY

① $\boxed{\lim_{x \to 4} f(x) = f(4)}$

② IF f IS CONTINUOUS ON THE INTERVAL $(-\infty, \infty)$ THEN THE FUNCTION f IS CONTINUOUS FOR AT EVERY REAL NUMBER. f IS CONTINUOUS AT EVERY X

③ a) f IS DISCONTINUOUS AT $X = -4, -2, 2,$ AND 4

- DISCONTINUOUS AT $X = -4$ BECAUSE $f(-4)$ IS UNDEFINED
- DISCONTINUOUS AT $a = -2, 2,$ AND 4 BECAUSE $\lim_{x \to a} f(x) \neq f(a)$

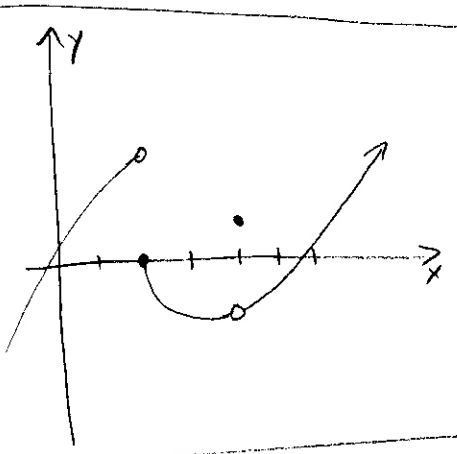
b) $X = -4$ NEITHER CONTINUOUS FROM THE LEFT AND RIGHT

$X = -2$ CONTINUOUS FROM THE LEFT

$X = 2$ CONTINUOUS FROM THE RIGHT

$X = 4$ CONTINUOUS FROM THE RIGHT

⑥



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(5) a) A TEMPERATURE FUNCTION IS CONTINUOUS BECAUSE THERE ARE NO UNDEFINED TEMPERATURES ON THE INTERVAL $(-273.15, \infty)$ °C AND THE FUNCTION DOES NOT HAVE ANY JUMP DISCONTINUITIES

b) TEMPERATURE AS A FUNCTION OF DISTANCE IS DEFINED FOR ALL DISTANCES.

c) ALTITUDE AS A FUNCTION OF DISTANCE IS DEFINED FOR ALL DISTANCES.

d) COST AS A FUNCTION OF DISTANCE CONTAINS JUMP DISCONTINUITIES.

e) CURRENT AS A FUNCTION OF TIME CONTAINS A JUMP DISCONTINUITIES WHEN SWITCHED FROM OFF TO ON AND VICE VERSA.

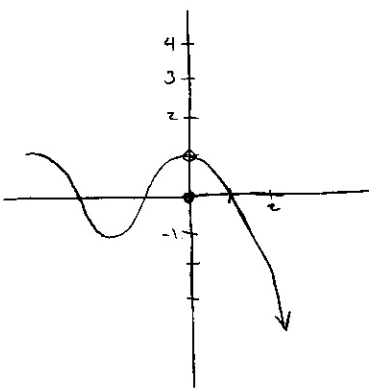
(19) $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$ $a = 0$

$\lim_{x \to 0^-} \cos x = 1$

$\lim_{x \to 0^+} 1 - x^2 = 1$

$\lim_{x \to 0} f(x) = 1$ $f(a) = 0$

$$\boxed{\lim_{x \to a} f(x) \neq f(a)}$$



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(39) $f(x) = \begin{cases} x+2 & \text{if } x<0 \\ e^x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x>1 \end{cases}$

$\lim_{x\to 0^-} x+2 = 2 \qquad \lim_{x\to 0^+} e^x = 1$

$\lim_{x\to 0} f(x)$ DOES NOT EXIST

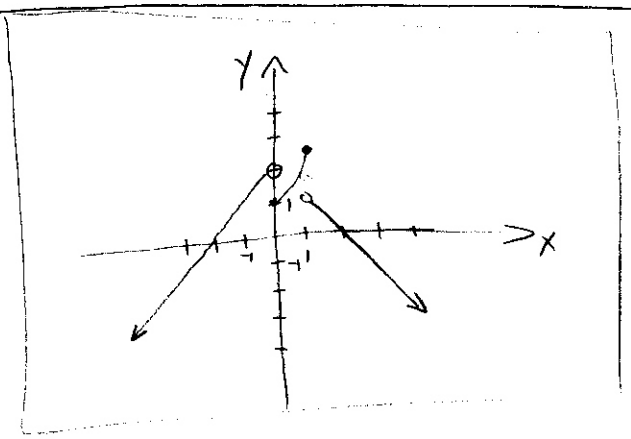
$\therefore$ DISCONTINUITY AT $x=0$

$\lim_{x\to 1^-} e^x = e \qquad \lim_{x\to 1^+} 2-x = 1$

$\lim_{x\to 1} f(x)$ DOES NOT EXIST

$\therefore$ DISCONTINUITY AT $x=1$

$a=0$	$\lim_{x\to a^-} f(x) = 2$	$f(0) = 2$	f CONTINUOUS FROM THE LEFT AT $x=0$
$a=1$	$\lim_{x\to a^+} f(x) = 1$	$f(1) = 1$	f CONTINUOUS FROM THE RIGH AT $x=1$



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(47) $x^4 + x - 3 = 0 \quad (1, 2)$

$f(x) = x^4 + x - 3$

$f(1) = 1^4 + 1 - 3 = -1 < 0$

$f(2) = 2^4 + 2 - 3 = 29 > 0$

$f(1) < 0 < f(2)$

$\therefore$ SINCE $f(x)$ IS CONTINUOUS BECAUSE IT IS A POLY NOMIAL, THERE MUST BE A NUMBER, N, THAT LIES BETWEEN 1 AND 2, SUCH THAT $f(N) = 0$.

N IS A ROOT OF $f(x)$ THAT LIES BETWEEN $X = 1$ AND $X = 2$

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(49) $\cos x = x \qquad (0, 1)$

$f(x) = \cos x$

$f(0) = \cos 0 = 1$

$f(1) = \cos 1 = 0$

$f(0) > x > f(1)$

$\therefore$ SINCE $f(x)$ IS CONTINUOUS OVER THE INTERVAL $(0, 1)$ THERE MUST BE A NUMBER, N, THAT LIES BETWEEN 0 AND 1 SUCH THAT $f(x) = x$.