SM 121 LESSON 19 25/29/31/34/39/41/46 9/23/2008

SECTION 2.7 DERIVATIVES & RATES OF CHANGE CONTINUED

(25) FIND $f'(a)$, $f(x) = 3 - 2x + 4x^2$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{3 - 2(a+h) + 4(a+h)^2 - (3 - 2a + 4a^2)}{h}$$

$$= \lim_{h\to 0} \frac{3 - 2a - 2h + 4(a^2 + 2ah + h^2) - 3 + 2a - 4a^2}{h}$$

$$= \lim_{h\to 0} \frac{3 - 2a - 2h + 4a^2 + 8ah + 4h^2 - 3 + 2a - 4a^2}{h}$$

$$= \lim_{h\to 0} \frac{8ah - 2h - 4h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(8a - 2 - 4h)}{h}$$

$$= \lim_{h\to 0} 8a - 2 - 4h \to 0$$

$$\boxed{f'(a) = 8a - 2}$$

(29) FIND $f'(a)$, $f(x) = \frac{1}{\sqrt{x+2}}$

$$f'(a) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{a+h+2}} - \frac{1}{\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{a+2} - \sqrt{a+h+2}}{(\sqrt{a+h+2})(\sqrt{a+2})}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{a+2} - \sqrt{a+h+2}}{h(\sqrt{a+h+2})(\sqrt{a+2})} \cdot \frac{\sqrt{a+2} + \sqrt{a+h+2}}{\sqrt{a+2} + \sqrt{a+h+2}}$$

$$= \lim_{h\to 0} \frac{(a+2) - (a+h+2)}{h(\sqrt{a+h+2})(\sqrt{a+2})(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{-h}{h(\sqrt{a+h+2})(\sqrt{a+2})(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{-1}{(\sqrt{a+h+2})(\sqrt{a+2})(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \frac{-1}{(\sqrt{a+2})(\sqrt{a+2})(\sqrt{a+2} + \sqrt{a+2})} = \frac{-1}{(\sqrt{a+2})^2(2\sqrt{a+2})} = \boxed{\frac{-1}{2(a+2)^{3/2}}}$$

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(31) $\lim_{h \to 0} \dfrac{(1+h)^{10} - 1}{h}$

$f(x) = x^{10}$

$a = 1$

BY DEFINITION 2

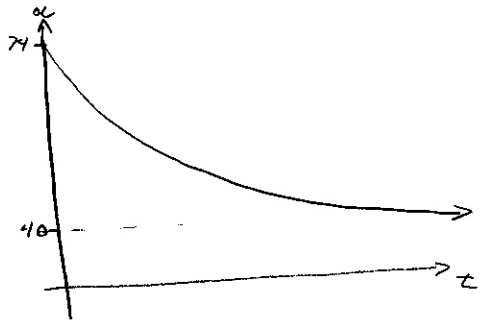
(34) $\lim_{x \to \frac{\pi}{4}} \dfrac{\tan x - 1}{x - \frac{\pi}{4}}$

$f(x) = \tan x$

$a = \frac{\pi}{4}$

BY DEFINITION 1

(39)



INITIAL RATE OF CHANGE OF TEMPERATURE IS GREATER THAN THE RATE OF CHANGE IN TEMPERATURE AFTER ONE HOUR.

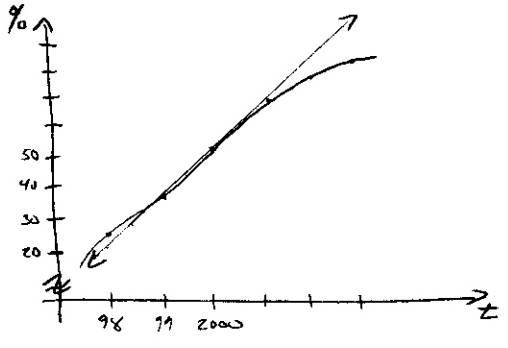
(41) a) i) $\dfrac{77 - 55}{2002 - 2000} = \dfrac{22}{2} =$ 11%/YEAR

ii) $\dfrac{68 - 55}{2001 - 2000} = \dfrac{13}{1} =$ 13%/YEAR

iii) $\dfrac{55 - 39}{2000 - 1999} = \dfrac{16}{1} =$ 16%/YEAR

b) $\dfrac{13 + 16}{2} = \dfrac{29}{2} =$ 14.5 %/YEAR

c)



ESTIMATING POINTS ON THE TANGENT LINE

(1998, 40)

(2001, 70)

$\dfrac{70 - 40}{2001 - 1998} = \dfrac{30}{2} =$ 15%/YR

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(46) a) $f'(5)$ INDICATES THE RATE OF GROWTH OF THE BACTERIA COLONY AT 5 HOURS.

b) WITH UNLIMITED SPACE AND FOOD $f'(10)$ WOULD BE GREATER THAN $f'(5)$, $f'(10) > f'(5)$.

WITH LIMITED SUPPLIES GROWTH WILL EVENTUALLY SLOW OR BECOME NEGATIVE. $f'(5)$ MAY BE GREATER THAN $f'(10)$.