SM 121 LESSON 27 1/5/8/9/13/15/30/40/49 10/7/2008

SM 3.4 CHAIN RULE

① $y = \sin 4x$

$u = 4x$

$y = \sin u$

$$\frac{d}{dx}(\sin 4x) = \frac{d}{du}\sin u \cdot \frac{d}{dx}u$$

$$= (\cos u)\left(\frac{d}{dx}4x\right)$$

$$= (\cos u)(4)$$

$$\boxed{\frac{dy}{dx} = 4\cos 4x}$$

⑤ $y = e^{\sqrt{x}}$

$u = \sqrt{x}$

$y = e^u$

$$\frac{d}{dx}(e^{\sqrt{x}}) = \frac{d}{du}e^u \cdot \frac{d}{dx}u$$

$$= (e^u)\left(\frac{d}{dx}\sqrt{x}\right)$$

$$= \left(e^{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$$

⑧ $F(x) = (4x - x^2)^{100}$

$F(x) = u^{100}$ $\quad u = 4x - x^2$

$F'(x) = 100u^{99} \cdot \frac{d}{dx}(4x - x^2)$

$$\boxed{F'(x) = 100(4x - x^2)^{99}(4 - 2x)}$$

Lesson 27 10/7/2008

(9) $F(x) = \sqrt[4]{1+2x+x^3}$

$F(x) = u^{\frac{1}{4}}$ $\quad u = 1+2x+x^3$

$F'(x) = \frac{1}{4}u^{-\frac{3}{4}} \cdot \frac{d}{dx}(1+2x+x^3)$

$= \frac{1}{4}(1+2x+x^3)^{-\frac{3}{4}}(2+3x^2)$

$$\boxed{F'(x) = \frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}}}$$

(13) $y = \cos(a^3+x^3)$

$y = \cos u$ $\quad u = a^3+x^3$

$y' = -\sin u \cdot \frac{d}{dx}(a^3+x^3)$

$y' = -\sin(a^3+x^3)(3x^2)$

$$\boxed{y' = -3x^2\sin(a^3+x^3)}$$

(15) $y = xe^{-kx}$

$y = uv$ $\quad u = x$ $\quad v = e^{-kx} = e^m$ $\quad m = -kx$

$u' = 1$ $\quad v' = e^m \cdot \frac{d}{dx}(-kx) = -ke^m = -ke^{-kx}$

$y' = u'v + uv'$

$y' = (1)(e^{-kx}) + x(-ke^{-kx})$

$$\boxed{y' = e^{-kx}(1-kx)}$$

LESSON 27 10/7/2008

(30) $G(y) = \left(\frac{y^2}{y+1}\right)^5$

$u = \frac{y^2}{y+1} = \frac{m}{n}$ $m = y^2$ $m' = 2y$

$u' = \frac{m'n - mn'}{n^2}$ $n = y+1$ $n' = 1$

$n^2 = (y+1)^2$

$G(y) = u^5$

$u' = \frac{(2y)(y+1) - (y^2)(1)}{(y+1)^2} = \frac{2y^2+2y-y^2}{(y+1)^2} = \frac{y^2+2y}{(y+1)^2}$

$G'(y) = 5u^4 \cdot \frac{d}{dx} u$

$G'(y) = 5\left(\frac{y^2}{y+1}\right)^4\left(\frac{y^2+2y}{(y+1)^2}\right) = 5\left(\frac{y^8(y^2+2y)}{(y+1)^4(y+1)^2}\right)$

$= 5\left(\frac{y^{10}+2y^9}{(y+1)^6}\right)$

$$\boxed{G'(y) = \frac{5y^9(y+2)}{(y+1)^6}}$$

(40) $y = \sin(\sin(\sin x))$

$y = \sin u$ $u = \sin(\sin x)$

$u = \sin v$ $v = \sin x$

$y' = \frac{d}{du}\sin u \cdot \frac{d}{dv}\sin v \cdot \frac{d}{dx}\sin x$

$y' = (\cos u)(\cos v)(\cos x)$

$$\boxed{y' = \cos(\sin(\sin x))(\cos(\sin x))(\cos x)}$$

Lesson 27 10/7/2008

(49) $y = e^{ax}\sin\beta x$

$m = e^{ax} = e^{u}$ $\quad u = ax$

$m' = e^{u}\cdot\frac{d}{dx}(ax) = ae^{u} = ae^{ax}$

$y = mn$

$y' = m'n + mn'$

$n = \sin\beta x = \sin v$ $\quad v = \beta x$

$n' = \cos v \cdot \frac{d}{dx}(\beta x) = \beta\cos v = \beta\cos\beta x$

$y' = (ae^{ax})(\sin\beta x) + (e^{ax})(\beta\cos\beta x)$

$$\boxed{y' = e^{ax}(a\sin\beta x + \beta\cos\beta x)}$$

$y'' = fg$

$y'' = f'g + fg'$

$f = e^{ax} = e^{u}$ $\quad u = ax$

$f' = e^{u}\cdot\frac{d}{dx}(ax) = ae^{u} = ae^{ax}$

$g = a\sin\beta x + \beta\cos\beta x$

$g = a\sin u + \beta\cos v$ $\quad u = \beta x \quad u' = \beta$

$v = \beta x \quad v' = \beta$

$g' = a\cos u\cdot\frac{d}{dx}u + \beta(-\sin v)\frac{d}{dx}v$

$= a\beta\cos\beta x - \beta(\sin\beta x)\beta$

$g' = a\beta\cos\beta x - \beta^2\sin\beta x$

$y'' = (ae^{ax})(a\sin\beta x + \beta\cos\beta x) + (e^{ax})(a\beta\cos\beta x - \beta^2\sin\beta x)$

$y'' = e^{ax}(a^2\sin\beta x + a\beta\cos\beta x + a\beta\cos\beta x - \beta^2\sin\beta x)$

$y'' = e^{ax}(a^2\sin\beta x + 2a\beta\cos\beta x - \beta^2\sin\beta x)$

$y'' = e^{ax}(a^2\sin\beta x - \beta^2\sin\beta x + 2a\beta\cos\beta x)$

$$\boxed{y'' = e^{ax}((a^2-\beta^2)\sin\beta x + 2a\beta\cos\beta x)}$$