SM 121 LESSON 28 56/61/63/65/66/74/81/84 10/8/2008

SECTION 3.4 CHAIN RULE CONTINUED

(56) $y = \dfrac{|x|}{\sqrt{2-x^2}}$ FIND TANGENT @ (1,1)

$f(x) = |x| = x \quad x > 1$

$y = \dfrac{x}{\sqrt{2-x^2}}$

$y' = \dfrac{u'v - uv'}{v^2}$

$u = x \quad u' = 1$

$v = m^{1/2} \quad m = 2-x^2 \quad m' = -2x$

$v' = \frac{1}{2}m^{-\frac{1}{2}} \cdot m'$

$= \frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)$

$v' = \dfrac{-x}{\sqrt{2-x^2}} \qquad v^2 = \left(\sqrt{2-x^2}\right)^2 = 2-x^2$

$y' = \dfrac{1(2-x^2)^{1/2} - x\left(\frac{-x}{\sqrt{2-x^2}}\right)}{2-x^2}$

$y' = \dfrac{\sqrt{2-x^2}\left(\sqrt{2-x^2}\right) + \frac{x^2}{\sqrt{2-x^2}}}{2-x^2}$

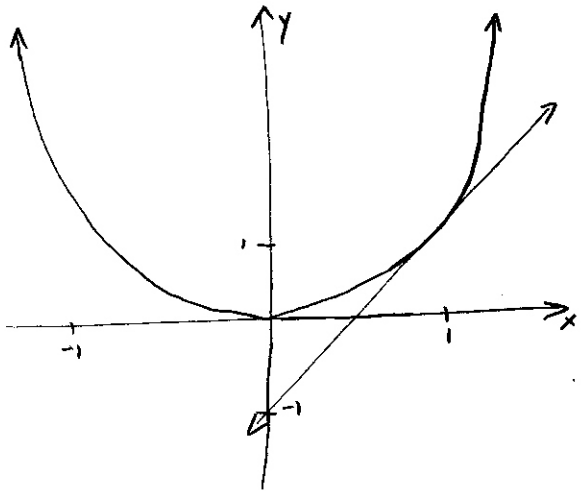
$y' = \dfrac{\frac{2-x^2}{\sqrt{2-x^2}} + \frac{x^2}{\sqrt{2-x^2}}}{2-x^2}$

$\dfrac{\frac{2}{\sqrt{2-x^2}}}{2-x^2}$

$\frac{2}{(2-x^2)^{\frac{1}{2}}} \cdot \frac{1}{(2-x^2)} = \boxed{\frac{2}{(2-x^2)^{3/2}}}$

$\dfrac{2}{(2-1^2)^{3/2}} = \dfrac{2}{1^{3/2}} = 2$

(b)



$y - 1 = 2(x-1)$

$y - 1 = 2x - 2$

(a) $\boxed{y = 2x - 1}$

TANGENT AT (1,1)

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(61) GIVEN $F(x) = f(g(x))$

$f(-2) = 8$ $\quad f'(5) = 3$

$f'(-2) = 4$ $\quad g(5) = -2$

$g'(5) = 6$

FIND $F'(5)$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$F'(5) = f'(g(5)) \cdot g'(5)$$

$$= f'(-2)(6)$$

$$= (4)(6)$$

$$\boxed{F'(5) = 24}$$

(63) a) $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1))g'(1)$

$= f'(2)(6)$

$= (5)(6)$

(a) $\boxed{h'(1) = 30}$

b) $H(x) = g(f(x))$

$H'(x) = g'(f(x))f'(x)$

$H'(1) = g'(f(1))f'(1)$

$= g'(3)(4)$

$= (9)(4)$

$\boxed{H'(1) = 36}$

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(66) a) $h(x) = f(f(x))$

OUT OF ORDER

$h'(x) = f'(f(x))f'(x)$

$h'(2) = f'(f(2))f'(2)$

$= f'(1)(-1)$

$= (-1)(-1)$

(4)

$\boxed{h'(2) = 1}$

b) $g(x) = f(x^2)$

$g'(x) = f'(x^2)2x$

$g'(2) = (2)(2)f'(2^2)$

$= (4)(2)$

$\boxed{g'(2) = 8}$

(65) a) $u(x) = f(g(x))$

$u'(x) = f'(g(x))g'(x)$

$u'(1) = f'(g(1))g'(1)$

$= f'(3)(-3)$

$= \left(\frac{-1}{4}\right)(-3)$

$\boxed{u'(1) = \frac{3}{4}}$

b) $v(x) = g(f(x))$

$v'(x) = g'(f(x))f'(x)$

$v'(1) = g'(f(1))f'(1)$

$= g'(2)(2)$

$=$ DOES NOT EXIST

$g'(2)$ DOES NOT EXIST

c) $w(x) = g(g(x))$

$w'(x) = g'(g(x))g'(x)$

$w'(1) = g'(g(1))g'(1)$

$= g'(3)(-3)$

$= \left(\frac{2}{3}\right)(-3)$

$\boxed{w'(1) = -2}$

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(74) FOR WHAT VALUES OF r DOES $y = e^{rx}$ SATISFY $y'' + 5y' - 6y = 0$?

$y' = e^u \frac{d}{dx} u \qquad u = rx \qquad u' = r$

$y' = re^{rx}$

$y'' = re^u \frac{d}{dx} u \qquad u = rx \qquad u' = r$

$y'' = r(re^{rx})$

$y'' = r^2 e^{rx}$

$y'' + 5y' - 6y = 0$

$r^2 e^{rx} + 5re^{rx} - 6e^{rx} = 0$

$e^{rx}(r^2 + 5r - 6) = 0$

$e^{rx}(r+6)(r-1) = 0$

$r + 6 = 0 \qquad r - 1 = 0$

$r = -6 \qquad r = 1$ SATISFY $y'' + 5y' - 6y = 0$

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(81) $s(t) = 2e^{-1.5t} \sin 2\pi t$

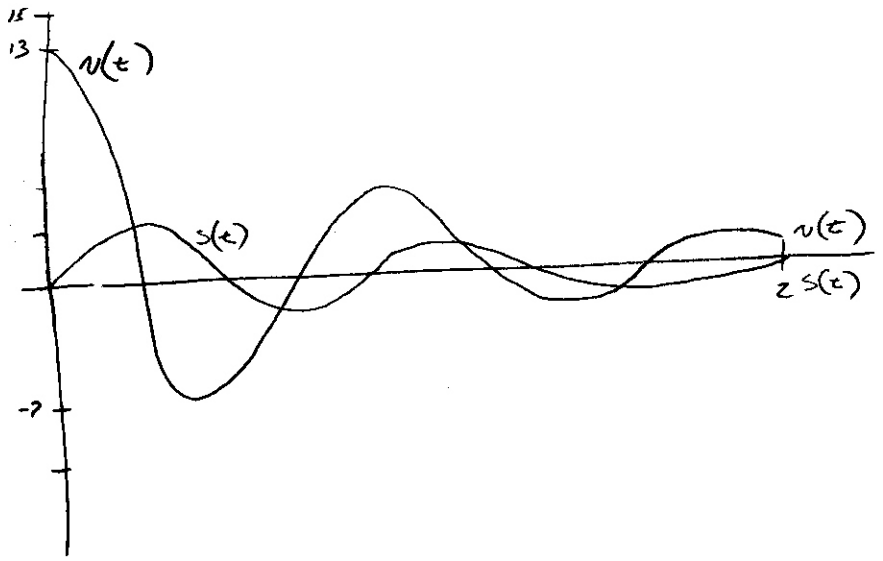
$v'(t) = m'n + mn'$ $\quad m = 2e^{-1.5t}$ $\quad n = \sin 2\pi t$ $\quad u = 2\pi t$

$m' = 2(e^u)\frac{d}{dx}u$ $\quad u = -1.5t$ $\quad u' = -1.5$ $\quad n' = \cos u \cdot \frac{d}{dx}u$ $\quad u' = 2\pi$

$m' = -3e^{-1.5t}$ $\quad n' = 2\pi \cos 2\pi t$

$v(t) = (-3e^{-1.5t})(\sin 2\pi t) + (2e^{-1.5t})(2\pi \cos 2\pi t)$

$= 2e^{-1.5t}\left[2\pi \cos(2\pi t) - \frac{3}{2}\sin(2\pi t)\right]$



(89) a) PROVE THE DERIVATIVE OF AN EVEN FUNCTION IS AN ODD FUNCTION

IF f IS EVEN $f(x) = f(-x)$ ← DEFINITION OF EVEN FUNCTION

$$f'(x) = f'(u)\frac{d}{dx}(u) \qquad u = -x \quad u' = -1$$

$$= f'(u)(-1)$$

$$f'(x) = -f'(u)$$

(a)

$$f'(x) = -f'(-x)$$

DEFINITION OF ODD FUNCTION:

$$f(x) = -f(-x)$$

∴ DERIVATIVE OF AN EVEN FUNCTION IS ODD

b) IF f IS ODD $f(x) = -f(-x)$

$$f'(x) = -f'(-x)\left(\frac{d}{dx}-x\right)$$

$$= -f'(-x)(-1)$$

$$f'(x) = f'(-x)$$

DEFINITION OF EVEN FUNCTION:

$$f(x) = f(-x)$$

∴ DERIVATIVE OF AN ODD FUNCTION IS EVEN