SM 121 LESSON 33 1/10/13/15/17/33/35 10/20/2008

SECTION 3.7 RATE OF CHANGE

① $s = f(t) = t^3 - 12t^2 + 36t$

ⓐ $\boxed{v = s' = f'(t) = 3t^2 - 24t + 36}$

b) $v(3) = 3(3)^2 - 24(3) + 36$

$= 27 - 72 + 36$

ⓑ $\boxed{v(3) = -9 \text{ ft/s}}$

c) $v(t) = 0 = 3t^2 - 24t + 36$

$0 = 3(t^2 - 8t + 12)$

$0 = 3(t-6)(t-2)$

$t - 6 = 0 \qquad t - 2 = 0$

$t = 6 \qquad t = 2$

ⓒ PARTICLE IS AT REST WHEN $t = 2$ AND $t = 6$

d) $v(t) > 0$

$3(t-2)(t-6) > 0$

$t - 2 < 0 \qquad t - 6 > 0$

$t < 2 \qquad t > 6$

ⓓ PARTICLE MOVING IN A POSITIVE DIRECTION WHEN $0 \leq t < 2$ OR $t > 6$

e) $|f(2) - f(0)| = |32 - 0| = 32$

$|f(6) - f(2)| = |0 - 32| = 32$

$|f(8) - f(6)| = |32 - 0| = 32$

ⓔ TOTAL DISTANCE TRAVELED IN THE FIRST 8 SECONDS IS 96 ft.

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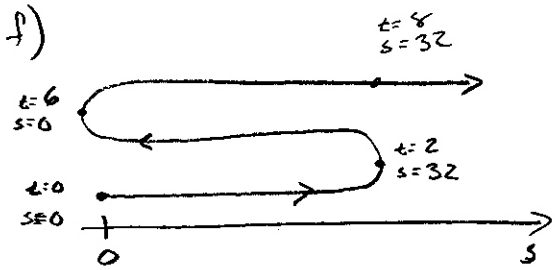
f)

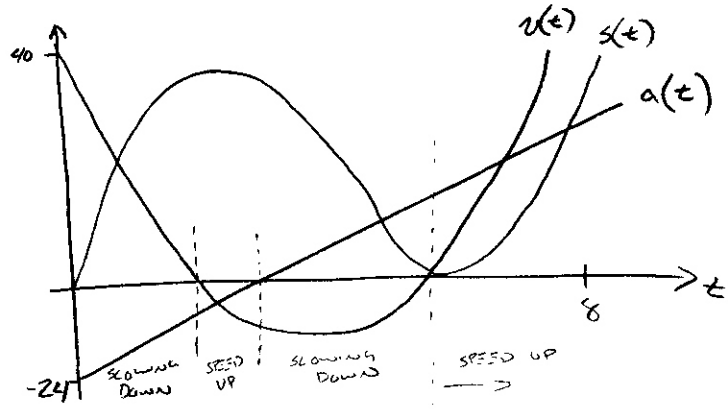


g) $a(t) = v'(t) = 6t - 24$

$a(3) = 6(3) - 24 = 18 - 24 = \boxed{-6 \text{ ft/s}^2}$

9

h)



i) THE PARTICLE IS SPEEDING UP WHEN a AND v HAVE THE SAME SIGN. THIS OCCURS WHEN $2 < t < 4$ (v AND a ARE NEGATIVE), AND WHEN $t > 6$ (v AND a ARE BOTH POSITIVE). THE PARTICLE IS SLOWING DOWN WHEN v AND a HAVE OPPOSITE SIGNS. THE PARTICLE IS SLOWING DOWN WHEN $0 \leq t < 2$ AND $4 < t < 6$

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(10) $s(t) = 80t - 16t^2$

a) WHAT IS THE MAXIMUM HEIGHT REACHED?

$v(t) = 80 - 32t$

$0 = 80 - 32t$

$-80 = -32t$

$t = \frac{-80}{-32} = \frac{5}{2}$ s

$s(\frac{5}{2}) = 80(\frac{5}{2}) - 16(\frac{5}{2})^2$

$= 200 - 16(\frac{25}{4})$

$= 200 - 4(25)$

$= 200 - 100$

(9) $\boxed{s(\frac{5}{2}) = 100 \text{ ft}}$

b) v OF BALL WHEN IT IS 96 ft IN THE AIR? GOING UP AND DOWN.

$s(t) = 96 = 80t - 16t^2$

$0 = -96 + 80t - 16t^2$

$= -16(6 - 5t + t^2)$

$= -16(t-3)(t-2)$

$t - 3 = 0 \quad t - 2 = 0$

$t = 3 \quad t = 2$

$v(2) = 80 - 32(2)$

$= 80 - 64 = \boxed{16 \text{ ft/s}}$ ON ITS WAY UP

$v(3) = 80 - 32(3)$

$= 80 - 96 = \boxed{-16 \text{ ft/s}}$ ON ITS WAY DOWN

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③ a) i) $A(r) = \pi r^2$ $\quad \dfrac{A(3) - A(2)}{3-2} = \dfrac{9\pi - 4\pi}{3-2} = \boxed{5\pi}$ (a.)

a) ii) $\dfrac{A(2.5) - A(2)}{2.5-2} = \dfrac{\frac{25\pi}{4} - 4\pi}{2.5-2} = \boxed{\dfrac{9\pi}{2}}$ (a.)

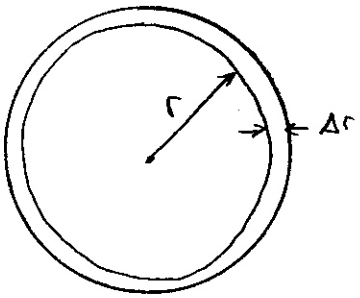
a) iii) $\dfrac{A(2.1) - A(2)}{2.1-2} = \dfrac{4.41\pi - 4\pi}{2.1-2} = \boxed{4.1\pi}$ (a.)

b) $A(r) = \pi r^2$

$A'(r) = 2\pi r$ $\quad A'(2) = 2\pi(2) = \boxed{4\pi}$ (b)

c) SHOW THAT THE RATE OF CHANGE OF A WITH RESPECT TO ITS RADIUS IS EQUAL TO ITS CIRCUMFERENCE.

$C(r) = 2\pi r = A'(r)$



IF Δr IS SMALL, WE CAN STRAIGHTEN OUT THE RING AND GET A RECTANGLE WITH LENGTH $2\pi r$ AND WIDTH Δr. SO $\Delta A = 2\pi r \Delta r$

$$\Delta A = A(r+\Delta r) - Ar = \lim_{\Delta r \to 0} \frac{\pi(r+\Delta r)^2 - \pi r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} \frac{\cancel{\pi r^2} + 2\pi r \Delta r + \pi \Delta r^2 - \cancel{\pi r^2}}{\Delta r}$$

$$\Delta A = \lim_{\Delta r \to 0} \frac{2\pi r \Delta r + \pi \Delta r^2}{\Delta r}$$

$$\Delta A = \lim_{\Delta r \to 0} \frac{\cancel{\Delta r}(2\pi r + \pi \Delta r)}{\cancel{\Delta r}}$$

$$\Delta A = \lim_{\Delta r \to 0} 2\pi r + \pi \Delta r^{\nearrow 0} = \boxed{2\pi r = \Delta A}$$

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(15) $\frac{d}{dr}(S = 4\pi r^2)$

$S'(r) = 8\pi r$

a) $S'(1) = 8\pi(1) = \boxed{8\pi \text{ ft}^3/\text{ft}}$

b) $S'(2) = 8\pi(2) = \boxed{16\pi \text{ ft}^2/\text{ft}}$

c) $S'(3) = 8\pi(3) = \boxed{24\pi \text{ ft}^2/\text{ft}}$

(b) AS RADIUS GROWS, THE SURFACE AREA OF THE BALLOON GROWS AT AN INCREASING RATE. THE RATE OF CHANGE OF SURFACE ARE IS LINEAR WITH RESPECT TO RADIUS

(17) $M(x) = 3x^2$

$\rho = \frac{dM}{dx} = 6x$

a) $\rho(1) = 6(1) = \boxed{6 \text{ kg/m}}$

b) $\rho(2) = 6(2) = \boxed{12 \text{ kg/m}}$

c) $\rho(3) = 6(3) = \boxed{18 \text{ kg/m}}$

SINCE ρ IS AN INCREASING FUNCTION, DENSITY IS HIGHEST AT THE RIGHT END OF THE ROD AND LOWEST AT THE LEFT END.

(33) $PV = nRT$

$P = 8$ atm

$V = 10$ L

$n = 10$ mol

$R = 0.0821$

$\frac{dP}{dt} = 0.10$ atm/min

$\frac{dV}{dt} = -0.15$ L/min

$\frac{d}{dt}(PV = nRT)$

$\frac{d}{dt}(PV) = nR\frac{d}{dt}T$

$V\frac{dP}{dt} + P\frac{dV}{dt} = nR\frac{dT}{dt}$

$\frac{dT}{dt} = \frac{V\frac{dP}{dt} + P\frac{dV}{dt}}{nR} = \frac{(10)(0.10) + (8)(-0.15)}{(10)(0.0821)} \approx \boxed{-0.2436 \text{ °K/min}}$

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(a) $\boxed{\text{POPULATION IS STABLE IF } \frac{dC}{dt} = 0 \text{ and } \frac{dW}{dt} = 0}$

(b) $\boxed{\text{"CARIBOU GO EXTINCT" IS REPRESENTED MATHEMATICALLY AS } C = 0}$

(c) $\frac{dC}{dt} = 0 = (0.05)C - (0.001)CW$

$+ \quad \frac{dW}{dt} = \left[0 = -(0.05)W + (0.0001)CW\right]10$

$0 = 0.05C - (0.5)W$

$0.5W = 0.05C$

$C = 10W$

SUBSTITUTE $10W$ FOR C IN $aC - bCW$

$(0.05)(10W) - (0.001)(10W)W = 0$

$(0.05)10W - 0.01W^2 = 0$

$W(0.5 - 0.01W) = 0$

$W = 0 \qquad 0.5 - 0.01W = 0$

$0.01W = -0.5$

$W = 50$

$W = 0 \Rightarrow C = 10(W) = 10(0) = 0$

$W = 50 \Rightarrow C = 10(W) = 10(50) = 500$

THUS THE POPULATION PAIRS (C, W) THAT LEAD TO STABLE POPULATIONS ARE $(0, 0)$ AND $(500, 50)$. SO IT IS POSSIBLE TO LIVE IN BALANCE.