SM121 LESSON 34 1/11/15/18 10/21/2008

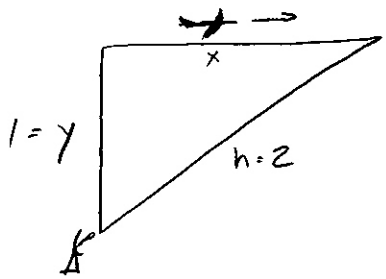
SECTION 3.9 RELATED RATES

(I) $\frac{d}{dt}\left(V = x^3\right)$

$$\boxed{\frac{dV}{dt} = 3x^2 \frac{dx}{dt}}$$

(II) GIVENS:

(c)



(a)

$y = 1$ mile $\quad \frac{dy}{dt} = 0$ m/h

$x = \sqrt{3}$ mile $\quad \frac{dx}{dt} = 500$ m/h

$h = 2$ mile

(b)

UNKNOWN: $\boxed{\frac{dh}{dt}}$

$x^2 + y^2 = h^2$

$x^2 = 2^2 - 1^2$

$x^2 = 4 - 1$

$x = \sqrt{3}$

(d) $\boxed{h^2 = y^2 + x^2}$

(e) $\frac{d}{dt}\left(h^2 = y^2 + x^2\right)$

$$2h\frac{dh}{dt} = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$$

$$\frac{dh}{dt} = \frac{2y\frac{dy}{dt} + 2x\frac{dx}{dt}}{2h} = \frac{\cancel{2(1)(0)}^{\,0} + \cancel{2}(\sqrt{3})(500)}{\cancel{2}(2)} = \boxed{250\sqrt{3}\text{ mi/h}}$$

$$\boxed{\approx 433\text{ mi/h}}$$

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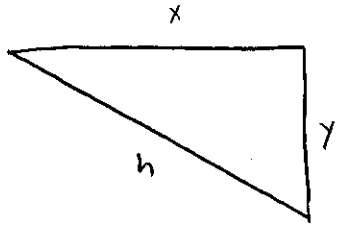
(15)



$x = 50$ mi $\qquad \frac{dx}{dt} = 25$ mi/h

$y = 120$ mi $\qquad \frac{dy}{dt} = 60$ mi/h

$h = 130$ mi $\qquad \frac{dh}{dt} =$ ____

$$\frac{d}{dt}\left(h^2 = x^2 + y^2\right)$$

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dh}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2h}$$

$$= \frac{2(50)(25) + 2(120)(60)}{2(130)}$$

$$= \frac{2500 + 14400}{260}$$

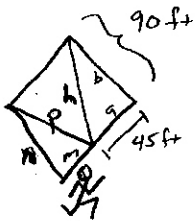
$$\boxed{\frac{dh}{dt} = 65 \text{ mi/h}}$$

$h^2 = x^2 + y^2$

$h = \sqrt{50^2 + 120^2}$

$h = 130$ mi

(18)



$a = 45$ ft $\qquad \frac{da}{dt} = -24$ ft/s

$b = 90$ ft $\qquad \frac{db}{dt} = 0$ ft/s

$h = 45\sqrt{5}$ ft $\qquad \frac{dh}{dt} =$ ____

$h^2 = a^2 + b^2$

$h = \sqrt{a^2 + b^2}$

$h = \sqrt{45^2 + 90^2}$

$h = 45\sqrt{5}$

$$\frac{d}{dt}\left(h^2 = a^2 + b^2\right)$$

$$2h\frac{dh}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} = \frac{2a\frac{da}{dt} + 2b\frac{db}{dt}}{2h} = \frac{2(45)(-24) + \cancel{2(90)(0)}^{\,0}}{2(45\sqrt{5})}$$

$$= \frac{-24}{\sqrt{5}}$$

(a) THE DISTANCE BTWN THE RUNNER AND 2B IS DECREASING AT APPROX. 10.7 ft/s

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$m = 45$ ft $\qquad \frac{dm}{dt} = 24$ ft/s

$n = 90$ ft $\qquad \frac{dn}{dt} = 0$ ft/s ← CONSTANT

$p = 45\sqrt{5}$ ft $\qquad \frac{dp}{dt} = ?$

$$\frac{d}{dt}\left(p^2 = m^2 + \cancel{90}^{\,0}\right)$$

$$2p\frac{dp}{dt} = 2m\frac{dm}{dt}$$

$$\frac{dp}{dt} = \frac{2m\frac{dm}{dt}}{2p} = \frac{m}{p}\frac{dm}{dt} = \frac{45}{45\sqrt{5}}(24) = \frac{24}{\sqrt{5}} \approx 10.7 \text{ ft/s}$$

(b) DISTANCE BTWN. RUNNER AND 3B IS INCREASING AT APPROX. 10.7 ft/s