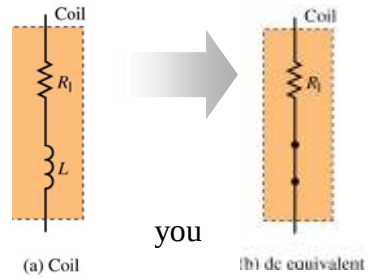
EE301 – Lesson 16
Reading: 11-5 to 11-10

INDUCTOR TRANSIENT ANALYSIS

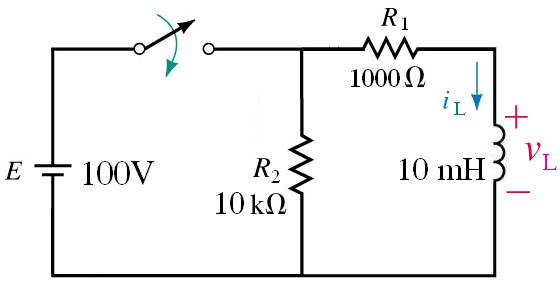
Learning Objectives

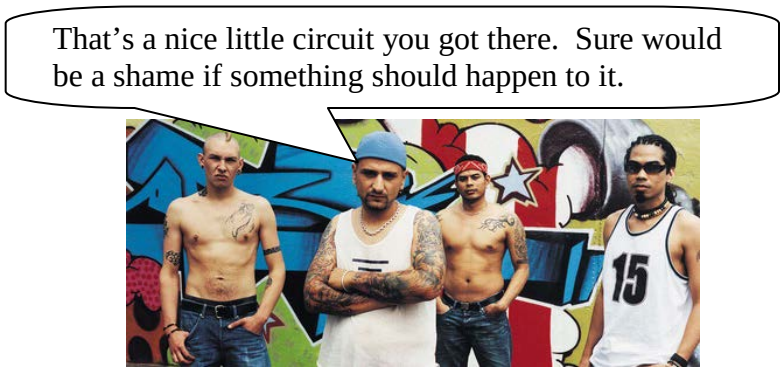
a. Calculate inductor voltage and current as a function of time.
b. Explain inductor DC characteristics.

Inductance and Steady State DC Recall that in steady state DC, an inductor looks like a short circuit. Stated another way, in steady state DC, the induced voltage across an inductor is equal to zero volts.



But what happens when we are not in steady state? Suppose we have the circuit shown below, in which the switch is initially open. Suppose someone in downtown Annapolis was threatening to beat you up unless you could tell them what happens when the switch is shut?



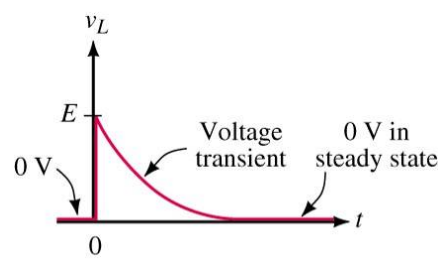


Since the safety and well-being of midshipmen is our primary concern, today we cover inductor transient analysis, so you will be safe next time you run into a gang of thugs in downtown Annapolis.

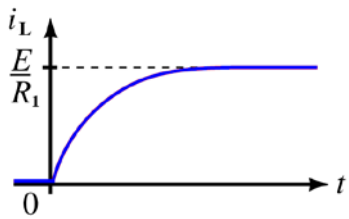
Transient Analysis: Inductor Storage Phase In the circuit above, when the switch is open, the inductor acts as a short circuit. But since the battery is not connected, no current flows anywhere.

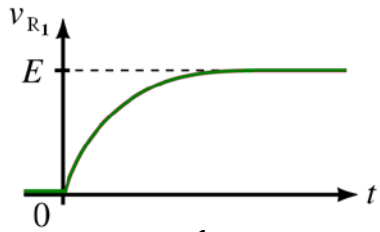
When switch is closed, the current through the branch with the inductor immediately "wants to change." The current through an inductor cannot change instantaneously (although the voltage across it can). In an attempt to keep the current through it from changing, the inductor immediately induces a voltage that opposes that change, which keeps the current near zero:

$$v_L = E$$



As the current i_L starts to build up, the voltage across the R1 resistor increases. As the voltage across R1 increases, the voltage drop across the inductor will decrease (since, by KVL) the voltage across R1 plus the voltage across the inductor must sum to the fixed battery voltage, E.





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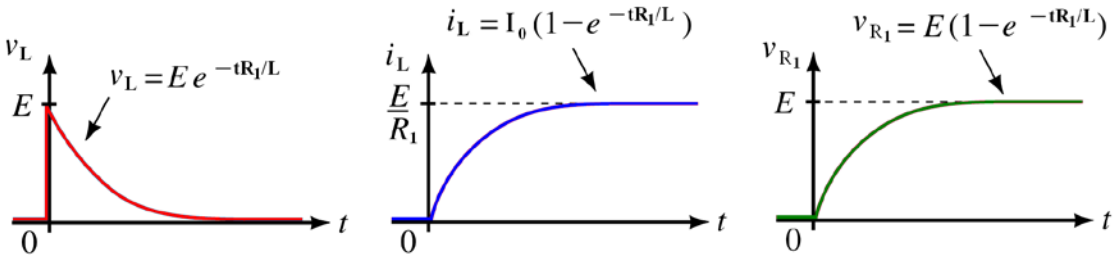
<u>Inductor Storage Phase Equations</u> The voltages and currents in the circuit above change exponentially over time, according to the equations shown below:



This phase is called the "storage phase" because the inductor is storing energy in its magnetic field (remember: this energy is equal to $\frac{1}{2}L\,i_L^{\,2}$). The circuit is at steady state when the voltage and current reach their final values and stop changing. At steady state the inductor acts as a short circuit, and the final inductor current (which is denoted I_0) will be:

$$i_L = I_0 = \frac{E}{R_1} = \frac{100}{1000} = 100\text{ mA}$$

<u>Example</u> (from text). For the circuit shown below, the switch shuts at time $t = 0$. Find the mathematical expressions for v_L and i_L from time $t = 0$ until your commissioning.

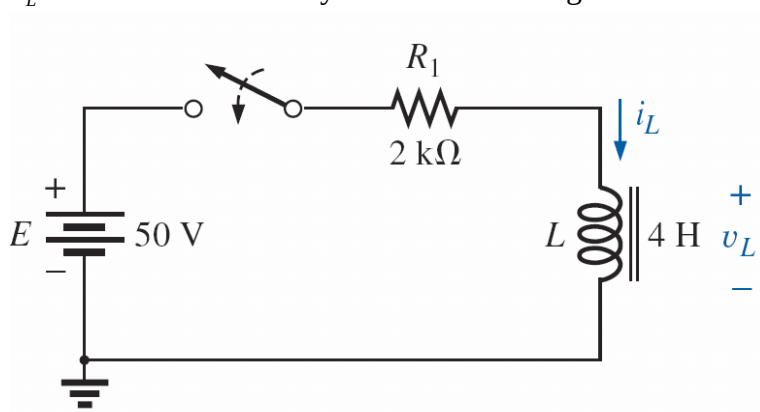


Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

<u>Solution</u>:

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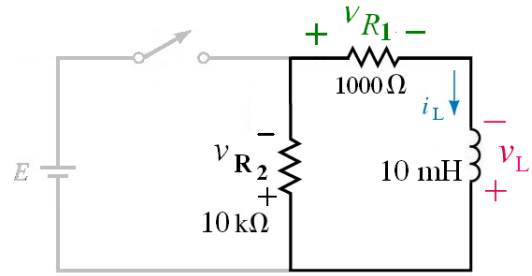
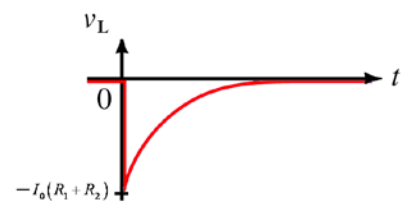
Inductor Release Phase Now, returning to the original circuit shown on page 1, when the switch has been shut for a long time (with a constant 100 ma current flowing through the inductor), suppose the switch then opens. What happens?



When the switch is opened, the inductor will immediately induce a voltage to keep the 100 mA current constant. What voltage will be needed to keep the current at 100 mA?

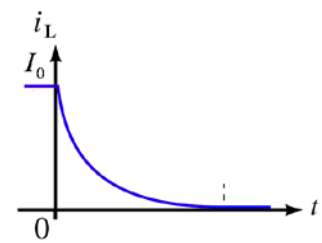
KVL can be used to calculate this induced voltage
Notice the polarity of the induced voltage!



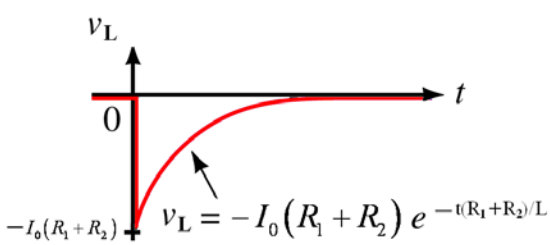
$$v_{R1} = I_0 R_1 = (0.1)1000 = 100 \text{ V}$$
$$v_{R2} = I_0 R_2 = (0.1)10{,}000 = 1000 \text{ V}$$
$$v_L = v_{R1} + v_{R2} = 1100 \text{ V}$$

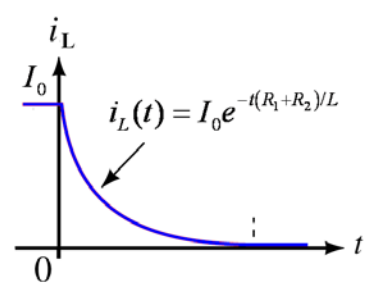
As stored energy is released, the induced voltage across the inductor drops.

This makes the voltage drop across the resistor drop, so current in the circuit drops.



Inductor Release Phase Equations Voltages and currents in a circuit in which an inductor is releasing energy also change exponentially over time:



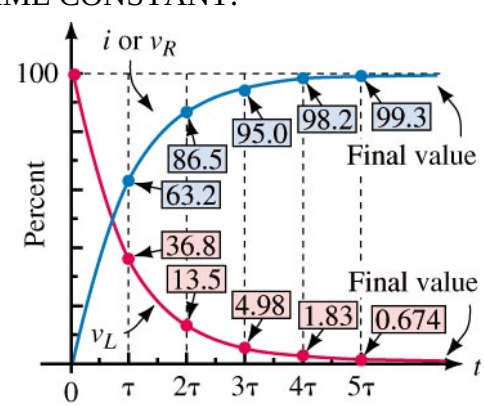


The Time Constant The rate at which an inductor's voltage/current changes as the inductor stores/releases energy depends on R and L, which is called the TIME CONSTANT:

$$\tau_{storage} = \frac{L}{R_1} = \frac{0.01}{1000} = 10\mu\text{sec}$$

$$\tau_{release} = \frac{L}{(R_1 + R_2)} = \frac{0.01}{11{,}000} = 0.91\mu\text{sec}$$

Transients can be considered to last for five time constants



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Example (from text). The circuit shown below has been in operation with the switch shut for a long time. The switch opens at time $t = 0$. Determine the equation for v_L and i_L.

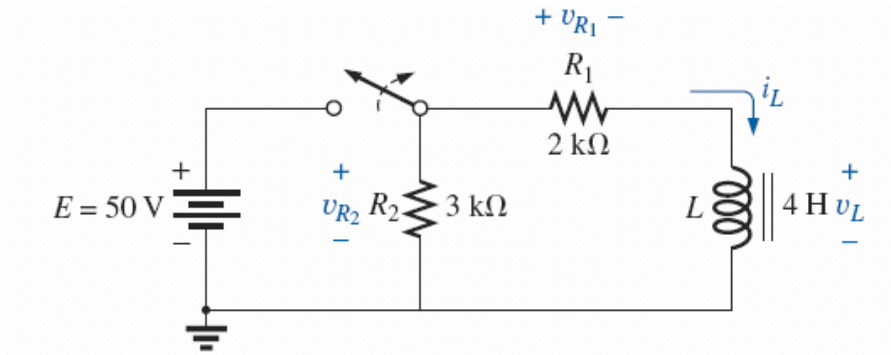
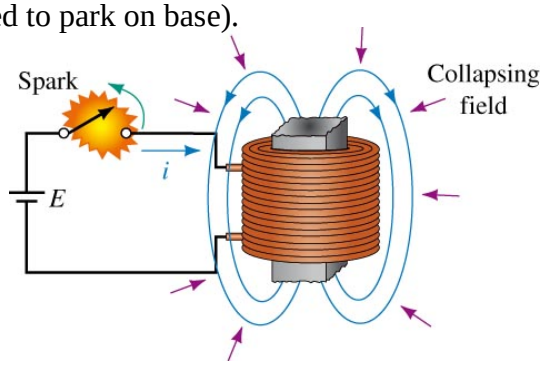


Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

Solution:

Interrupting Current in an Inductive Circuit When a switch opens in an RL circuit, the energy can be released in a short time, possibly creating a large voltage (as in the example on the prior page, where a voltage of 1100 volts was induced across the inductor). This induced voltage is called an *inductive kick*. As a practical matter, the abrupt opening of an inductive circuit may cause voltage spikes of thousands of volts. While these high voltages are generally undesirable, they can be controlled with proper engineering design and may, in fact, be useful in some applications (such as in automotive ignition systems, in which an inductor is used to generate a short-lived voltage of 25,000 volts to start the car that you are not allowed to park on base).



Example. The current in a 0.4 H inductor is changing at the rate of 200 A/sec. What is the voltage across it?

Solution:

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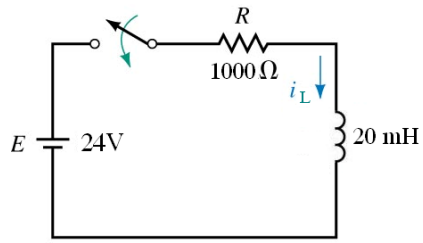
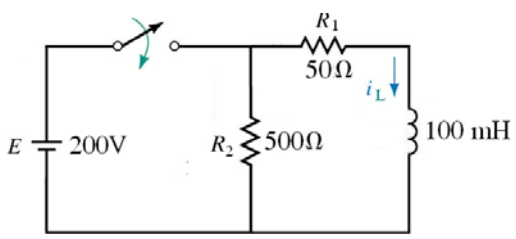
Example The switch is initially open and conditions are at steady-state. The switch is then shut.



(a) How long it will take for the inductor to reach a steady-state condition (>99% of final current).
(b) Write the equation for the current $i_L(t)$.
(c) Sketch the transient.

Solution:

Example The switch is initially shut and conditions are at steady-state. The switch then opens with a flourish of trumpets and fanfare.



(a) Determine the initial voltage induced across the inductor.
(b) How long will it take for the inductor to fully discharge (<1% of initial current)
(c) Write the equation for $i_L(t)$ and sketch the transient.
(d) If the discharging resistor R2 was changed to 1000Ω, how long would it take for the inductor to fully discharge?

Solution: