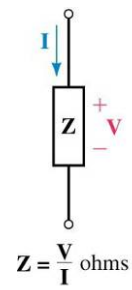
EE301 – Lesson 19
Reading: 15.1 - 15.2

IMPEDANCE

Learning Objectives

a. For purely resistive, inductive and capacitive elements define the voltage and current phase differences.
b. Define inductive reactance.
c. Understand the variation of inductive reactance as a function of frequency.
d. Define capacitive reactance.
e. Understand the variation of capacitive reactance as a function of frequency.
f. Define impedance.
g. Graph impedances of purely resistive, inductive and capacitive elements as a function of phase.

The Impedance Concept **Impedance** (**Z**) is the opposition that a circuit element presents to current in the phasor domain. If we view voltage and current as phasors (which are complex numbers in polar form), then the impedance is defined as



$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V}{I}\angle\theta = Z\angle\theta$$

This lends itself to an interpretation of Ohm's law for ac circuits:

$$\mathbf{V} = \mathbf{IZ}$$

In AC circuits, the impedance plays the same role that resistance (R) played in DC circuits.

Since impedance is a complex quantity it can also be expressed in rectangular form. In rectangular form, the impedance is made up of a real part, called the resistance R (the same resistance that we know and love from DC!), and an imaginary part called the reactance X:

$$\mathbf{Z} = R + j\,X \qquad (\Omega)$$

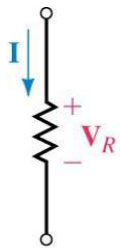
Note that the unit of impedance is ohms.

You are undoubtedly right this very minute completely paralyzed by the searing burning question: "*I know where resistance comes from: resistors! But where does 'reactance' come from? Where? What the heck is reactance? I simply cannot go on in life until this question is answered.*"

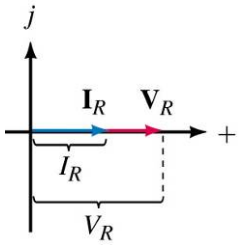
Since we do not want to see our midshipmen paralyzed, we will spend the next hour answering this burning question so that you can move forward with your lives. Lucky you.

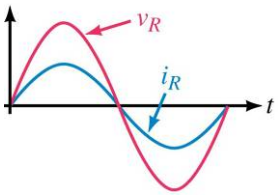
Resistance and Sinusoidal AC For a purely resistive circuit, current and voltage are in phase.

$$\mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{V_R\angle\theta}{I\angle\theta} = \frac{V_R}{I}\angle 0^\circ = R\angle 0^\circ = R$$



$$\boxed{\mathbf{Z}_R = R\angle 0^\circ}$$





(b) $i_R = v_R/R$. Therefore i_R is a sine wave also

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Example (from text). Using phasors, find the voltage v in the circuit below (the "q" in the equation should be ω—even books in their 12th edition can still have the occasional typo!).

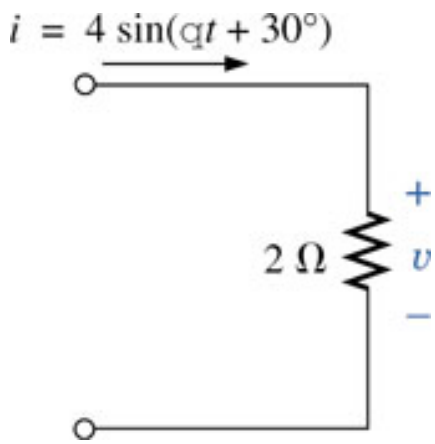

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

Solution:

Note the phasor diagram for the preceding example. In a purely resistive circuit, the current and the voltage are in phase.

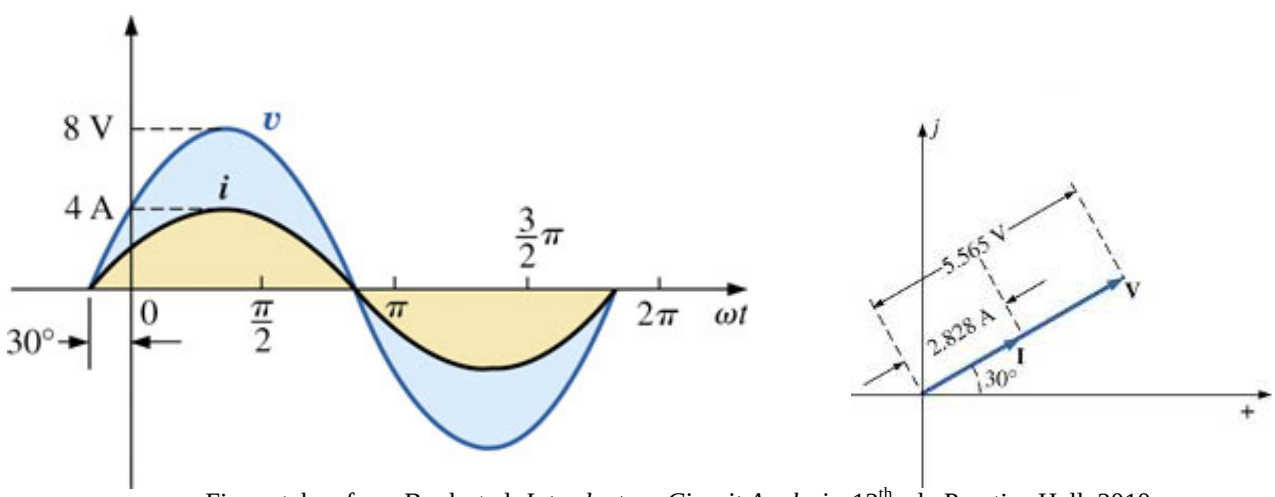

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

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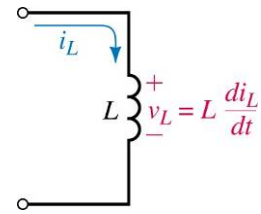
<u>Inductance and Sinusoidal AC</u> Suppose that the current through an inductor is sinusoidal:

$$i_L(t) = I_m \sin(\omega t)$$



and thus, the current expressed as a phasor is:

$$I_L = \frac{I_m}{\sqrt{2}}\angle 0^\circ$$

What is the voltage across this inductor? Recall the voltage-current relationship for an inductor:

$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}\left(I_m \sin\omega t\right) = \omega L I_m \cos\omega t = \omega L I_m \sin\left(\omega t + 90^\circ\right)$$

So, as a phasor, the voltage across the inductor is:

$$V_L = \frac{\omega L I_m}{\sqrt{2}}\angle 90^\circ$$

Thus, the impedance, which is the phasor voltage divided by the phasor current is

$$Z_L = \frac{V_L}{I_L} = \frac{\dfrac{\omega L I_m}{\sqrt{2}}\angle 90^\circ}{\dfrac{I_m}{\sqrt{2}}\angle 0^\circ} = \omega L\angle 90^\circ \quad (\Omega)$$

The impedance of an inductor can be written as a complex number (in polar or rectangular form):

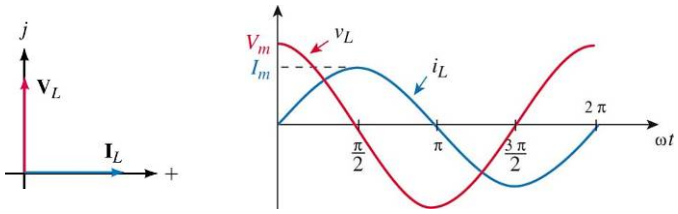
$$\boxed{Z_L = \omega L\angle 90^\circ = j\omega L \quad (\Omega)}$$

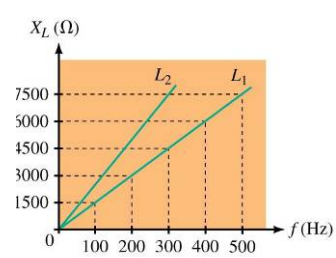
The boxed equation above should be memorized. Yea, yea, I know, it's on your formula sheet. **MEMORIZE IT**.

Since an ideal inductor has no real resistive component, this means the reactance of an inductor is the pure imaginary part:

$$\boxed{X_L = \omega L \quad (\Omega)}$$

It should be noted that *for a purely inductive circuit, the voltage leads the current by 90°*.





<u>Variation with Frequency:</u> Since $X_L = \omega L = 2\pi f L$, inductive reactance is *directly proportional to frequency*. For the extreme case when $f = 0$ Hz (DC), the inductor looks like a short circuit!

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Example (from text). Using the phasor transform, find the voltage *v* in the circuit below.

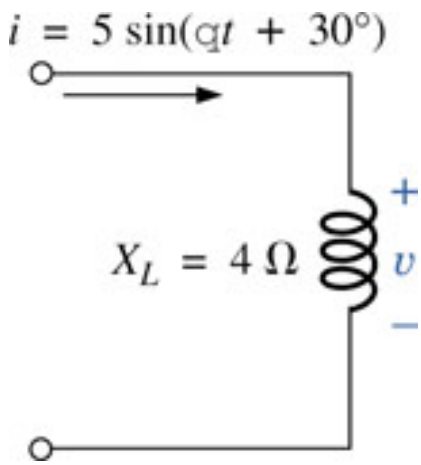

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

Solution:

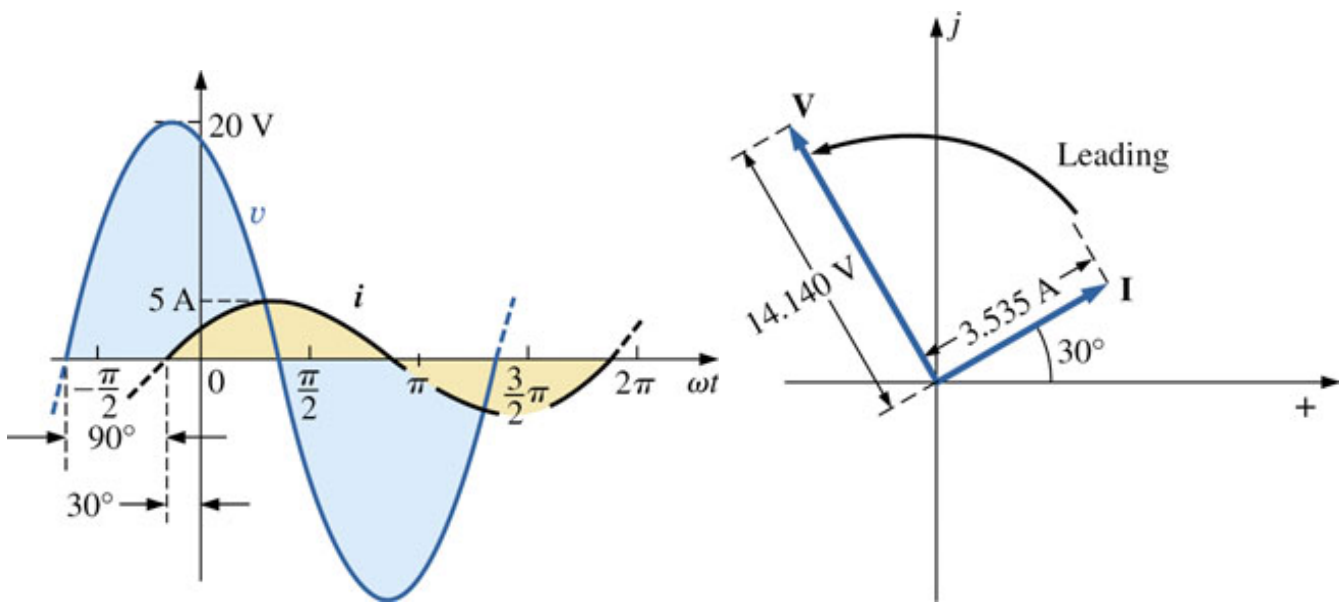

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

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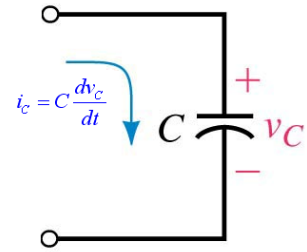
<u>Capacitance and Sinusoidal AC</u> Suppose that the voltage across a capacitor is sinusoidal:



$$v_C(t) = V_m \sin(\omega t)$$

and thus, the voltage expressed as a phasor is:

$$V_C = \frac{V_m}{\sqrt{2}}\angle 0^\circ$$

What is the current through this capacitor? Recall the voltage-current relationship for a capacitor:

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin\omega t) = \omega C V_m \cos\omega t = \omega C V_m \sin(\omega t + 90^\circ)$$

So, as a phasor, the current through the capacitor is:

$$I_C = \frac{\omega C V_m}{\sqrt{2}}\angle 90^\circ$$

Thus, the impedance, which is the phasor voltage divided by the phasor current is

$$Z_c = \frac{V_c}{I_c} = \frac{\frac{V_m}{\sqrt{2}}\angle 0}{\frac{\omega C V_m}{\sqrt{2}}\angle 90} = \frac{1}{\omega C}\angle -90^\circ \ (\mathbf{\Omega})$$

Impedance can be written as a complex number (in polar or rectangular form):

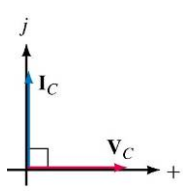
$$\boxed{Z_c = \left(\frac{1}{\omega C}\right)\angle -90^\circ = -j\left(\frac{1}{\omega C}\right) \quad (\mathbf{\Omega})}$$

The boxed equation above should be memorized. Yea, yea, I know, it's on your formula sheet. **MEMORIZE IT**.

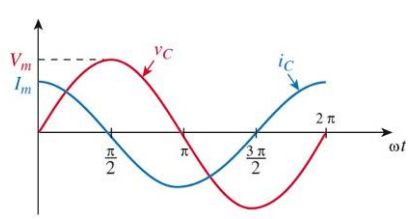
Since a capacitor has no real resistive component, this means the reactance of a capacitor is the pure imaginary part:

$$\boxed{X_c = \left(\frac{1}{\omega C}\right)}$$

It should be noted that, *for a purely capacitive circuit current leads the voltage by 90º*.



(c) $\mathbf{V}_C$ at 0°



(b) Waveforms with v_C as reference

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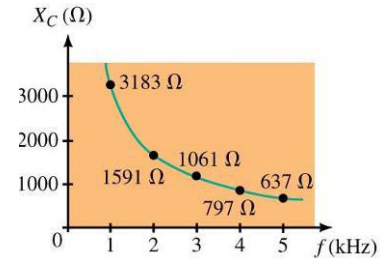

Variation with Frequency Since $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$, capacitive reactance is *inversely proportionally to frequency*. As an extreme case, if $f = 0$ Hz (DC), the capacitor looks like an open circuit!

Example (from text). Find the voltage v in the circuit below.

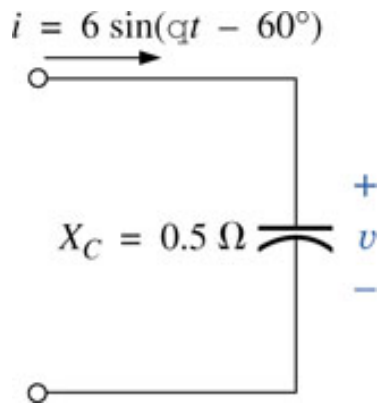

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

Solution:

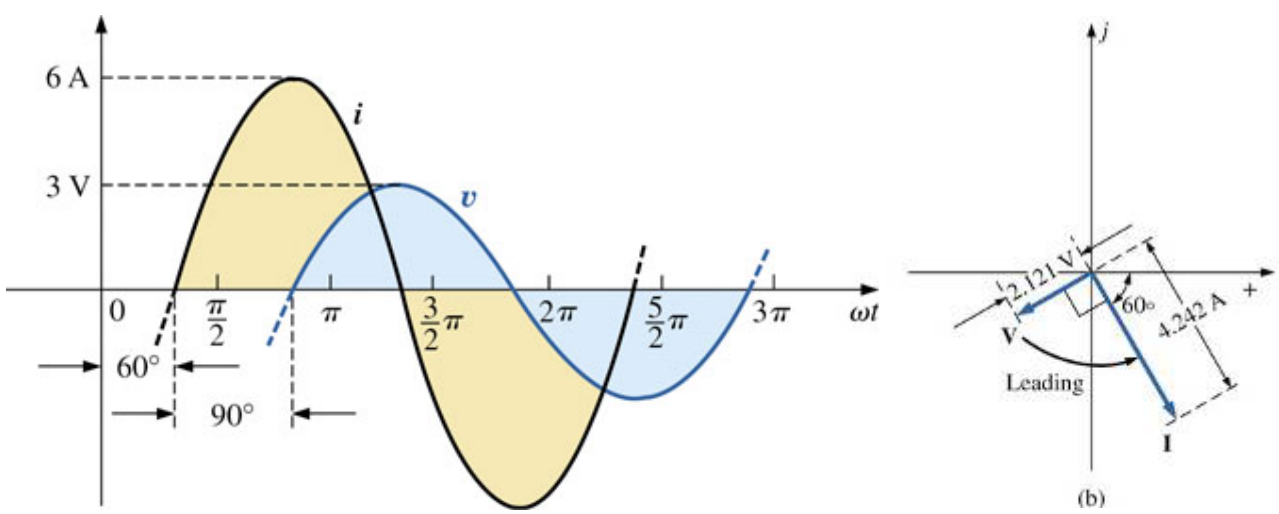

Figure taken from Boylestad, *Introductory Circuit Analysis*, 12th ed., Prentice Hall, 2010

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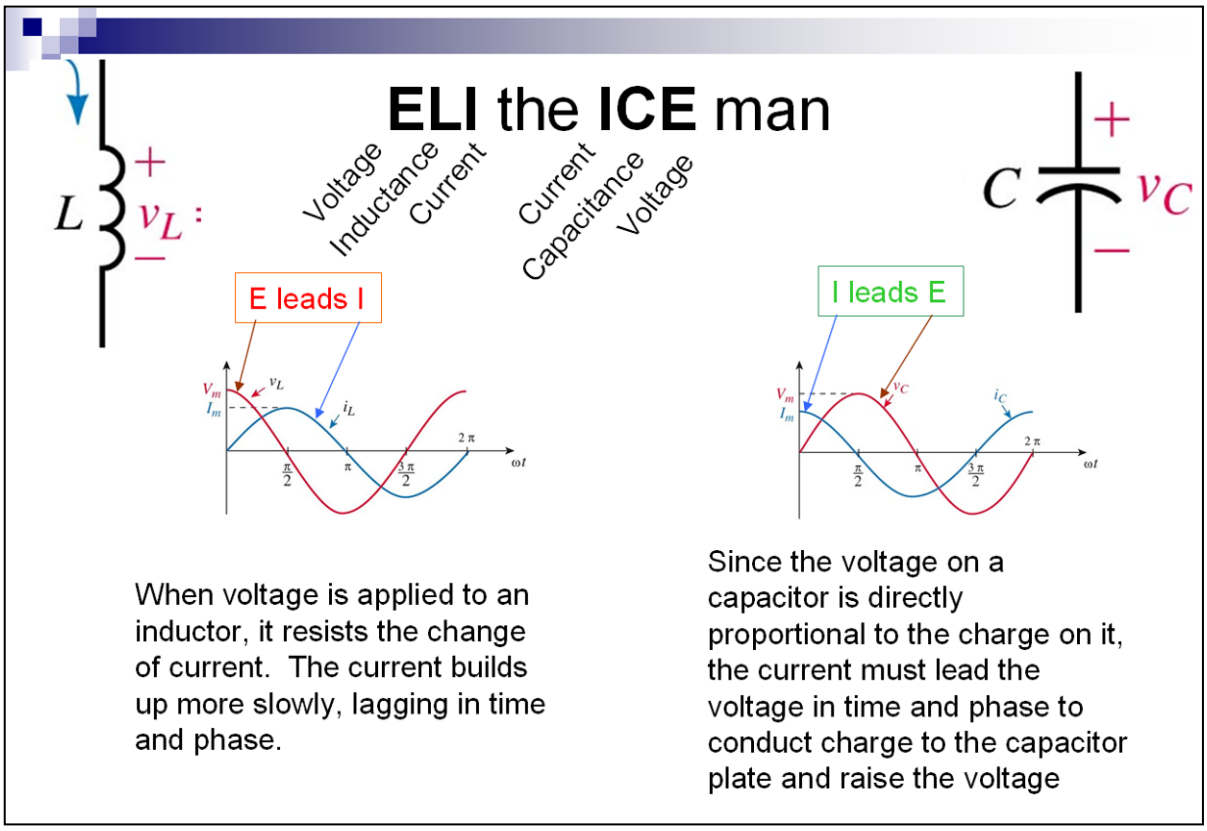
Monday

Hi, I'm Eli. What a happy day! Isn't EE301 great! I came up with a great memory device for remembering phase relationships. Hey, what's the weather supposed to be tomorrow? It's not supposed to be too hot, right?

Tuesday

But… we still have Eli's legacy:



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Impedance and AC Circuits solution technique

1. Transform the time domain currents and voltages into phasors
2. Calculate the impedances for circuit elements
3. Perform all calculations using complex math in the phasor domain
4. Transform the resulting phasors back to time domain if required

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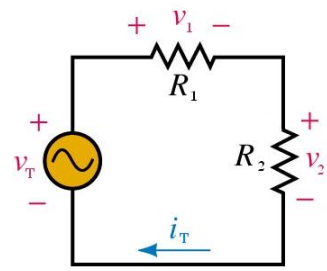
Example Consider the circuit shown which as two resistors R_1=10 kΩ and R_2=12.5 kΩ in series. The current is $i(t) = 14.7 \sin(\omega t + 39^\circ)$ mA.



(a) Compute $\mathbf{V}_{R1}$ and $\mathbf{V}_{R2}$

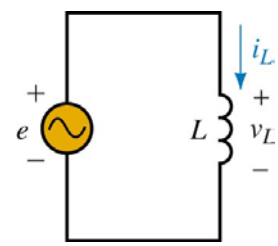
(b) Compute $\mathbf{V}_T = \mathbf{V}_{R1} + \mathbf{V}_{R2}$

(c) Calculate $\mathbf{Z}_T$

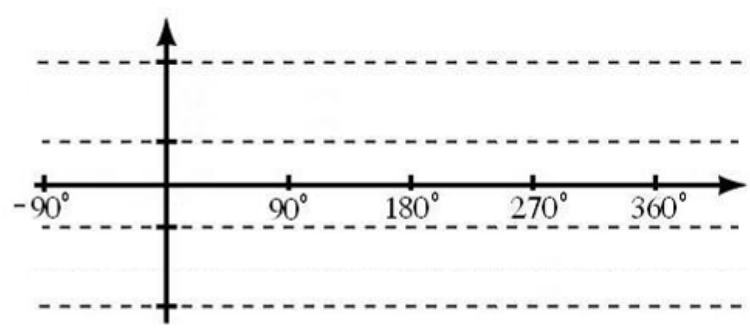
(d) Compare $\mathbf{V}_T$ to the results of $\mathbf{V}_T = \mathbf{I}\,\mathbf{Z}_T$

Solution:

Example For the inductive circuit shown, $v_L = 40 \sin(\omega t + 30^\circ)$ V , *f=26.53 kHz* and L = 2 mH. Determine V_L and I_L. Graph i_L and v_L .



Solution:



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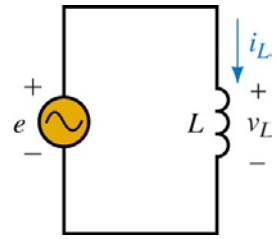
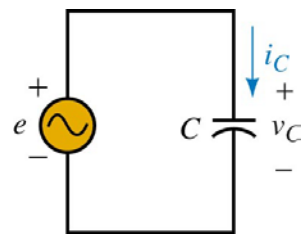


Example: For the inductive circuit, $v_L = 40 \sin(\omega t + \Theta)$ V and $i_L = 250 \sin(\omega t + 40^\circ)$ μA. The frequency is $f = 500$ kHz.

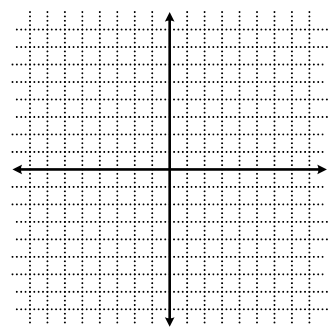
Determine:
(a) L
(b) Θ

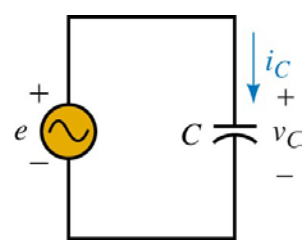
Solution:



Example For the capacitive circuit, $v_C = 3.6 \sin(\omega t - 50)$ V, C=*1.29 uF* and $f = 12$ kHz. Determine V_C and I_C and then plot V_C and I_C as phasors.

Solution:





Example For the capacitive circuit: $v_C = 362 \sin(\omega t - 33^\circ)$ V, $i_C = 94 \sin(\omega t + 57^\circ)$ mA and C = 2.2 μF. Determine the frequency.

Solution: