EE301 – Lesson 21
Reading: 15.7-15.9 and 15.11

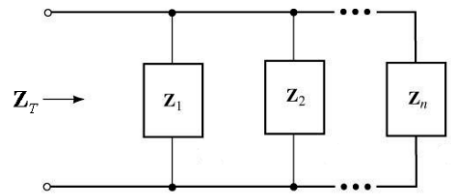
AC PARALLEL CIRCUITS

Learning Objectives

a. Compute the total impedance for AC parallel circuits.
b. Apply Kirchhoff's Current Law and the current divider rule to AC parallel circuits.
c. Graph impedance, voltages and current as function of phase.
d. Graph voltages and current as a function of time.

Parallel AC Circuit Impedances in parallel can be combined into a single impedance using the same formula that applied for resistors in parallel; we have only to replace the resistors with impedances:

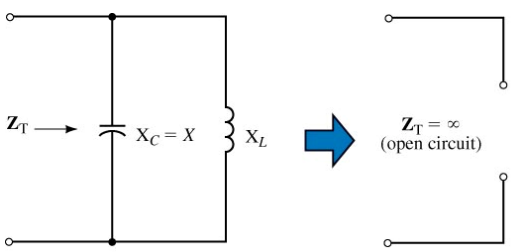
$$\mathbf{Z}_T = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_n}} \qquad (\Omega)$$



If we have only two impedances in parallel, we have the familiar "product over sum" rule:

$$\boxed{\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad (\Omega)}$$

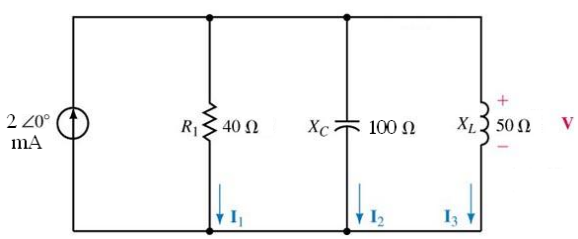
If $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are *equal* inductive and capacitive reactances, the resulting total impedance is infinite, so the equivalent is an **open circuit**.



$$Z_C = 0 - 2j = 2\angle -90$$
$$Z_L = 0 + 2j = 2\angle 90$$
$$Z_T = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{(2\angle 90)(2\angle -90)}{(0+2j)+(0-2j)}$$
$$= \frac{(2x2)\angle(90-90)}{0 + j(2-2)} = \frac{4\angle 0}{0} = \infty \qquad (\Omega)$$

Current Divider Rule (CDR) Continuing the carryover of concepts that you learned in DC: for AC circuits we have the Current Divider Rule:

$$\boxed{\mathbf{I}_x = \frac{\mathbf{Z}_T}{\mathbf{Z}_x}\mathbf{I}_{TOT}}$$



$$\frac{1}{Z_T} = \frac{1}{Z_R} + \frac{1}{Z_c} + \frac{1}{Z_L} = \frac{1}{40\angle 0} + \frac{1}{100\angle -90} + \frac{1}{50\angle 90} \rightarrow Z_T = 37.1\angle 21.8\ \Omega$$

<u>CDR</u>

$$I_1 = I_T\left(\frac{Z_T}{Z_R}\right) = 2\angle 0^\circ mA\left(\frac{37\angle 21.8}{40\angle 0}\right) = 1.86\angle 21.8 mA$$

$$I_2 = I_T\left(\frac{Z_T}{Z_C}\right) = 2\angle 0^\circ mA\left(\frac{37\angle 21.8}{100\angle -90}\right) = 0.743\angle 111.8 mA = 743\angle 111.8 \mu A$$

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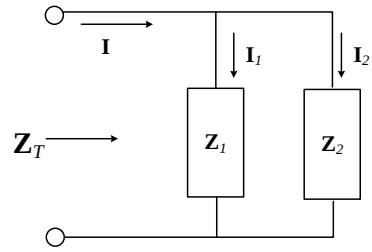
<u>Current Divider (two impedances)</u> Hopefully, you won't be too shocked to learn that we have the same simplification of the CDR in the case of two impedances that we saw in DC:

$$\boxed{\begin{aligned}\mathbf{I}_1 &= \mathbf{I}_T\left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}\right)\\ \mathbf{I}_2 &= \mathbf{I}_T\left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}\right)\end{aligned}}$$



<u>Kirchhoff's Current Law (KCL)</u> Would you believe that KCL doesn't hold for AC? Nah, of course it does! *The summation of current phasors entering and leaving a node is equal to zero.*

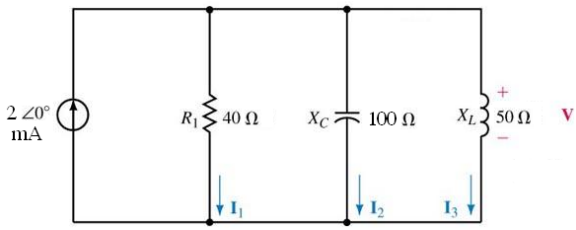
$I_1 = 1.86\angle 21.8mA$

$I_2 = 743\angle 111.8\mu A$

<u>*KCL*</u>

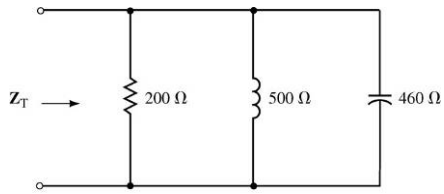
$$\sum I_{IN} = \sum I_{OUT} \rightarrow I_T = I_1 + I_2 + I_3$$

$$I_3 = I_T - I_1 - I_2\left(\frac{Z_T}{Z_L}\right) = 2\angle 0 - 1.86\angle 21.8 - 0.74\angle 111.8mA = 1.49\angle -68.2mA$$



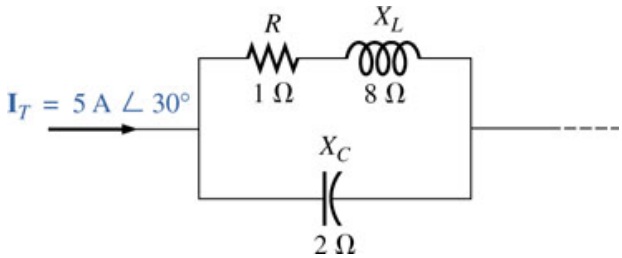
<u>Example.</u> Determine Z_T.

<u>Solution:</u>



(a)

<u>Example</u> (from text). Use the current divider rule to find the current through each branch in the circuit below:



<u>Solution:</u>

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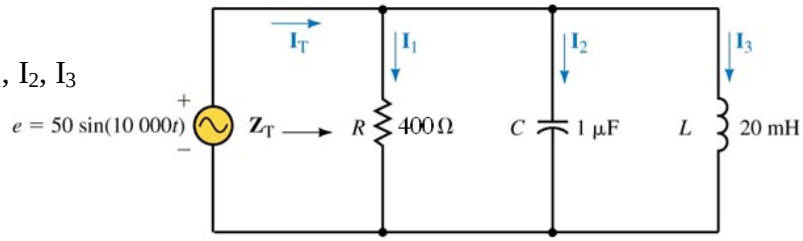
Example For the circuit shown:

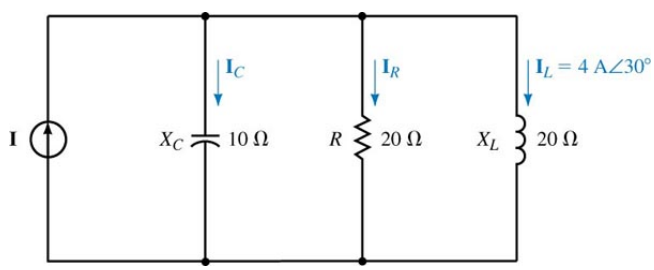
(a) Find Z_T, I_T, I_1, I_2, I_3
(b) Sketch the phasor diagrams showing E, I_T, I_1, I_2, I_3



Solution:

Example For the circuit shown:

(a) Find the total impedance $\mathbf{Z}_T$.
(b) Determine the supply current ($\mathbf{I}_T$,) using the current divider rule
(c) Calculate $\mathbf{I}_C$ and $\mathbf{I}_R$.
(d) Graph $\mathbf{I}_T$, $\mathbf{I}_C$, $\mathbf{I}_R$, $\mathbf{I}_L$ in the time domain.
(e) Verify KCL.



Solution: