EE301 – Lesson 25
Reading: 19.1-19.8

AC POWER AND POWER TRIANGLE

Learning Objectives

a. Define real (active) power, reactive power and average power.
b. Calculate the real and reactive power in AC series parallel networks.
c. Graph the real and reactive power of purely resistive, inductive, or capacitive loads in AC series parallel networks as a function of time.
d. Determine when power is dissipated, stored, or released in purely resistive, inductive, or capacitive loads in AC series parallel networks.
e. Use the power triangle to determine relationships between real, reactive and apparent power

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AC Power Recall that AC impedance is a complex quantity made up of real resistance and imaginary reactance.

Note the crazy coloring in the sentence above (which I am sure shows up wonderfully on your black and white printer!): Real is blue, imaginary is gold (get it…blue and gold…get it…?). And complex is green.

$$\vec{\mathbf{Z}} = R + jX \qquad (\Omega)$$

AC Apparent Power is a complex quantity made up of real active power and imaginary reactive power:

$$\vec{\mathbf{S}} = P + jQ \qquad (VA)$$

- It is the real resistance that leads to the dissipation of real active power (blue).
- It is the imaginary reactance that leads to the reactive power (gold).
- The sum of the real power and the reactive power yields the apparent power (green).

We will discuss real active power first, then imaginary reactive power second.

AC Real (Active) Power (P) The Active power is the power that is dissipated in the resistance of the load. It uses the same formula used for DC:

$$P = I^2 R = \frac{V^2}{R} \qquad \text{[watts, W]}$$

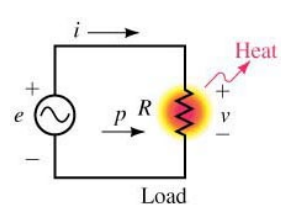
WARNING! **#1 mistake with AC power calculations!** The **Voltage** in the above equation is the Voltage drop across the resistor, not across the entire circuit!

CAUTION!
REAL value of resistance (R) is used in REAL power calculations, not Impedance!

In ac circuits, voltage and current are functions of time.
Power at a particular instant in time is given by:

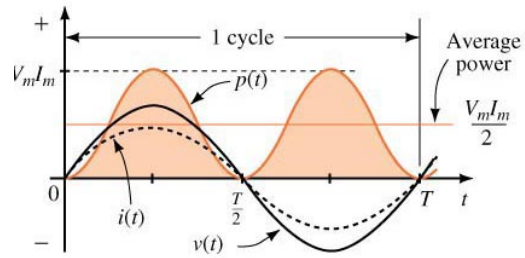
$$p = vi = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$= V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$



So, the power as a function of time is

$$p(t) = \frac{V_m I_m}{2}\left(1 - \cos 2\omega t\right)$$



This is shown as the gold portion of the plot on the right.

Again, this is called **real (active) power** and note for the graph that p is always positive.

In this case (with this purely resistive circuit) the power delivered by the source is absorbed by the load.

Now, we are usually interested in the average power, not necessarily the instantaneous power. Integrating the equation for the instantaneous power over a period, we find the average power is

$$P = V_m I_m / 2$$

Recalling that the RMS values of V and I are:

$$V_{RMS} = \frac{V_m}{\sqrt{2}} \qquad \text{rms value of voltage}$$

$$I_{RMS} = \frac{I_m}{\sqrt{2}} \qquad \text{rms value of current}$$

we can write the average power as

$$P = \frac{V_m I_m}{2} = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) = V_{RMS} I_{RMS} \qquad \text{(watts)}$$

The Active power is the average value of instantaneous power.

In an analogous fashion, we can use Ohm's Law to write $P = \frac{V_{RMS}^{\ 2}}{R} = I_{RMS}^{\ 2} R$

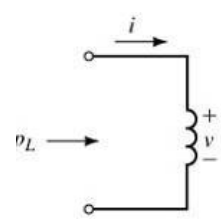
(This is why we use RMS values in phasors! We can use the phasor magnitude directly in these power calculations.)

<u>AC Imaginary (Reactive) Power (Q)</u> The reactive power is the power that is exchanged between reactive components (inductors and capacitors). The formulas look similar to those used by the active power, but use reactance instead of resistances.

$$Q = I^2 X = \frac{V^2}{X} \qquad \text{[VAR]}$$

The units of reactive power are Volts-Amps-Reactive (VAR). By convention, Q is negative for a capacitor and positive for an inductor.

<u>Power to an Inductive Load</u> Consider the following circuit where $i = I_m \sin \omega t$.



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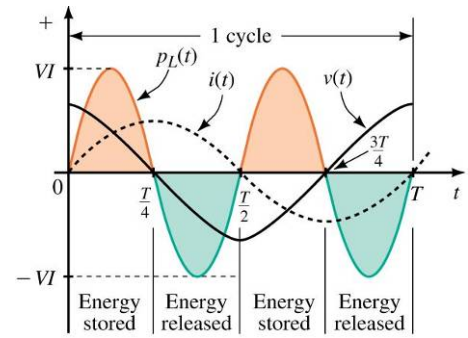
The expression for **instantaneous power** or $p_L(t)$:

$$i = I_m \sin \omega t$$
$$v = V_m \sin(\omega t + 90)$$
$$p = vi = (V_m \sin \omega t + 90^\circ)(I_m \sin \omega t) = V_m I_m \cos \omega t \sin \omega t$$
$$= \frac{V_m I_m}{2}(\sin 2\omega t) = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\sin 2\omega t = V_{RMS} I_{RMS} \sin 2\omega t$$



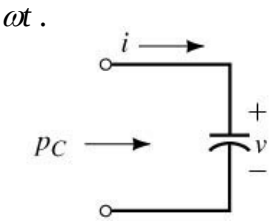
So, the instantaneous power p is equally positive and negative.

All of the power delivered by the source is returned.

The average power P_L =0.

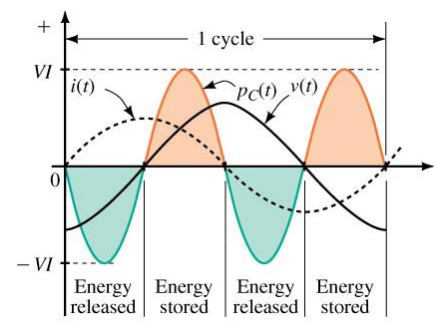
Reactive power is the portion of power that flows into load and then back out. It contributes nothing to average power. *The power that flows into and out of a pure inductor is reactive power only*.

<u>Power to an Capacitive Load</u> Consider the following circuit where $i = I_m \sin \omega t$.



Expression for **instantaneous power** or $p_C(t)$:

$$i = I_m \sin \omega t$$
$$v = V_m \sin(\omega t - 90)$$
$$p = vi = (V_m \sin \omega t - 90^\circ)(I_m \sin \omega t) = -V_m I_m \cos \omega t \sin \omega t$$
$$= -\frac{V_m I_m}{2}(\sin 2\omega t) = -\left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\sin 2\omega t = -V_{RMS} I_{RMS} \sin 2\omega t$$



So, the instantaneous power p is equally positive and negative
All of the power delivered by the source is returned (no power losses with a pure reactive load).

The average power P_C = 0 W

<u>AC Apparent Power (S)</u>
The apparent power is the power that is "appears" to flow to the load.
The magnitude of apparent power can be calculated using similar formulas to those for active or reactive power:

$$S = VI = I^2 Z = \frac{V^2}{Z} \qquad \text{[VA]}$$

Units: Volts-Amps (VA)

$$\vec{\mathbf{Z}} = R + jX \quad (\Omega)$$

Apparent power calculated with Z
Real power calculated with R
Reactive power calculated with X

$$\vec{\mathbf{S}} = P + jQ \quad (VA)$$

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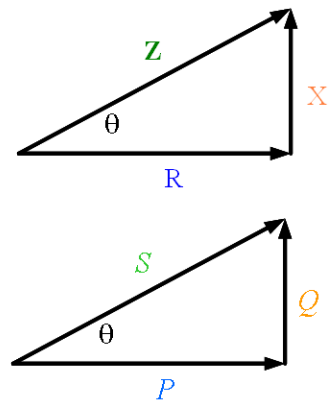
Power Triangle

The power triangle graphically shows the relationship between real (P), reactive (Q) and apparent power (S).



$$S = \sqrt{P^2 + Q^2}$$

$$\vec{\mathbf{S}} = P + jQ_L$$

$$\vec{\mathbf{S}} = S\angle\theta$$

The power triangle also shows that we can find real (P) and reactive (Q) power, given S and the impedance angle θ.

$$S = IV \qquad \text{(VA)}$$

$$P = S\cos\theta \qquad \text{(W)}$$

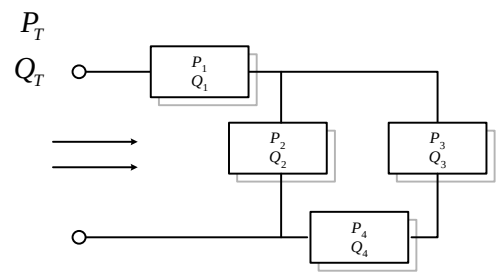
$$Q = S\sin\theta \qquad \text{(VAR)}$$

NOTE: The impedance angle and the "power factor angle" are the same value!

Total Power in AC Circuits

The total power real (P_T) and reactive power (Q_T) is simply the sum of the real and reactive powers for all of the individual circuit elements.

How elements are connected does not matter for computation of total power.

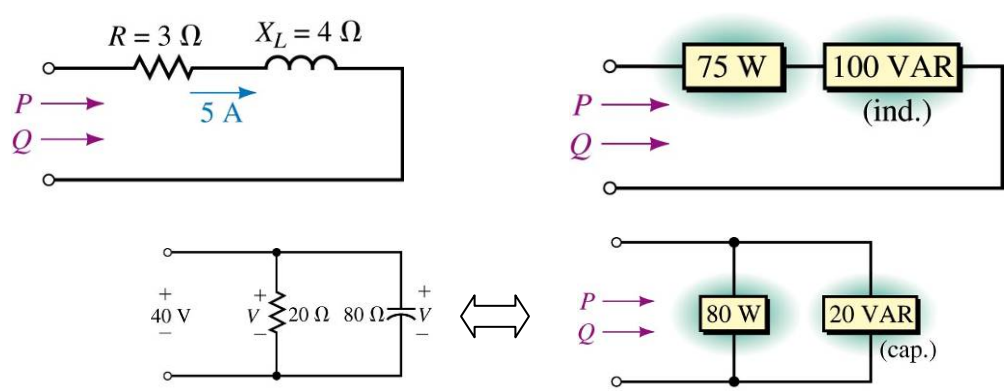


$$P_T = P_1 + P_2 + P_3 + P_4$$

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4$$

Total Power in AC Circuits

Sometimes it is useful to redraw the circuit to symbolically express the real and reactive power loads



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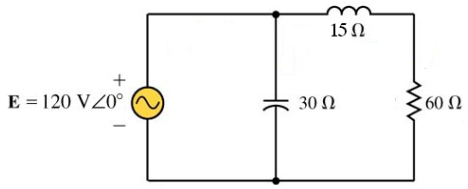
Example For the circuit shown:

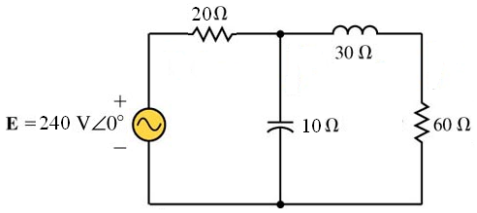
a. Determine real and reactive power of each component.
b. Determine the apparent power delivered by the source.



Solution:

Example Determine the apparent power, total real and reactive power using the following equations:

$$S = VI \qquad \text{(VA)}$$
$$P = S\cos\theta \qquad \text{(W)}$$
$$Q = S\sin\theta \qquad \text{(VAR)}$$



Solution:

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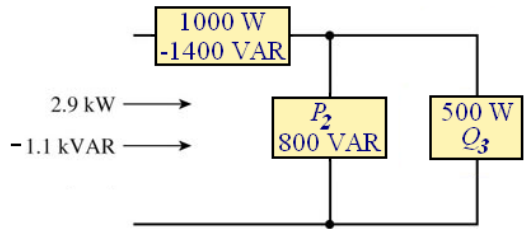
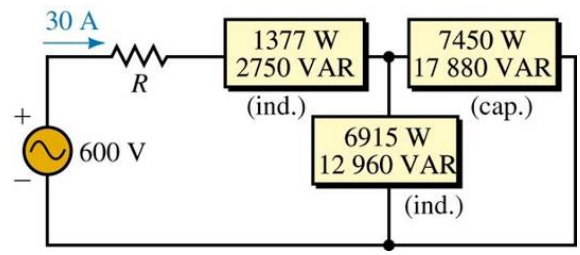
Example For the circuit shown

a. Determine the unknown real (P2) and reactive powers (Q3) in the circuit.
b. Determine total apparent power
c. Draw the power triangle
d. Is the unknown element in Load #3 an inductor or capacitor?

Solution:



Example For the circuit shown

a. Determine the value of R, P_T and Q_T
c. Draw the power triangle and determine S.

Solution: