Can Reducing the Size of the Pie Enhance Bargaining Position? The Case of the Cable Television Industry

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Abstract

A cable operator chooses to bundle or provide programs *à la carte* by striking a balance between maximizing total surplus and minimizing transfer payments to program providers. We show, using general demand and cost functions, that a cable operator's decision to bundle maximizes total producer surplus if the cable operator's bargaining power is sufficiently high, and that a cable operator in a weak bargaining position might strategically choose to unbundle viewer channels in order to enhance its bargaining position with individual program suppliers, even when this decision reduces total surplus. Thus, it is plausible that regulations that cap market share or impose *à la carte* on cable operators may reduce total surplus. Under more restrictive conditions, we extend the analysis and explore consumer and social welfare.

Keywords: Bundling, Division of Surplus, Nash Bargaining, Regulation

JEL Classification: L12, L82, L50

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1 Introduction

Product bundling is widespread in markets where sellers have market power, and firms routinely utilize bundling to facilitate consumer price discrimination and possibly deter entry. Adams and Yellen (1976) identified two types of bundling, pure and mixed, and developed a two-product monopoly bundling model that demonstrated firms could benefit from mixed bundling, i.e., products sold in combination or separately. McAfee et al. (1989) and Salinger (1995) provided useful extensions of this early model, while Nalebuff (2002) explored the possible use of bundling to deter entry.

In what follows, we model bundling in the cable television industry, where a cable operator (such as Comcast or Time Warner Cable) purchases programming from program suppliers (such as CNN or ESPN) to package and re-sell to consumers, either as a bundle or à la carte. We suggest that the cable operator uses bundling or unbundling of its programs to enhance its bargaining position with program suppliers, which implies that the cable operator's "packaging decision" is influenced by factors other than economies of scale or consumer price discrimination.

Because a cable operator pays each program supplier a share of the supplier's marginal contribution to total surplus, the cable operator has an incentive to package programs in such a way that it reduces program suppliers' marginal contributions to total surplus, which, in turn, decreases the total amount of transfer payments paid by the cable operator. A higher total surplus, however, increases the amount of surplus available to the cable operator. Thus, the cable operator chooses to bundle or to provide programs à la carte to consumers by comparing the effects of its decision on both the *amount of transfer payments* paid to suppliers and on *total surplus*. If a cable operator keeps a relatively small portion of total surplus, then minimizing transfer payments has a larger impact on profitability. Thus, we suggest that when a cable operator's bargaining power is low, the cable operator finds it profitable to unbundle programs, even when such behavior reduces total surplus.[1] This finding contrasts with the traditional prediction that

[1] Each program supplier tries to claim a portion of the total surplus created when it joins the bundle. In this sense the bundling choice creates a common resource and each program supplier makes a claim

negotiating parties choose the outcome that maximizes total surplus.[2]

Bundling or unbundling programs to enhance bargaining position between cable operators and content providers is not uncommon. For example, in 2009, Comcast, a cable operator, added the NFL Network to a large bundle of channels to prevent an increase in its per-subscriber transfer to the content provider NFL Network.[3] Moreover, efforts to exploit bargaining power to extract a greater share of surplus during negotiations are routine. In fact, 3 million Cablevision customers in the New York area missed the first ten minutes of the Academy Awards broadcast in 2010 because the Walt Disney Company and Cablevision did not reach a timely agreement.[4]

The efforts of O'Brien and Shaffeer (2005), Milliou et. al. (2009), and Crawford and Yurukoglu (2010) are related to our analysis. O'Brien and Shaffer analyze how a merger between upstream producers impacts downstream prices and social welfare, conditional upon bargaining power and bundling of upstream products. Their main finding is that merger is efficient and consumer welfare is unaffected when the merged firm can bundle. They note, however, that downstream prices may increase when the merged firm's bargaining power is high and it cannot bundle. In contrast to our model, O'Brien and Shaffer explore bundling by an upstream firm while we explore bundling by a downstream firm. Moreover, their paper focuses on traditional (non-information) goods. Milliou et. al. explore upstream and downstream firms bargaining over both contract type (linear pricing, two-part tariff, price-quantity bundle) and terms of trade. In their model,

on the surplus. If the program suppliers have a strong bargaining position they will claim so much of the bundled surplus that the cable operator will prefer getting a larger portion of the unbundled surplus, even if that surplus is smaller.

[2]Generally, Nash bargaining provides a Pareto efficient outcome. We find that surplus may not be maximized since our bargaining outcome emerges in the context of multiple, uncoordinated bilateral negotiations. We assume that each negotiated outcome is not observed by the other bargaining parties so that it is not possible to credibly offer a contract which is conditioned on all possible permutations of channel lineups and transfers.

[3]"Comcast Settles Dispute With NFL Network," CBSNews.com, published online May 19, 2009.

[4]Brian Stelter and Brooks Barnes, "WABC Returns to Cablevision Programming," *The New York Times*, published online, March 7, 2010.

changes in bargaining power affect outcomes through both the terms of trade and the contract type. They find that bargaining can lead, under certain conditions, to a choice of contract type that does not maximize producer surplus. Crawford and Yurukoglu empirically estimate the effects of à la carte on consumer welfare via simulations of a structural model and find that à la carte may not be welfare-enhancing. Unlike our model, where packaging decisions and transfers are simultaneously determined, Crawford and Yurukoglu assume the cable operator chooses bundling after transfers have been determined.[5] Their empirical findings are consistent with our theoretical predictions in the case where a cable operator has high bargaining power but low advertising revenue rates.

In the next section, we present our model, using very general demand and cost functions. This framework is a useful contribution to a literature that typically relies on very specific assumptions and functional forms regarding demand and cost parameters, and our baseline model presents a flexible framework for empirical work. After we present the core model, we explore the implications of bundling programs on consumer and social welfare under a conventional utility framework. Our initial qualitative results remain unchanged.

2 Model and Results

2.1 Model

In our model, there are n program suppliers (upstream firms), indexed by $i = 1, ..., n$, and a single cable operator (downstream firm) that purchases programs from suppliers

[5] While the contracts between cable operators and program providers are almost always private, Wiley and Stansbury (2010) offer a window into the experience of Belo Corporation, a large U. S. program supplier. Belo Corporation's negotiations with cable operators involved financial elements along with non-monetary considerations such as program rebroadcast quality as well as channel and tier placement. This implies that the financial terms of program carriage are coordinated with the bundling outcome.

and provides programming to consumers in its franchise area.[6] Let Z denote the set of all programs, i.e., $Z = \{1, 2, ..., n\}$.

The model is structured as a two-stage game. In stage one, the cable operator identifies a subset K ($K \subset Z$) of programs that it would like to bundle. In stage two, the cable operator and network providers engage in simultaneous bilateral negotiations. If network provider j belongs to subset K, it has three choices: (a) accept the cable operator's offer to be included in the bundle and negotiate a transfer payment T_j to be paid from the cable operator to the network provider, (b) reject the cable operator's offer to be included in the bundle and negotiate a transfer payment for its program that is offered *à la carte*, (c) reject the cable operator's bundling offer and do not sell the program to the cable operator *à la carte*, i.e., the network provider walks away from the negotiations.[7] If network provider j does not belong to subset K, it has two choices: (a) negotiate a transfer payment for its program that is offered *à la carte*, or, (b) do not sell its program to the cable operator. An *equilibrium* here denotes a subgame perfect equilibrium for the stage game. Thus, in equilibrium, each network provider's decision maximizes its profit (keeping K fixed) and the cable operator's choice of K maximizes its profit. Graphical representation of the game is presented Figure 1.

We assume that a transfer payment denotes the total expected value of the revenue received by a program supplier[8] and that the amount of a transfer payment is determined through an asymmetric Nash Bargaining solution, which will be subsequently discussed.[9]

[6]The television subscriber market has become more competitive since the entry of satellite broadcasters and telecommunications firms. Chu (2010) presents evidence that there is heterogeneity in the impact of satellite providers on cable operators. In some markets satellite entry increases prices and quality while in others prices and quality fall. Given the difficulty associated with introducing satellite competition in our model we chose to leave exploration of a more competitive downstream market to future research and focus on the monopoly case.

[7]If the operator proposes a bundle K and provider j does not agree then the operator offers the bundle K_{-j}.

[8]The transfer payment can be made in terms of direct payments, a share of advertising time/revenue, or a combination of both direct payments and advertising time.

[9]We do not employ the Shapley value solution because the Shapley value is designed for cooperative games; our approach is non-cooperative. That said, some recent research suggests that the Shapley value

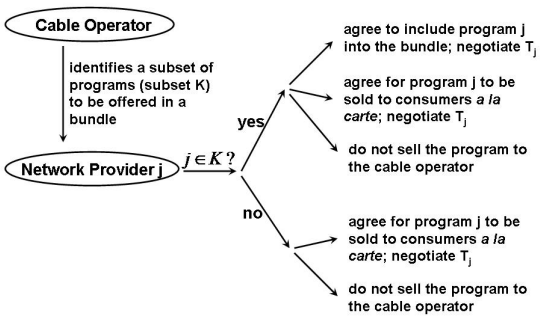

Figure 1: The Timing of Decisions.

Let $U = \{K, S\}$ denote the programming package offered by the cable operator, where K is the set of programs that are bundled and S is the set of programs offered to consumers *à la carte*.[10] The total revenue generated from this programming package is $R(U)$, which denotes the sum of total advertising and subscription revenue from offering programming package U. (Section 3 explains how this revenue function can be obtained from consumer demand functions.) Further, let i denote supplier *i's* program that is included in programming package U. Then, the marginal revenue generated from including supplier *i's* program in programming package U is denoted by $R(U) - R(U_{-i})$, where U_{-i} is a programming package that does not include supplier *i's* program but includes all other programs that were in programming package U while keeping bundled programs still bundled and unbundled programs unbundled.

The cable operator's cost of providing programming package U is denoted by $\gamma(U)$

may approximate the non-cooperative outcome (Gul (2006) and Harsanyi (1985)). We doubt, however, that this is the likely case for the cable industry. First, it is unlikely in the cable industry that a 50-50 bargaining solution obtains. Second, individual program producers/channels do not know the payment amounts designated for other producers/channels. In short, the Shapley value solution concept is ill-equipped, relative to the approach taken here, to accommodate important strategic interactions within the cable industry.

[10]For simplicity, we omit the possibility of mixed bundling.

and the cable operator's marginal cost of including program i into programming package U is denoted by $\gamma(U) - \gamma(U_{-i})$. Similarly, supplier *i's* cost of producing program i is denoted by c_i. For simplicity, it is assumed that c_i is specific to firm i and does not depend upon the whole programming package. Without loss of generality, the cable operator's cost of not providing any programs and a supplier's cost of not producing a program are normalized to zero. In other words, the cable operator's and supplier's costs here denote incremental costs, not absolute costs.

Total producer surplus created from programming package U is defined as the difference between total revenue and total costs of the cable operator and program suppliers. This surplus can be expressed as $v(U) = R(U) - \gamma(U) - \sum c_i$. Then, the marginal surplus created from including program i into programming package U is $v(U) - v(U_{-i})$. Further, the cable operator's profit from programming package U is $\pi(U) = R(U) - \gamma(U) - \sum_{i \in U} T_i$ and supplier *i's* profit is $\pi_i = T_i - c_i$.

It is assumed that the transfer payment from negotiations between a program supplier and the cable operator is determined by the asymmetric Nash Bargaining solution. Specifically, it is assumed that the cable operator keeps the share α_i of the marginal surplus created from including program i into programming package U. Supplier i keeps the remaining $1 - \alpha_i$ share of the marginal surplus. In other words, $\alpha_i \in (0, 1)$ is the cable operator's bargaining power when negotiating with supplier i.[11] Correspondingly, supplier *i's* bargaining power is $\beta_i = 1 - \alpha_i$.

Further, we adopt the following assumptions.

Assumption 1 [More is better]: Let K denote the set of programs that are bundled and S denote the set of programs that are unbundled. Then, $v(K, S) > v(K_{-i}, S)$ if $i \in K$ and $v(K, S) > v(K, S_{-i})$ if $i \in S$.

Assumption 2 [Independence of unbundled programs]: Let K and K^* denote the sets of programs that are bundled and S and S^* denote the

[11] Given the novelty of our model we don't have corresponding empirical estimates of these parameters. Future research might seek to obtain estimates of these parameters in order to establish which of the model's predictions are most relevant for a particular market.

sets of programs that are unbundled. Then, $v(K,S)-v(K_{-i},S) = v(K,S^*)-v(K_{-i},S^*)$ for $i \in K$ and $v(K,S) - v(K,S_{-j}) = v(K^*,S^*) - v(K^*,S^*_{-j})$ for $j \in S \cap S^*$.

Assumption 3 [Convexity]: Let K denote the set of programs that are bundled and S denote the set of programs that are unbundled. For any subset P of S, if $v(K \cup P, S \cap \neg P)$ denotes total producer surplus when the cable operator adds P to the bundle, then $v(K \cup P, S \cap \neg P) > v(K,S)$.

The first assumption states that adding new programs into a programming package, while keeping the cable operator's program packaging decision for other programs unchanged, increases total producer surplus. One can interpret this assumption as a "more is better" assumption, or, equivalently, that the analysis is restricted to the program suppliers that generate positive levels of marginal total producer surplus. The second assumption implies that the value of the channel provided *à la carte* is not affected by the cable operator's decisions to bundle other channels.[12] In addition, this assumption implies that the value of a bundled channel is not altered when some new channel is provided *à la carte*. However, if a new channel is added to the bundle, the marginal surplus generated from other channels in the bundle could be affected. We also note that the second assumption implies that $v(K,S) = v(K) + \sum_{j \in S} v(j)$, i.e., total surplus equals the surplus generated from the bundle, plus the sum of individual surpluses from programs not in the bundle. This observation is useful when calculating marginal surplus levels and the cable operator's profit levels under different program packages. The third assumption states that bundling increases total producer surplus. Adams and Yellen (1976), McAfee et al. (1989), Salinger (1995), and Schmalensee (1984) describe how bundling could increase total producer surplus and Bakos and Brynjolfsson (1999 and 2000) describe why bundling information goods increases total surplus. In Section 3, we describe how this result can be derived endogenously from consumer and producer

[12] A valuable future extension might explore how our model's results change when unbundled programs are less independent.

choices.

2.2 Bargaining Power and Producer Surplus

This subsection considers the relationship between the cable operator's bargaining power and the cable operator's choice to bundle or unbundle programs. Claim 1 below implies that an equilibrium exists when assumptions 1-3 hold.

Claim 1. If assumptions 1-3 hold and $\alpha_p \in (0,1)$ for all suppliers, then there exists an equilibrium. In equilibrium, if K denotes the set of bundled programs and S denotes the set of unbundled programs, the transfer payment T_i to supplier i is $T_i = (1-\alpha_i)(v(K) - v(K_{-i})) + c_i$ for $i \in K$ and $T_i = (1-\alpha_i)v(i) + c_i$ for $i \in S$. The cable operator's profit in equilibrium is $v(K) - \sum_{i \in K}(1-\alpha_i)(v(K) - v(K_{-i})) + \sum_{j \in S} \alpha_j v(j)$ and supplier p's profit is $T_p - c_p$.

Proof of Claim 1. Given in the Appendix.

First, consider the case with two program suppliers.

Proposition 1. Suppose $n = 2$ and that assumptions 1-3 hold. Then, total producer surplus is maximized in equilibrium if $\alpha_1 + \alpha_2 > 1$ and total producer surplus is not maximized in equilibrium if $\alpha_1 + \alpha_2 < 1$.

Proof of Proposition 1. Suppose $n = 2$ and that assumptions 1-3 hold. This implies that total producer surplus is maximized if the programs are bundled. The cable provider strictly prefers to bundle the two programs in equilibrium ($K = \{1,2\}$) to unbundling ($K = \emptyset$) if $\pi(K = \{1,2\}) > \pi(K = \emptyset)$. This condition can be re-written as:

$$v(\{1,2\}^{bundle}) - (1-\alpha_1)(v(\{1,2\}^{bundle}) - v(2)) - (1-\alpha_2)(v(\{1,2\}^{bundle}) - v(1)) >$$

$$\alpha_1 v(1) + \alpha_2 v(2)$$

$$\Leftrightarrow (v(\{1,2\}^{bundle}) - v(1) - v(2))(\alpha_1 + \alpha_2 - 1) > 0 \qquad (1)$$

Assumption 3 implies that $(v(\{1,2\}^{bundle}) - v(1) - v(2)) > 0$. Then, inequality (1) holds if $\alpha_1 + \alpha_2 > 1$. Therefore, the cable operator chooses to bundle programs ($K = \{1,2\}$) and thus maximizes producer surplus when $\alpha_1 + \alpha_2 > 1$. On the other hand, the cable operator chooses to provide programs *à la carte* ($K = \emptyset$) if $\alpha_1 + \alpha_2 < 1$, and thus does not maximize producer surplus. ■

Proposition 1 implies that when the cable operator's bargaining power is high enough, total producer surplus is maximized in equilibrium. Maximizing total producer surplus does not necessarily imply the socially optimal level of surplus because it does not guarantee that the cable operator's pricing policy is socially optimal. The effect of bundling on consumer surplus is discussed in Section 3.

The intuition behind Proposition 1 is that greater bargaining power implies the cable operator keeps a larger portion of producer surplus, and thus, the cable operator's incentives are more aligned with an incentive to maximize producer surplus. Note that the cable operator also has an incentive to minimize the sum of infra-marginal surplus from program suppliers in order to minimize transfer payments to suppliers. Thus, the cable operator chooses to bundle or to unbundle programs by striking a balance between the incentives to maximize surplus and to minimize transfer payments. Those payments represent each program supplier's claim of a portion of the total surplus created when it joins the bundle. In this sense the bundling choice creates a common resource and each program supplier makes a claim on the surplus. If the program suppliers have a strong bargaining position they will claim such a high portion of the bundled surplus that the cable operator will prefer bargaining over the a la carte surplus, even if that surplus is smaller. There is a coordination problem among the program suppliers since multiple bilateral negotiations tend to dissipate the bundled surplus.

The following example further illustrates this intuition.

Example 1. Suppose $n = 2$, $v(\{1,2\}^{bundle}) = 5$, $v(1) = 2$, $v(2) = 2$,

and $\alpha_1 = \alpha_2 = 1/3$. If assumptions 1-3 hold, bundling maximizes total producer surplus. The cable operator's profit from bundling programs is $\pi(\{1,2\}^{bundle}) = 5 - (2(5-2)/3 + 2(5-2)/3) = 1$. The cable operator's profit from unbundling programs is $\pi(K = \emptyset) = (1/3)(2) + (1/3)(2) = 4/3$. Under these conditions, the cable operator prefers to unbundle programs even though producer surplus is maximized by bundling programs. The reason is as follows. If the cable operator decides to bundle programs instead of providing programs *à la carte*, total producer surplus increases by one unit $(5 - 4 = 1)$. At the same time, each seller's marginal contribution to producer surplus goes up by one unit as well. Since the cable operator keeps only one-third of sellers' marginal contributions, the cable operator's transfer payment to each seller increases by 2/3. Thus, bundling decreases the cable operator's profit by $2/3 + 2/3 - 1 = 1/3$.

Now, suppose that $\alpha_1 = \alpha_2 = 2/3$ so that condition $\alpha_1 + \alpha_2 > 1$ is satisfied. Then, the cable operator prefers to bundle programs and thus maximizes total producer surplus. The reason is as follows. If the cable operator decides to bundle programs instead of providing programs *à la carte*, total producer surplus increases by one unit $(5 - 4 = 1)$. At the same time, each seller's marginal contribution to producer surplus goes up by one unit as well. Since the cable operator keeps two-thirds of sellers' marginal contributions, the cable operator's transfer payment to each seller increases only by 1/3. Therefore, bundling increases the cable operator's profit by $1 - 1/3 - 1/3 = 1/3$. Thus, greater bargaining power increases the cable operator's incentive to maximize total producer surplus because the cable operator keeps a higher portion of the surplus.

A two supplier case is a special case because the cable operator's disagreement surplus is the same whether the cable operator bundles or unbundles programs, i.e., $v(\{1,2\}_{-i}^{bundle}) = v(\{1,2\}_{-i}^{unbundle})$. Propositions 2 and 3 describe how total surplus is

affected by bargaining power for an arbitrary number of program suppliers. Specifically, Proposition 2 describes conditions when total producer surplus is not maximized in equilibrium and Proposition 3 describes conditions when total producer surplus is maximized in equilibrium. For $n > 2$, the cable operator's decision to bundle or unbundle programs depends not only on its bargaining power but also on the marginal surplus values generated by different network programs. Thus, a generalization of Proposition 1 stating that total producer surplus is maximized (not maximized) when the sum of bargaining power is greater (lower) than a fixed cutoff value q no longer applies. However, the intuitive result that total surplus is maximized (not maximized) when bargaining power is high (low) enough still holds. What constitutes "high enough" or "low enough" varies depending on the distribution of bargaining power and marginal surplus values.

Proposition 2. Suppose $n \geq 2$, assumptions 1-3 hold, and $\alpha_i \in (0,1)$ for all i. Then, there exists $\alpha^* \in (0,1)$ such that total producer surplus is not maximized in equilibrium if $\alpha_j < \alpha^*$ for some j for given α_i's, $i \neq j$.

Proof of Proposition 2. The proof is by induction. Let $n = 2$. Then, the proposition holds if we select $\alpha^* = 1 - \alpha_{-j}$. Assume the proposition holds for $n = k$. Now, we need to show that it holds for $n = k + 1$.

Let $n = k + 1$ and without loss of generality assume that $j = k + 1$. We need to show that total producer surplus is not maximized when α_{k+1} is small enough. Let $Y = \{1, ..., k, k + 1\}$ denote the programming package when $n = k + 1$ and all programs are bundled, and let $X = \{1, ..., k\}$ denote the programming package when $n = k$ and first k programs are bundled. Then, the necessary condition for the cable operator to bundle all programs in equilibrium, and thus to maximize total surplus, is $\pi(K = Y) \geq \pi(K = X, S = \{k + 1\})$. This condition can be rewritten as:

$$v(Y) - \sum_{i=1}^{k+1}(1-\alpha_i)(v(Y) - v(Y_{-i})) \geq v(X) + \alpha_{k+1}v(k+1) - \sum_{i=1}^{k}(1-\alpha_i)(v(X) - v(X_{-i}))$$

$$\Leftrightarrow v(Y) - (1 - \alpha_{k+1})(v(Y) - v(X)) - \alpha_{k+1}v(k + 1) - v(X) \geq$$

$$\sum_{i=1}^{k}(1-\alpha_i)((v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i})))$$

The condition given above simplifies to:

$$\alpha_{k+1}(v(Y)-v(X)-v(k+1)) \geq \sum_{i=1}^{k}(1-\alpha_i)((v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i}))) \tag{2}$$

Assumption 3 implies that $v(Y)-v(X)-v(k+1)>0$ and that $(v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i}))>0$. Therefore, condition (2) does not hold if $\alpha_{k+1}<\alpha^*$, where $\alpha^*=\frac{\sum_{i=1}^{k}(1-\alpha_i)((v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i})))}{v(Y)-v(X)-v(k+1)}$. ■

Proposition 3. Suppose $n \geq 2$ and assumptions 1-3 hold. Then, there exist α_i^*'s such that total producer surplus is maximized in equilibrium if $\alpha_i > \alpha_i^*$ for $i = 1, 2, ..., n$.

Proof of Proposition 3. The proof is by induction. Let $n = 2$. Then, the proposition holds if we choose $\alpha_i^* = 0.5$. Assume the proposition holds for $n = k$. Now, we need to show that it holds for $n = k + 1$, i.e., we need to show that the cable operator decides to bundle all programs for $n = k + 1$.

Let $n = k+1$. Because the proposition holds for $n = k$, the cable operator's profit is maximized either when there are k or $k+1$ programs in the bundle. Then, the cable operator adds program $k + 1$ into the bundle instead of selling it *à la carte* if it increases the cable operator's profit. This condition is described below.

$$v(Y)-\sum_{i=1}^{k+1}(1-\alpha_i)(v(Y)-v(Y_{-i})) > v(X)+\alpha_{k+1}v(k+1)-\sum_{i=1}^{k}(1-\alpha_i)(v(X)-v(X_{-i})) \tag{3}$$

In condition (3), $X = \{1, ..., k\}$ denotes the programming package when $n = k$ and first k programs are bundled, and $Y = \{1, ..., k, k + 1\}$ denotes the programming package when $n = k + 1$ and all programs are bundled.

Condition (3) above further simplifies to:

$$\alpha_{k+1}(v(Y)-v(X)-v(\{k+1\})) > \sum_{i=1}^{k}(1-\alpha_i)((v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i}))) \tag{4}$$

Assumption 3 implies that $v(Y)-v(X)-v(k+1) > 0$ and $(v(Y)-v(Y_{-i}))-(v(X)-v(X_{-i})) > 0$. Then, condition (4) holds when $\alpha_i \to 1$ for all i because the left-hand side approaches a positive number and the right-hand side approaches zero as the α_i's approach one. Because both sides of the inequality are continuous in the α_i's, there exist α_i^*'s such that (4) holds if $\alpha_i > \alpha_i^*$ for all i. ■

As noted earlier, the intuition behind Propositions 2 and 3 is similar to the intuition in Proposition 1: greater (lesser) bargaining power on the part of the cable operator implies that the cable operator is more (less) likely to make a program packaging decision that maximizes total producer surplus.

In the next section, we extend the analysis, and introduce consumers and advertising revenue into the model. Unlike the previous sections, we assume certain functional forms to carry out the analysis, and we employ standard forms commonly used in analyzing the industry. Using these standard forms, we show conditions under which the interplay of bargaining power and advertising revenue determine both consumer welfare and social welfare.

3 Model Extension: Incorporating Consumer Behavior

The revenue function described in Section 2 is derived from demand for programming and demand for advertising. Specifically, the revenue function when the cable operator provides n programs separately is:

$$R_{Separate} = \sum_{i=1}^{n}(q_i(\bar{p}^*)p_i^* + A_i(q_i(\bar{p}^*))) \tag{5}$$

In (5), $q_i(.)$ is a consumer demand function for program i, $A_i(.)$ is an advertising revenue function for program i, and $\bar{p}^*$ is a vector of prices that maximize the cable operator's profit. Specifically, $\bar{p}^*$ maximizes the marginal surplus from each program and is derived from the optimization problem:

$$\bar{p}^* = argmax[\sum_{i=1}^{n}(q_i(\bar{p})p_i + A_i(q_i(\bar{p})) - c_i) - \gamma(q_1(\bar{p}), ..., q_n(\bar{p}))] \tag{6}$$

As noted in Section 2, c_i is supplier i's cost of producing program i and $\gamma(.)$ is a cable operator's cost function. Assuming there is an interior solution, the first-order conditions are:

$$q_i(\bar{p}) + \frac{\partial q_i(\bar{p})}{\partial p_i}p_i + \frac{\partial A_i}{\partial q_i}\frac{\partial q_i(\bar{p})}{\partial p_i} - \sum_{j=1}^{n}\gamma_j(q_1(\bar{p}), ..., q_n(\bar{p}))\frac{\partial q_j(\bar{p})}{\partial p_i} = 0 \quad for \quad i = 1, ..., n \tag{7}$$

In (7), the first two terms denote the marginal subscription revenue from increasing the price of program i, the third term denotes the marginal advertising revenue, and the last term denotes the marginal cost.

The revenue function and optimal price levels when the cable operator bundles n programs are:[13]

$$R_{Bundle} = q_B(p_B^*)p_B^* + A_B(q_B(p_B^*)) \tag{8}$$

$$p_B^* = argmax[\, q_B(p_B)p_B + A_B(q_B(p_B)) - \sum_{j=1}^{n} c_i - \gamma(q_B(p_B), ..., q_B(p_B))] \tag{9}$$

Assuming there is an interior solution, the first-order conditions are:

$$q_B(p_B) + \frac{\partial q_B(p_B)}{\partial p_B}p_B + \frac{\partial A_B}{\partial q_B}\frac{\partial q_B(p_B)}{\partial p_B} - \sum_{j=1}^{n}\gamma_j(q_1(p_B), ..., q_n(p_B))\frac{\partial q_B(p_B)}{\partial p_B} = 0 \tag{10}$$

In (10), the first two terms denote the marginal subscription revenue from increasing the price of the bundle, the third term denotes the marginal advertising revenue, and the last term denotes the marginal cost. Consumer surplus levels when programs are unbundled and bundled are calculated as:

$$CS^{Separate} = \sum_{i=1}^{n}\int_{p_i^*}^{\infty} q_i(p_1^*, ..., p_{i-1}^*, p_i, p_{i+1}^*, ..., p_n^*)dp_i \tag{11}$$

[13] We use subscript 'B' to denote quantity and price levels under bundling.

$$CS^{Bundle} = \int_{p_B^*}^{\infty} q_B(p_B)dp_B \tag{12}$$

Without further assumptions regarding consumer demand and advertising revenue functions, we cannot determine which pricing system is preferred by consumers.[14] To proceed then, we adopt conventional functional forms from the bundling literature.[15] Note that our approach is distinguished from these earlier efforts since we incorporate advertising revenue and bargaining power.

Suppose there are two network suppliers. The consumer chooses among three products: product (program) 1 that is produced by supplier 1, product 2 that is produced by supplier 2, and an outside good M (a numeraire). The consumer's indirect utility function is:

$$U = x_1 D_1 + x_2 D_2 + M \tag{13}$$

In (13), x_i is a random variable that measures a consumer's preference (valuation) for product i, where x_i is distributed uniformly on $[0, z]$.[16] It is assumed that consumers' valuations of the two goods are independent. D_j is an indicator variable that equals one if a consumer buys product j and zero otherwise. M is the quantity of the outside good.

Suppose the cable operator decides to sell the two programs *à la carte*. Let r denote per subscriber per channel advertising revenue. We assume that r is exogenously determined in a competitive advertising market, and that $(r < z)$. Let c_i denote supplier i's marginal cost of producing product i and γ_i denote the cable operator's marginal cost of providing product i. We assume that the c_i's and γ_i's are sufficiently low, such that assumption 1 holds. Then, the profit maximizing price levels solve the following

[14] The results presented earlier in the main text, however, are not sensitive to the choice of functional forms.

[15] See for example, Adams and Yellen (1976), Bakos and Brynjolfsson (1999 and 2000), McAfee et al. (1989), and Salinger (1995).

[16] The assumption of independent preferences is used for tractability purposes. Correlated preferences would be a valuable future extension of the current model.

optimization problem:[17]

$$(p_1, p_2) = argmax \sum_i (R_i - \gamma_i - T_i)$$

$$\Rightarrow (p_1, p_2) = argmax \sum_i q_i(p_i + r)$$

$$\Rightarrow (p_1, p_2) = argmax \sum_i (\frac{z - p_i}{z})(p_i + r) \tag{14}$$

The first-order conditions and profit-maximizing price levels are:[18]

$$\frac{1}{z}(z - 2p_i - r) = 0 \tag{15}$$

$$\Rightarrow p_i = \frac{z - r}{2} \tag{16}$$

Next, consider profit-maximizing price levels when the cable operator bundles the two programs.[19]

$$p_B = argmax(R_B - \sum_i (\gamma_i + T_i))$$

$$\Rightarrow p_B = argmax(q_B)(p_B + 2r) \tag{17}$$

In (17), $q_B = (2z - p_B)^2/(2z^2)$ if $2z > p_B > z$ and $q_B = (z^2 - p_B^2/2)/(z^2)$ if $p_B \leq z$. In equilibrium we should have $p_B \leq z$.[20]

[17] γ_i's do not vary with q_i's and T_i's are taken as fixed when choosing optimal price levels.

[18] The first-order conditions are necessary and sufficient because the second order condition is satisfied:

$$\frac{\partial^2 R_i}{\partial p_i^2} = -\frac{2}{z} < 0$$

[19] Again, γ_i's do not vary with q_i's and T_i's are taken as fixed when choosing optimal price level.

[20] If $p_B > z$, then p_B solves:

$$\max_{p_B} \frac{(2z - p_B)^2}{2z^2}(p_B + 2r) \tag{18}$$

The first derivative of (18) is:

$$\frac{dR_B}{dp_B} = \frac{1}{2z^2}((2z - p_B)^2 - 2(2z - p_B)(p_B + 2r)) =$$

$$= \frac{1}{2z^2}(2z - p_B)(2z - 4r - 3p_B) \tag{19}$$

Note that $\frac{dR_B}{dp_B} < 0$ because $2z - p_B > 0$ and $2z - 4r - 3p_B < 0$. Thus, $p_B > z$ cannot be the case in equilibrium, because the value function is decreasing in p_B.

Given $p_B \leq z$, we calculate the optimal p_B:

$$\max_{p_B} \frac{z^2 - p_B^2/2}{z^2}(p_B + 2r) \tag{20}$$

The first-order condition implies that:[21]

$$\frac{1}{2z^2}(2z^2 - p_B^2 - 2p_B^2 - 4p_B r) = 0 \tag{21}$$

$$\Rightarrow p_B = \frac{\sqrt{4r^2 + 6z^2} - 2r}{3} \tag{22}$$

Based on the functional forms we have adopted, it is clear that Assumptions 1 and 2 hold. Next, we show that Assumption 3 holds, and can be derived endogenously from traditional functional forms used in the bundling literature.

Claim 2. Under the assumptions and functional forms described above, Assumption 3 holds.

Proof of Claim 2. Given in the appendix.

Proposition 4 describes conditions under which consumer prices and consumer surplus values are determined under bundling.

Proposition 4. Under the assumptions and functional forms described above:

(a) $p_B > p_1 + p_2$ and $CS^B < CS_1 + CS_2$ if $r \in ((\sqrt{2} - 1)z, z)$,

(b) $p_B = p_1 + p_2$ and $CS^B < CS_1 + CS_2$ if $r = (\sqrt{2} - 1)z$,

(c) $p_B < p_1 + p_2$ if $r \in [0, (\sqrt{2} - 1)z)$,

(d) there exists $r^* \in [0, (\sqrt{2} - 1)z)$ such that $p_B < p_1 + p_2$ and $CS^B > CS_1 + CS_2$ if $r < r^*$.

Proof of Proposition 4. Given in the appendix.

[21] The first-order condition is necessary and sufficient because $\frac{\partial^2 R_B}{\partial p_B^2} = \frac{1}{2z^2}(-6p_B - 4r) < 0$.

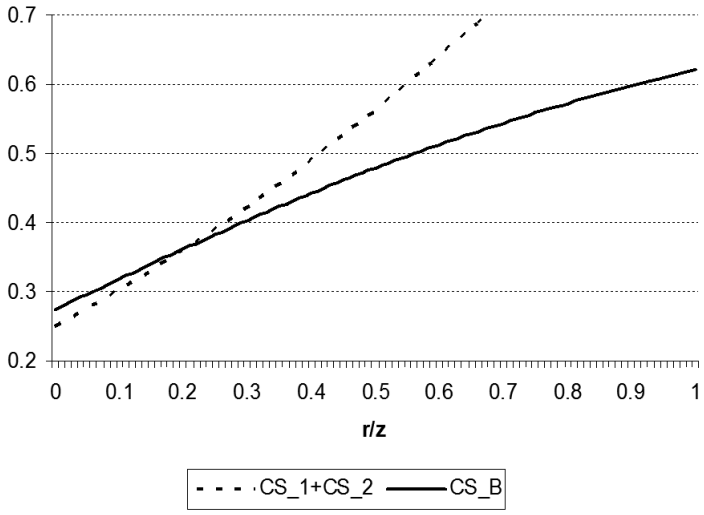

Figure 2: Consumer surplus levels as a function of r.

Proposition 4 implies that the average price per program is lower, and consumer surplus higher, under *à la carte* (bundling) pricing when r is sufficiently high (low), i.e., when a larger (smaller) share of revenue comes from advertising.[22] Figure 2 presents the results of numerical simulation of the model. The upper cutoff value of parameter r for consumer surplus to be greater under bundling is approximately $0.21z$.

Proposition 5 considers how bundling affects social welfare, i.e., the sum of producer and consumer surplus.

Proposition 5. Under the assumptions and functional forms described above:

(a) there exists $r^* > 0$ such that social welfare is maximized in equilibrium

[22] Consumer surplus is higher under *à la carte* pricing if the average price per program is not higher than the average price per program under bundling. The reasoning is that if the average price per program is the same under both systems, consumers are better off with *à la carte* pricing because consumers can still afford to buy the bundle, but consumers still have a choice of buying just one program instead of the bundle.

if $\alpha_1 + \alpha_2 > 1$ and $r \in [0, r^*)$,

(b) there exists $r^* > 0$ such that social welfare is not maximized in equilibrium if $\alpha_1 + \alpha_2 < 1$ and $r \in [0, r^*)$,

(c) there exists $r^* > 0$ such that social welfare is not maximized in equilibrium if $\alpha_1 + \alpha_2 > 1$ and $r \in (r^*, z]$,

(d) there exists $r^* > 0$ such that social welfare is maximized in equilibrium if $\alpha_1 + \alpha_2 < 1$ and $r \in (r^*, z]$.

Proof of Proposition 5. Given in the appendix.

In Proposition 5, maximizing social welfare does not refer to the first-best outcome achieved under the social planner. Rather, it refers to the second-best outcome in which social welfare is higher in equilibrium as compared to the alternative equilibrium that could have resulted from the negotiations between the cable operator and program suppliers. The intuition behind Proposition 5 is that the cable operator's decision to bundle or unbundle programs depends on its bargaining power when negotiating with network providers, while social welfare levels under bundling and unbundling depend on advertising rates. Thus, depending on the values of bargaining power and advertising rates, social welfare may or may not be maximized in equilibrium.

While we cannot analytically calculate the cutoff value of r when social welfare is higher under bundling or *à la carte* pricing, we provide a numerical approximation. The result, shown in Figure 3, suggests that social welfare is higher (lower) under bundling for low (high) values of r. Social welfare levels are equal under bundling and *à la carte* pricing when the parameter r is approximately $0.775z$.

4 Conclusion

Cable operators choose to bundle or unbundle programs by striking a balance between the incentives to maximize surplus and minimize transfer payments. Greater bargaining power on the part of a cable operator implies that a cable operator is more likely to make

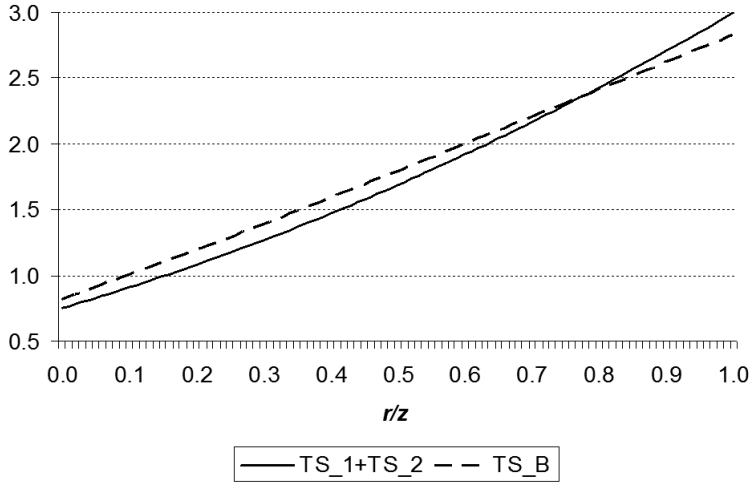

Figure 3: Social welfare levels as a function of r.

a program packaging decision that maximizes total producer surplus. Regulatory efforts to limit the market share of cable operators or force cable operators to provide programs *à la carte* may reduce total producer surplus. Moreover, absent offsetting increases in consumer welfare, such policy measures may reduce total welfare. This relationship between bargaining power and the effects of *à la carte* regulation has not previously been investigated in the bundling literature.

As a policy matter, our results imply that a monopolist does not necessarily increase deadweight loss, and under certain circumstances a monopolist's bargaining outcomes will yield higher social welfare. From Section 3 we note that when advertising revenue is low, a monopolist with sufficient bargaining power chooses to bundle programming, which limits increases in program prices, increases the subscriber base, and increases social welfare relative to *à la carte*.[23] Conversely, if advertising revenue is high, a mo-

[23]Recall from equations 16 and 22 that P_b is concave in advertising, while P_a is linear. Since bundling increases the subscriber base a given subscription price change has a large impact on revenues, relative to a la carte. For this reason advertising prices have a smaller (and inverse) impact on subscription prices under bundling.

nopolist with limited bargaining power chooses a la carte programming, as profitable advertising generates a "cross-subsidy" under which program prices drop, increasing the subscriber base and social welfare relative to bundling. Our findings suggest some lessons for policymakers: (1) a monopoly market structure is not a sufficient condition for social welfare losses, and (2) regulatory efforts to circumscribe monopoly bargaining power should account for advertising markets.

We emphasize, again, that our baseline results are obtained under general assumptions regarding demand and cost functions. This is an important contribution to a literature that often relies on specific assumptions and functional forms regarding demand and cost parameters when analyzing the implications of regulation. Thus, the baseline model provides a rich and flexible framework for empirical researchers to estimate the magnitudes of various effects that influence bundling. When we augment our analysis with conventional consumer utility functions from the bundling literature our results are qualitatively unchanged. As a technical matter, we employ Assumptions 2 and 3 in our model for tractability; the qualitative results of the model still hold when these assumptions are relaxed. Suppose, for example, that Assumption 3 does not hold and that some programs increase total surplus when bundled, while others do not. If we restrict the cable operator's decision to bundle to the subset of programs that increase total surplus, the model results apply directly and the relationship between bargaining power and the decision to bundle still obtains.

Our model assumes that firms engage in bilateral, rather than multilateral, negotiations. This assumption is supported by empirical observation since contracts between cable operators and program suppliers are private and non-observable by other parties. Furthermore, multilateral negotiations, in this case, might violate antitrust regulations. Therefore, we believe that our modeling approach is reasonable. Multilateral negotiations would have complex implications for our results. On the one hand, multilateral bargaining tends to maximize joint surplus, which could alter our findings. On the other hand, modeling multilateral negotiations is difficult and equilibrium could depend

on a specific modeling choice.[24] For these reasons prior research has suggested that multilateral bargaining models should be tailored to specific real-life situations.[25]

While our paper reveals a previously unexplored aspect of bargaining power on bundling decisions, this complex topic provides numerous additional opportunities to explore cable operators' and program providers' decisions. One might, for example, extend the model to allow for mixed bundling. Another extension might analyze the effect of vertical mergers on bundling decisions.

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A Appendix

Proof of Claim 1. To show that an equilibrium exists, we need to show that every subgame has an equilibrium. First, we fix K and consider stage two of the game. Suppose supplier i is in the bundle. Then, if the supplier decides to remain in the bundle, the solution to the asymmetric Nash bargaining problem implies that the cable operator's share of the marginal producer surplus from including program i into bundle K is α_i and supplier *i's* share of the marginal producer surplus is $1-\alpha_i$. The marginal surplus from including program i into bundle K is $v(K)-v(K_{-i})$.[26] Thus, the cable operator keeps $\alpha_i(v(K)-v(K_{-i}))$ and supplier i keeps $(1-\alpha_i)(v(K)-v(K_{-i}))$. This implies that the transfer payment received by supplier i is $T_i = (1-\alpha_i)(v(K) - v(K_{-i})) + c_i$.[27] Supplier i's profit is $\pi_i(i \in K) = T_i - c_i = (1-\alpha_i)(v(K) - v(K_{-i}))$. If supplier i decides to sell its program separately instead of being in the bundle, marginal surplus from program i is $v(i)$. Then, the transfer

[26] Note that assumption 2 implies that $v(K,S) = v(K) + \sum_{j\in S} v(j)$.

[27] Explicitly, one can model the Nash bargaining outcome between supplier i and the cable operator as a solution to the following maximization problem (keeping the transfer payments from other suppliers fixed):

$$\max_{T_i} (\pi - d)^{\alpha_i} (\pi_i - d_i)^{1-\alpha_i} \tag{23}$$

where d and d_i are the disagreement profits of the cable operator and program supplier i. Because only the bargaining welfare gains in excess of the disagreement values are relevant, supplier i's disagreement profits can be normalized to zero. The cable operator's disagreement profit when negotiating with supplier i to include program i into the programming package U is $R(U_{-i}) - \gamma(U_{-i}) - \sum_{-i} T_k$. Then, the transfer payment to supplier i is $T_i = (1-\alpha_i)(R(U) - \gamma(U) - \sum_{-i} T_k - R(U_{-i}) + \gamma(U_{-i}) + \sum_{-i} T_k) + \alpha_i c_i = (1-\alpha_i)(R(U) - \gamma(U) - R(U_{-i}) + \gamma(U_{-i}) - \sum c_k + \sum_{-i} c_k) = (1-\alpha_i)(v(U) - v(U_{-i})) + c_i$.

payment to supplier i is $T_i = (1-\alpha_i)v(i) + c_i$ and the supplier's profit is $\pi_i(separate) = T_i - c_i = (1-\alpha_i)v(i)$. If supplier i chooses not to sell its program to the cable operator, its profit is zero. Assumption 1 implies that supplier i's profit is positive when supplier i sells its program to the cable operator as a part of the bundle. Furthermore, $v(K)-v(K_{-i}) > v(i)+v(\emptyset) = v(i)$ from assumption 3. Therefore, $\pi_i(i \in K) > \pi_i(separate)$ and supplier i accepts the cable operator's offer to be included in the bundle. Now consider supplier i's choice when $i \in S$. If supplier i chooses not to sell its program to the cable operator, its profit is zero. If the supplier sell's its program to the cable operator, it keeps share $1-\alpha_i$ of the marginal surplus $v(i)$. Thus, the transfer payment to supplier i is $T_i = (1-\alpha_i)v(i) + c_i$ and its profit is $(1-\alpha_i)v(i)$. Then, proposition 1 implies that the supplier is better off by selling its program to the cable operator. Thus, each supplier's choices and corresponding profit levels are well defined for all possible K and S and there exists a choice that maximizes the supplier's profit.

Next, consider stage one. For any K and S, the cable operator's profit is derived below.

$$\pi = R(U) - \gamma(U) - \sum_{p \in U} T_p = R(U) - \gamma(U) - \sum_{i \in K} T_i - \sum_{j \in S} T_j$$

$$= R(U) - \gamma(U) - \sum_{i \in K} c_i - \sum_{i \in K} (1-\alpha_i)(v(K) - v(K_{-i})) - \sum_{j \in S} c_j - \sum_{j \in S} (1-\alpha_j)v(j)$$

$$= v(U) - \sum_{i \in K} (1-\alpha_i)(v(K) - v(K_{-i})) - \sum_{j \in S} (1-\alpha_j)v(j)$$

$$= v(K) + \sum_{j \in S} v(j) - \sum_{i \in K} (1-\alpha_i)(v(K) - v(K_{-i})) - \sum_{j \in S} (1-\alpha_j)v(j)$$

$$= v(K) - \sum_{i \in K} (1-\alpha_i)(v(K) - v(K_{-i})) + \sum_{j \in S} \alpha_j v(j)$$

Because the cable operator has finite choices and its profit is well defined for each choice, there exists a programming package that maximizes the cable

operator's profit. Thus, we have shown that an equilibrium exists, as well as corresponding profit levels and transfer payments. ■

Proof of Claim 2. To prove the claim, we need to show that $v(\{1,2\}^{bundle}) > v(1) + v(2)$. $v(1) + v(2)$ is calculated as:

$$v(1) + v(2) = \sum_i (q_i(p_i + r) - \gamma_i - c_i) = \frac{1}{2z}(z + r)^2 - \sum_i (\gamma_i + c_i)$$

Since p_B maximizes total surplus from bundling for all possible price levels, $v(\{1,2\}^{bundle}) \geq v(\{1,2\}^{bundle}|p \neq p_B)$. Then,

$$\begin{aligned} v(\{1,2\}^{bundle}) &\geq v(\{1,2\}^{bundle}|p = z - r) = \\ &= \frac{1}{2z^2}(2z^2 - (z-r)^2)(z - r + 2r) - \sum_i (\gamma_i + c_i) = \\ &= \frac{(z+r)}{2z}\frac{(z^2 + zr + r(z-r))}{z} - \sum_i (\gamma_i + c_i) > \\ &> \frac{2(z+r)^2}{4z} - \sum_i (\gamma_i + c_i) = v(1) + v(2) \end{aligned} \tag{24}$$

Therefore, assumption 3 holds. ■

Proof of Proposition 4. First, to compare price levels under different programming packaging decisions, we compare monotonic transformations of prices. Let $f(x) = (3x + 2r)^2$. Then, $f(p_B) = 4r^2 + 6z^2$ and $f(p_1 + p_2) = 9z^2 + r^2 - 6zr$. Then, $p_B > p_1 + p_2$ if and only if $f(p_B) > f(p_1 + p_2)$. $f(p_B) > f(p_1 + p_2)$ if $r > (\sqrt{2} - 1)z$. Similarly, $f(p_B) < f(p_1 + p_2)$ if $0 \leq r < (\sqrt{2} - 1)z$, and $f(p_B) = f(p_1 + p_2)$ if $r = (\sqrt{2} - 1)z$. This completes the proof regarding price levels under different programming packaging decisions. Next, consider consumer surplus levels.

$$CS_1 + CS_2 = 2\int_{p_i}^{z} \frac{t - p_i}{z} dt = \frac{2}{z}(\frac{t^2}{2} - \frac{z-r}{2}t)|_{\frac{z-r}{2}}^{z} = \frac{(z+r)^2}{4z} \tag{25}$$

$$CS^B = z - p_B q_B - \int_0^{p_B} \int_0^{p_B - t_1} (\frac{t_1 + t_2}{z^2}) dt_2 dt_1 =$$

$$= z - p_B q_B - \frac{1}{z^2}\int_0^{p_B}(t_1(p_B - t_1) + \frac{(p_B - t_1)^2}{2})dt_1 =$$

$$= z - p_B q_B - \frac{1}{2z^2}\int_0^{p_B}(p_B^2 - t_1^2)dt_1 =$$

$$= z - p_B q_B - \frac{p_B^3}{3z^2} = z - p_B + \frac{p_B^3}{6z^2} \tag{26}$$

Because consumer surplus under bundling is decreasing in price, $CS^B \leq CS^B(p = z - r)$ when $r \geq (\sqrt{2} - 1)z$. Then,

$$CS_1 + CS_2 - CS^B \geq CS_1 + CS_2 - CS^B(p = z - r) =$$

$$= \frac{(z+r)^2}{4z} - \frac{z^3 + 3z^2 r + 3zr^2 - r^3}{6z^2} = \frac{(z-r)^2(z+2r)}{12z^2} > 0 \tag{27}$$

Therefore, $CS_1 + CS_2 > CS^B$ if $r \geq (\sqrt{2} - 1)z$.

Finally, consider part (d) of the proposition. If when $r = 0$, $CS_1 + CS_2 = 0.25z$ and $CS_B = z - p_B + \frac{p_B^3}{6z^2} \approx 0.274z$. Thus, $CS_1 + CS_2 < CS^B$ when $r = 0$. Because consumer surplus levels are continuous in r, there exists r^* such that $CS_1 + CS_2 < CS^B$ when $r < r^*$. ■

Proof of Proposition 5. Claim 2 implies that producer surplus is greater under bundling and Proposition 4 implies that consumer surplus is higher under bundling if r is low enough. Thus, when r is low enough, social welfare is maximized under bundling. As implied by Proposition 1, programs are bundled in equilibrium when $\alpha_1 + \alpha_2 > 1$ and sold separately when $\alpha_1 + \alpha_2 < 1$. Therefore, social welfare is maximized in equilibrium if $\alpha_1 + \alpha_2 > 1$ and r is low enough, and social welfare is not maximized in equilibrium if $\alpha_1 + \alpha_2 < 1$ and r is low enough. This completes the proof for parts (a) and (b).

Now suppose that $r = z$. Then, price levels under separate pricing are zero, while a price level under bundling is positive. Because not all consumers are served under bundling, total advertising revenue is higher under separate pricing and the sum of consumer surplus and subscription revenue is higher under separate pricing as well. Because consumer surplus and profit levels

are continuous with respect to r, there exists an r^* such that social welfare is maximized under separate pricing for $r > r^*$. As implied by Proposition 1, programs are bundled in equilibrium when $\alpha_1 + \alpha_2 > 1$ and sold separately when $\alpha_1 + \alpha_2 < 1$. Therefore, social welfare is maximized in equilibrium if $\alpha_1 + \alpha_2 < 1$ and r is high enough, and social welfare is not maximized in equilibrium if $\alpha_1 + \alpha_2 > 1$ and r is high enough. This completes the proof for parts (c) and (d) of the proposition. ■