Can Financing Constraints Explain the Asset Pricing Puzzles in Production Economies? *

Katherine A. Smith†
U.S. Naval Academy

January 18, 2008

Abstract

General Equilibrium asset pricing models have a difficult time simultaneously delivering a sizable equity premium, a low and counter-cyclical real risk free rate, as well as cyclical variation in return volatility. To explain these stylized facts, this paper introduces occasionally binding financing constraints that impede producers' ability to invest in an otherwise standard real business cycle model. These financing constraints increase the marginal cost of investing without altering the marginal rate of substitution directly, generating a sizable equity premium as well as other standard business cycle quantity and price moments. The financial frictions drive a wedge between the marginal rate of substitution and firms' internal stochastic discount factors so that the shadow value of capital is no longer tied to the average price of capital serving to increase asset price volatility.

JEL Codes: G12;E22;C63

Key Words: Financial Frictions, Equity Premium, Return to Investment

*Thanks to Enrique Mendoza, Jonathan Heathcote, Kent Kimbrough and Michelle Connolly for many helpful comments on an earlier version of this paper.

†Department of Economics, 589 McNair Road, Annapolis, MD 21402, USA. Email: ksmith@usna.edu.

1 Introduction

There are several asset pricing stylized facts that prove difficult to capture in a General Equilibrium (GE) setting. The most notable is that equity returns are high while the risk free rate is simultaneously low. Campbell (1999) estimates the ex-post premium to be just under seven percent and the risk free rate roughly two percent. While the large equity premium and low risk free rate are well documented and have been extensively explored in the literature, less emphasis has been placed on the counter-cyclical variation in return volatility (Schwert (1989)) and short term real interest rates (King and Rebelo (1999)).

To explain these stylized facts, this paper introduces financial frictions that impede producers' ability to invest in an otherwise standard real business cycle (RBC) model. Since returns tend to vary with the business cycle it makes sense to examine business cycle models to understand asset price dynamics [1]. While RBC models may seem like good candidates to aid in the understanding of these aggregate risk factors, standard models with standard preferences do an abysmal job as Rouwenhorst (1995) among others have explored. After calibrating and simulating a standard RBC model, Rouwenhorst (1995) shows how the quantity dynamics are quite accurate (with highly persistent aggregate productivity shocks) but the model fails to capture price dynamics. Without frictions, consumers can easily smooth their consumption by investing. Thus, consumption is flat and the marginal product of capital not particularly volatile since firms can easily adjust their capital stock to take advantage of the expected productivity shocks.

For a GE model to successfully explain asset price dynamics, investment must be restricted. Forward looking firms want to invest when investment returns are expected to be high next period. Assuming diminishing marginal returns to capital, greater investment delivers lower ex-post returns. The key to consistently high returns to capital then is to limit capital accumulation. In a GE framework, investment is naturally limited because the interest rate is not constant. Higher demand for capital must mean higher savings, achieved through lower consumption today. Households only accept lower consumption if the interest rates increase, which slows capital accumulation.

[1] For a more detailed discussion of the role business cycle models may play in understanding asset prices, see Cochrane (2008).

While frictionless GE models are therefore able to deliver high returns to capital, these returns are realized through higher interest rates. Even if one increases the elasticity of investment demand by introducing adjustment costs or increases the inter-temporal elasticity of substitution by altering preferences, these changes impact investment returns by altering the inter-temporal price of consumption directly.

Counter-cyclical risk free rates and return volatility also hinge on investment being constrained. In the extreme, if no investment occurred, the interest rate would be highly counter-cyclical because the expected productivity shocks would completely drive the inter-temporal price of consumption. In the opposite extreme, with perfectly elastic investment the interest rate would be acyclical as households could perfectly offset changes in their expected income stream through investing. To produce counter-cyclical real short term interest rates in an RBC model, investment must be restricted to keep agents from trying to excessively smooth consumption and force the inter-temporal price of consumption to respond to expected productivity shocks. In particular, an asymmetric restriction on investment can explain why return volatility tends to vary counter-cyclically. Schwert (1989) shows a significant increase in volatility for both equity returns and short term interest rates during recessions.

The model presented here restricts investment (thus accounting for observed asset price features) by exploring the dynamic effects on producers' decisions if firms face a lower bound on their sources of financing. Rather than focusing on the financing choice and the costs that may drive that choice, this paper simply supposes there is a limit to these financing options. In the economy, this restriction on financing is evident in stable debt to equity ratios, dividends being bounded by zero, and the fact that virtually no equity issuance occurs during recessions. Knowing an upper-bound on financing exists, firms must manage their internal resources so as not to become constrained.

The model works in a similar manner to Gomes, Yaron, and Zhang (2006), where investment is impeded not just by capital adjustment costs but also financing costs. Gomes, Yarron and Zhang (2006) examine the asset pricing implications from endogenously determined debt constraints found in the financial accelerator literature [2]. Like the financial accelerator literature, they are able to get

[2] Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and Carlstrom and Fuerst (1997).

nice hump shape quantity dynamics, however, due to risk-neutral entrepreneurs and permanently binding constraints the frictions have little impact on mean returns. In addition, they find that if one forces agency costs to be pro-cyclical (as bond spreads seem to indicate) the equity premium would actually be negative. In contrast to Gomes et al (2006), the financial frictions in this model are specified as an occasionally binding constraint rather than a cost function. Under this alternative set-up, the marginal rate of substitution is not affected directly which allows the risk free rate to remain low while the return to investment and equity are driven higher.

Unlike other dynamic GE models with non-trivial production sectors (Jermann (1998) and Boldrin, Christiano, and Fisher (2001)), the model is able to deliver interesting asset price dynamics without altering preferences. Jermann (1998) modifies preferences to account for habit formation, forcing preferences for consumption to be sticky/persistent and by adding adjustment costs to investing forcing firms to desire sticky/persistent investment. With sticky desired consumption and investment, Jermann (1998) matches quite well the observed mean returns of assets, however he gets the unfortunate by product of very volatile interest rates which is not a feature of the data. In a similar vain, Boldrin, Christiano, and Fisher (2001) show how the combination of preferences for habit formation and adjustment costs to production help to alter both the consumers and producers optimality conditions and therefore match mean returns. Instead of simply including quadratic adjustment costs, they add frictions that limit capital mobility between sectors. Like Jermann (1998), they are able to match mean returns but at a cost of introducing increased volatility in those returns.

Occasionally binding financial frictions have been used in partial equilibrium settings to try and generate greater asset price volatility. By adding margin requirements, trading costs, and short selling constraints to a consumption based asset pricing model, Heaton and Lucas (1996) qualitatively succeed in getting an equity premium but miss quantitatively. Not only have these partial equilibrium models been ineffective at capturing the asset price dynamics, conceptually they do not link the price of risk to macro factors. Adding financial frictions on investment in a GE setting, on the other hand, ties changes in the time varying price of risk to key macro variables like leverage, the marginal product of capital, and expected changes in the capital stock.

In the model presented here, firms face a lower bound on financing for two reasons. First, long term debt is fixed. Second, short term debt cannot exceed the short term obligations to equity and debt holders. Firms are assumed to hold long term debt to satisfy a target debt to equity ratio. Trade-off theory suggests, the long term target debt-equity ratio is based upon varying costs and benefits of issuing debt which are only in part dependent on phase of the business cycle. Benefits of holding debt include tax advantages and reduced agency costs. Costs of holding debt include increased risk of financial distress and increased monitoring/contracting costs associated with higher debt levels. Since many of the factors that drive the target debt to equity ratio depend on non-state contingent characteristics, in the model long term debt is assumed to be independent of the business cycle and therefore constant at the business cycle frequency.

Figure (1) shows the justification for this assumption. Using the Flow of Funds (FOF) data, the graph plots the variation in long and short term new issues. Because the FOF data reports outstanding debt levels rather than new issues specifically, the long term new issues have to be backed out of the level data as the note on Figure (1) explains. While long term debt outstanding makes up the majority of long term debt, new issues of long term debt as a percent of all credit is quite small and does not seem to show a strong business cycle pattern. In fact the correlation with GDP growth is nearly zero.

While long term debt is assumed constant in the model, total debt to equity does vary due to changes in short term debt. Firms may take on short term debt (one period) to cover cyclical financing needs. By introducing non-state dependent long term debt and state-dependent short term debt, the model is able to match not only the high leverage ratios observed for large capitalization firms but also capture the business cycle impacts on deviations from their target. Firms can issue short term debt in the model, however, the amount of funds borrowed cannot exceed the short term obligations to equity and debt holders. If the short term debt decision were not modeled explicitly, this constraint would simplify to the restriction that dividends could not be negative. While other models have also introduced this type of non-negativity constraint, the difference in this model is that given firms' long term debt payments the presence of the constraint has a much larger impact on firms' optimal capital decisions.

If firms' liquidity constraints are currently binding or expected to bind in the future, the marginal cost of investing increases, limiting investment. The liquidity constraint causes the borrowing costs for firms to differ from the return on savings to the households. As a result, the shadow value of installed capital does not equal the average price of capital. While the return to investment drives the return to equity, these two returns are only equivalent if constraints are never expected to bind. Like an adjustment cost model, the introduction of the financing constraints provide a channel through which investment may be limited other than through the interest rate. While financing constraints impact consumption indirectly through investment, they do not directly impact consumption through the resource constraint. Adjustment costs models, in contrast, have a difficult time capturing asset price dynamics since consumption is determined by output less investment and less costs.[3] Investment is slowed by the costs, but these costs directly impact consumption, forcing consumption to be more volatile. Occasionally binding constraints, in contrast, impede optimal investment without having a secondary impact on consumption via a cost term. Investment is hindered, the marginal rate of transformation altered, but consumption and thus the interest rate is not overly volatile.

Occasionally binding financing constraints restrict investment in an asymmetric manner. The constraints have a higher probability of binding when firms are expecting a positive productivity shock next period but their capital stock is currently low, which typically occurs at the trough of the business cycle. Near the peak of the business in contrast, future constraints may bind but otherwise the model behaves like a standard RBC model. In this manner the financial frictions force the volatility of asset returns to vary in a counter-cyclical fashion.

While consumption, investment, and output behave in a similar manner as the standard RBC model, asset prices not only depend on the business cycle quantities but occasionally binding constraints and therefore differ dramatically from the frictionless model. The financing constraints generate a wedge between the cost of borrowing and the return from savings delivering high returns to equity, low returns on bonds, and counter-cyclical variation in both returns. In this manner, the model is able to match business cycle quantity dynamics as well as asset pricing properties

[3] This is true in both capital adjustment costs models like Jermann (1998) and investment adjustment costs models like Boldrin, Christiano, and Fisher (2001)

observed in the data. In many ways this model can be interpreted as a GE version of Aiyagari & Gertler (1999). They show how margin requirements for investors and portfolio costs for savers (consumers) drive equity prices above their fundamental value, generating excess volatility to equity returns over the exogenous risk free rate. By fixing the risk free rate, however, Aiyagari & Gertler (1999) is unable to explain why the return on the two savings vehicles behave differently.

2 The Structure of the Model

The model is set up as decentralized dynamic stochastic GE problem. Firms possess the production technology and make optimal investment decisions. However, in certain states of nature, capital market imperfections force the supply of financing to be perfectly inelastic. Households work and save by purchasing claims on the value of firms, in the form of stocks and bonds.

2.1 Households

This closed economy is characterized by a large number of identical infinitely-lived households. The households choose consumption, equity holdings in firms, and bonds respectively to maximize their lifetime utility:

$$\mathbf{Max}\ \ \mathbf{E_o} \sum_{\mathbf{t=0}}^{\infty} \beta^{\mathbf{t}}\ \mathbf{u}[\mathbf{c_t}] \tag{2.1}$$

.

Households make their decisions based on the following period budget constraint:

$$\frac{b_{t+1}}{(1+r_t)} + p_t(s_{t+1} - s_t) + c_t \leq w_t l_t + b_t + div_t s_t \ \ , \tag{2.2}$$

where s_t represent the purchase of stocks between households and b_t represents the borrowing or lending by households. They earn income by working for w_t and receiving dividends on their

equity holdings. They are able to transfer income through time by lending or buying stocks and bonds to smooth their consumption. Households take the price of the stock, p_t, and the return on the risk free bond, $(1 + r_t)$, as given. Since households do not receive utility from leisure, they choose to work their full endowment of time.

2.2 Firms

A large number of identical firms produce an identical good using both labor (l_t) and capital (k_t). Labor and capital are combined using constant returns to scale technology $exp(\epsilon_t)F(k_t, l_t)$. This production function is subject to productivity shocks. Firms must pay labor at a market rate of w_t as well as make investments (i_t).

Productivity shocks follow a symmetric Markov process, exhibiting simple persistence. This specification minimizes the size of the exogenous state space without restricting the variance and first-order autocorrelation of the shocks. The shocks take a high or low value, $E(e_H, e_L)$. Symmetry implies that $e_L = -e_H$, and that the long-run probabilities of each state satisfy $\Pi(e_L) = \Pi(e_H) = 1/2$. Transition probabilities follow the simple persistence rule (see Backus, Gregory and Zin (1989)). Under these assumptions, the shocks have zero mean, their variance is $(e_H)^2$. Firms pay out dividends according to the following budget constraint:

$$div_t = \ exp(\epsilon_t)F(k_t, l_t) - w_t l_t - i_t - \overline{RB} \ - b_t + \frac{b_{t+1}}{(1 + r_t)}. \tag{2.3}$$

Firms receive income from producing $F(k_t, l_t)$ and pay workers $w_t l_t$ as well as invest i_t in order to alter the amount of capital used in production next period.

Varying from a standard RBC model, it is assumed that firms hold a constant amount of long term debt on which it must pay interest each period $\overline{RB}$ as well as short term (one period) bonds. This implies that changes to the debt to equity ratio for the representative are due entirely to changes in short term financing. As the trade-off theory suggests, the long term target debt-equity ratio is based upon varying costs and benefits of issuing debt which are in large part unrelated

to the phase of the business cycle. Since many of the factors that drive the target debt to equity ratio depend on non-state contingent characteristics, long term debt is assumed to be constant. Masulis (1988) shows that historically debt to book value ranges from .53-.75 for all non-farm non-financial corporations. One feature of the data is that these target debt to equity ratios do vary tremendously by industry and firm size. Therefore, a heterogenous firm model would need to explain why the optimal capital structure varies across firms. In contrast, with a representative firm, it is less restrictive to assume firms have constant long term debt obligations.

While long term debt is assumed to be fixed in the model, total debt to equity does vary due to changes in short term debt. Firms may take on short term debt (one period) to cover any cyclical financing needs. The total short term debt is issued is captured in b_{t+1}. $(1 + r_t)$ is the risk free rate determined by the households.

Capital evolves according to the following standard equation of motion:

$$k_{t+1} = i_t + (1 - \delta)k_t. \tag{2.4}$$

As evident in (2.4) the firms do not face any direct costs to altering their capital stock. There is a large literature showing that adjustment costs to capital (Jermann (1998) and Boldrin, Christiano, and Fisher (2001)) as well as adjustment costs to altering investment directly (Beaubrun-Diant and Tripier (2005)) may be important for explaining investment dynamics. By leaving out adjustment costs, the financing constraints are forced to be the primary determinant of the investment dynamics. The model therefore can explore whether financing constraints themselves may eliminate the need for additional adjustment costs to slow down investment.

In choosing optimal capital to maximize its value, firms face the following non-negativity constraint:

$$div_t + b_t \geq \frac{b_{t+1}}{(1 + r_t)}. \tag{2.5}$$

Equation (2.5) effectively limits short term debt positions to be less than current obligations to equity and debt holders. By rearranging (2.5), this constraint can be interpreted as the ratio of short term debt to current obligations must be less than one:

$$1 \geq \frac{b_{t+1}}{(1+r_t)(div_t + b_t)}. \tag{2.6}$$

This type of constraint is consistent with a trade-off theory of firms' capital structure. If firms have some target debt to equity ratio then it is likely they would want to limit their new debt obligations to be less then their current obligations to all claimants. This would work to keep the firm in the neighborhood of the target. If short-term debt is not explicitly modeled (so that households borrow/lend to each other and purchase shares in the firms), this constraint reduces to a simple more traditional constraint that dividends cannot be negative (Gomes, Yaron and Zhang (2003)). [4]

Combining (2.3), (2.4), and (2.5) the financing constraint can also be seen as a limitation on the use of internal resources of firms. In a similar manner to Gross (1994), the constraint can be interpreted as a non-negativity constraint on firms' cash flows:

$$exp(\epsilon_t)F(k_t, l_t) - w_t l_t - k_{t+1} + (1-\delta)k_t - \overline{RB} \ \geq 0. \tag{2.7}$$

The firms' objective is to maximize the discounted value of cash flows. The internally generated cash flow gets paid out in the form of dividends as follows:

[4] For simplicity new equity offerings are not explicitly modeled. As long as one assumes new equity offerings also face a limit, the basic implications of the model would still hold. Given the limits on financing, it could be interesting to think about how firms manage their holdings of cash balances in order to avoid hitting their constraint. Currently, firms must pay out all positive cash flow in the form of dividends to households.

$$\mathbf{E_t} \sum_{i=0}^{\infty} \theta_{t+i}[div_{t+i}]. \tag{2.8}$$

Since the households own the firms, adjusted dividends will be discounted at the intertemporal marginal rate of substitution. So that $\theta_{t+i} = \beta^i(\frac{u'(c_t+i)}{u'(c_t)})$, which firms take as given. Firms choose i_t, b_{t+1}, div_t to maximize adjusted dividends, satisfying their budget constraint and the financing constraints.

2.3 Households' behavior

The first order condition with respect to equity reveals the equation for the price of equity as follows:

$$p_t = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \ (p_{t+1} + div_{t+1}) \right] \ . \tag{2.9}$$

Since it is assumed that firms cannot issue equity, the number of shares is normalized to one. Given these assumption, if (2.9) is iterated forward, the value of the firms next period is simply the discounted dividends which corresponds to the firms' objective function (equation (2.8)).

The first order condition (FOC) for bonds provides the following Euler equation:

$$1 = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right] \ (1 + r_t) \ . \tag{2.10}$$

While these equations may appear standard, there are two things to note. First, unlike a GE model with capital or investment adjustment costs the resource constraint which determines aggregate consumption and thus the marginal rate of substitution, is not affected by any costs, but is simply the residual of output less investment. Second, due to the limit on sources of financing a

wedge emerges between the firms' stochastic discount factor and the households' marginal rate of substitution, determined by (2.10).

2.4 Firms Behavior

When firms choose capital, dividends, and bonds next period taking the discount rate on dividends as given, the following first order conditions emerge.[5] η_t and η_{t+1} are the nonnegative multipliers associated with the current and future financing constraint respectively:

$$1 + \eta_t = E_t\left[\theta_{t+1}(1+\eta_{t+1})\left(exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)\right)\right] \tag{2.11}$$

$$1 = E_t\left[\theta_{t+1}\right]\left(1 + r_t\right). \tag{2.12}$$

The right hand side of (2.11) is the benefit of investing one unit today. Tomorrow, that unit provides firms with increased output, captured in the marginal product of capital. This benefit is then multiplied by the shadow value of capital tomorrow, $(1+\eta_{t+1})$. The left hand side represents the cost of investing using internal resources. If firms are investing then dividend payments are decreased. If the financing constraints are currently binding the marginal cost of financing rises above one as η_t would be non-zero.

The marginal cost of investing using debt is determined by (2.12). Looking at (2.12), the financing costs using debt depends on the endogenous risk free rate, r_t, and the marginal rate of substitution, θ_{t+1}, which firms take as given. Taking equations (2.11) and (2.12) and combinding the following equation emerges:

$$E_t\left[\theta_{t+1}\right]\left(1 + r_t\right) + \eta_t = E_t\left[\theta_{t+1}(1+\eta_{t+1})\left(exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)\right)\right]. \tag{2.13}$$

Equation (2.13) suggests that the marginal cost of investing using internal resources or debt is

[5] The FOCs also include the three constraints and the corresponding Kuhn Tucker conditions.

the same. Because both the amount of dividends paid out and short term debt obligations made are important for the financing constraint, the firm is constrained equally using either form of financing.

The cost of investing today differs from one if the financing constraint binds. If the financing constraint is not currently binding and is never expected to bind then the model collapses to the frictionless model where the shadow value of capital is always equal to one. Just like a capital adjustment cost model (Jermann (1998)), the financing constraint causes the cost of investing to be highest during the initial phase of an economic expansion, because that is when the occasionally binding constraints become relevant. To avoid being constrained, investment is delayed causing high returns to already installed capital.

Occasionally binding financing constraints impact the investment return but do not impact the marginal product of capital itself or the households' marginal rate of substitution directly. In an adjustment cost model, investment returns, which drive equity returns, have a first order dependence on adjustment costs. As firms invest, there is more capital next period, which lowers the cost of investing. These costs also directly impact households' marginal rate of substitution. Through the resource constraint, an increase in adjustment costs impede investment but are wasted resources and cannot be consumed, which reduces consumption today, causing the interest rate to rise. Adjustment costs in and of themselves can generate high investment returns but have the unfortunate by product of driving down the marginal rate of substitution and thus increasing the interest rate.

From (2.11) and (2.12) it is clear how these financing constraints impact the stochastic discount factor of the firms. Firms discount the return on investment not only by the households' marginal rate of substitution but also by next period's shadow value of capital which in turn depends on whether the constraints will bind next period. Therefore, the return on investment next period is driven by current and future financing constraints. In this manner, the cost of financing depends on the state of the economy and varies over the business cycle. Changes in endogenous quantities of the capital stock, the shadow value of capital next period, and the households' marginal rate of substitution affect the likelihood that the constraints binds and the marginal cost of investing.

2.5 Competitive Equilibrium

Given the exogenous stochastic process for the productivity shocks and initial states k_t, b_t, a competitive equilibrium is defined by sequences of state-contingent prices $w_t, p_t, \theta_t, 1 + r_t$ and allocations $k_{t+1}, b_{t+1}, c_t, i_t$ such that: (a) firms maximize the expected discounted dividends subject to CRS technology, the law of motion for capital, and the financing constraint; (b) households choose b_{t+1}, c_t, s_t to maximize expected discounted utility subject to their budget constraint; (c) the following markets clear.

the goods market,

$$c_t + i_t = F(k_t, l_t) \tag{2.14}$$

the bond market,

$$b_t = b_t^d \tag{2.15}$$

and the equity market,

$$s_t = s_t^d = 1. \tag{2.16}$$

2.6 Asset Prices and Returns

There are three key differences between this model and the standard RBC model that help match asset price dynamics. First, the investment demand function is kinked in some states of nature. Second, the price of equity depends on the probability of the constraints binding and not simply on the level of the desired capital stock. Last, the shadow value of capital does not equal the average value of capital generating more volatility in the capital stock.

2.6.1 The Risk Free Rate

The financing constraints generate a kink in investment demand in a similar manner to Fazzari, Hubbard, and Petersen (1988). Combining the FOC for the firms and the definition of dividend an

investment demand emerges [6]:

$$i_t = \frac{1}{(1+r_t)} \frac{E_t\left[(1+\eta_{t+1})\left(\tilde{div}_{t+1}\right)\right]}{[1-cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1})]} - (1-\delta)k_t \tag{2.17}$$

where $\tilde{div}_{t+1} = div_{t+1} + k_{t+2} + (1+r_t)b_{t+1} + \overline{RB} - b_{t+2}$. and $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$.

If the financing constraint does not currently bind, investment is influenced positively by higher expected dividends, more depreciated capital stock, and the probability the financing constraint may bind tomorrow ($(1+\eta_{t+1})$). While the first two factors are standard, the third indicates that if firms are expecting to be constrained in the future they invest more today to relax the future constraints. Looking at the denominator, investment depends negatively on the interest rate. The sensitivity of the investment to the interest rate, however, depends on the risk premium on investing, as captured by $[1-cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1})]$. A higher negative covariance between the stochastic discount factor and the return to investment lowers the amount of investment a firm is willing to take on at a given interest rate. If the financing constraint binds, investment demand becomes inelastic with respect to the interest rate. Firms simply invest what they can while satisfying the financing limitation:

$$i_t = exp(\epsilon_t)Fk_t, l_t - w_t l_t - \overline{RB}. \tag{2.18}$$

Due to this kink in the investment demand function, when the constraint binds the equilibrium interest rate and investment are less than they would be in a frictionless world, as seen on Figure 2.

On the other hand, the households' savings function is always upward sloping with respect to the risk free rate. Rearranging (2.10) we get the following standard savings function:

[6] The derivation of 2.17 is found in Appendix 6.1.

$$s_{t+1} = y_t - u_c^{(-1)}\Big(E_t(\beta u_c(c_{t+1}))(1+r_t)\Big) \tag{2.19}$$

where $y_t = div_t s_t + w_t l_t + b_t + \overline{RB}$.

As Figure 2 shows, the constraints generate a wedge between the rate firms are willing to pay to borrow funds and the price the households receive. Investment demand is inelastic with respect to the interest rate once the constraints bind. The marginal rate of substitution is higher than in the frictionless case, which translates into a lower risk free rate, $(1 + r^c)$. [7]

2.6.2 The Price of Equity

With occasionally binding financing constraints, the price of equity is driven by the level of the capital stock (as in a standard RBC model) but also depends on the probability of being constrained. This implies two important factors in the determination of the equity price relative to the frictionless model. First, since the probability of becoming constrained is state-dependent, additional asset price volatility is generated. Second, the average value of capital no longer has a one to one relationship with the shadow value of capital, resulting in a more volatile marginal cost of investing.

Working with the the households' FOC with respect to equity shares and the definition of dividends, one can link the price of equity to its endogenous factors, all of which are time varying at the business cycle frequency (see Appendix 6.2 for derivation):

$$p_t = k_{t+1} + \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(k_{t+i+1} - mpk_{t+i}k_{t+i})\right] - \frac{b_{t+1}}{(1+r_t)} - E_t\sum_{i=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB}). \tag{2.20}$$

If the financing constraint never binds then $\forall(t)\eta_t = 0$ and $p_t + \frac{b_{t+1}}{(1+r_t)} + E_t\sum_{t=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB}) = k_{t+1}$ or in other words, the total market value the firms (equity and debt) is equal to the replace-

[7] Even if the financing constraint does not currently bind, the potential for the constraints to bind in the future does impact the slope of investment demand as reflected in the partial derivative of the investment function with respect to the risk free rate.

ment costs. However, if the constraint binds today or has a positive probability of binding in the future, then the value the firms may differ substantially from the replacement value. Breaking the direct link between the capital stock and the market value of the firms is key for any model trying to capture observed asset price volatility. As Rouwenhorst (1996) points out, the capital stock is not particularly variable, therefore, any model which relies exclusively on the volatility in capital to explain asset price volatility will have a difficult time replicating stylized facts. Financing constraints introduce an additional source of volatility to equity prices. Because these are occasionally binding constraints, the shadow value on the constraint does not always bind. The constraints have a higher probability of binding when firms expect a positive productivity shock next period. Since the probability of becoming constrained depends on the stage in the business cycle, equity prices tend to move in a cyclical fashion.

2.6.3 Marginal q no longer equals average q

Another implication of (2.20) is that the shadow value of capital (marginal q) for firms does not equal the market value over the replacement value (average q). Under this scenario, investment should become more volatile, generating greater variance in the capital stock itself. If the contrary holds, there will be a smoothing impact on investment and the capital stock. To see the breakdown of the relationship of marginal to average q, define the total market value of firms (debt and equity claimants) as $V_t = p_t + \frac{b_{t+1}}{(1+r_t)} + E_t \sum_{i=1}^{\infty} (\prod_{j=1}^{i-1} \theta_j \overline{RB})$ then substitute into 2.20:

$$\frac{V_t}{k_{t+1}} = (1+\eta_t) + \frac{E_t\left[\sum_{i=1}^{\infty} \theta_{t+i}\eta_{t+i}(k_{t+i+1} - mpk_{t+i}k_{t+i})\right]}{k_{t+1}}. \tag{2.21}$$

The total value of firms over the replacement value will not only differ from unity but also from the shadow value of capital today, $(1+\eta_t)$. The market value divided by the replacement value $\frac{V_t}{k_{t+1}}$ depends on the marginal value of capital tomorrow $(1+\eta_{t+1})$ as well as the probability the financing constraint binds in future, captured in the second term. If the financing constraints are expected to bind today or any period of time in the future, the current market value of the firm rises above the replacement value. A unit of capital inside the form will be worth more than outside

the firm.

2.6.4 The Return to Investment

Given that the capital stock next period is not necessarily the same as the market value of the firms, it follows that the return to investing by firms and the return to equity the households receive will differ. Deriving each return separately we can determine the factors driving each and understand the magnitude of the wedge between them. If the model is to succeed in replicating stylized facts, this wedge should not be large given Cochrane (1991) which shows similar dynamics between the two ex-post returns.

The financing constraints are able to limit investment by generating a financing premium and thus driving up the return to investment. Rearranging the firms' FOC with respect to capital equation (2.11) the following equation emerges for the determination of the expected return to investing:

$$E_t[mpk_{t+1}] = \quad (1+r_t) \quad - \quad \frac{cov(\theta_{t+1}, mpk_{t+1})}{E_t[\theta_{t+1}]} - \frac{cov(\theta_{t+1}, \eta_{t+1} mpk_{t+1})}{E[\theta_{t+1}]} \quad + \frac{\eta_t}{E[\theta_{t+1}]} - E[\eta_{t+1} mpk_{t+1}] \tag{2.22}$$

or

$$E_t[mpk_{t+1}] = \quad (1+r_t) \quad - \quad \frac{cov(\theta_{t+1}, mpk_{t+1})}{E[\theta_{t+1}]} \quad + \quad FP_t, \tag{2.23}$$

where $FP_t = -\frac{cov(\theta_{t+1}, \eta_{t+1} mpk_{t+1})}{E[\theta_{t+1}]} \quad + \frac{\eta_t}{E[\theta_{t+1}]} - E[\eta_{t+1} mpk_{t+1}]$. In this case, the marginal product of capital represents the derivative of the production function with respect to next period's capital stock net depreciated capital, $mpk_{t+1} = exp(\epsilon_{t+1}) F_{k_{t+1}} + (1-\delta)$. From the FP_t term, we can see that the financing premium is time varying and tends to increase when the constraints are more likely to bind. Given this, the financing premium is highest at the trough of the business cycle when firms are expecting a positive productivity shock next period but currently have low desired capital stock due to bad shocks in the past.

Given a positive probability the constraints will bind in the future, the financing premium increases the return to investing relative to standard frictionless case. The return is no longer simply driven by the negative covariance between the marginal rate of substitution and the marginal product of capital, but also by the relationship between the relative returns and the financing constraints. More specifically, we can decompose the financing premium into several factors. First, there exists a negative covariance between the marginal rate of substitution and the product of the return to investing and the financing constraint. Second, if the constraint binds today firms cannot invest and returns to investing are expected to be higher.

2.6.5 The Return to Equity

While the firms' returns to investing directly impact the return households get from owning the firms, the relationship is no longer one to one. With the potential to hold both short and long term debt and financing constraints binding in certain states of nature, the difference between a value of a dollar inside and outside the firms is reflected in respective returns to investing and owning a firm. Given the households FOC with respect to equity (2.11) and the definition of dividends (2.3), the following equation emerges:

$$E_t[R^e_{t+1}] \;-\; R^f_t \;=\; -\;\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}\frac{k_{t+1}}{p_t} \;-\; \frac{cov[\theta_{t+1}, (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})})]}{p_t E[\theta_{t+1}]}, \quad (2.24)$$

where $R^f_t = (1 + r_t)$ and $R^e_{t+1} = \frac{div_{t+1}+p_{t+1}}{p_t}$. Without debt or financial frictions, the market value of capital is equal to replacement value of capital (2.20) so $\frac{k_{t+1}}{p_t} = 1$. In this case (2.24) reduces to the frictionless case where the equity premium is simply driven by the fact the equity pays off well in states of nature where the households do not care much for that additional payoff, captured in a negative covariance between the MRS and the the return to equity.

If firms have the option to hold debt but financing constraints are never binding then the equity premium is amplified by the amount of debt relative to the price of equity. Applying (2.20) without financial frictions, we get the following:

$$E_t[R^e_{t+1}] \;-\; R^f_t \;=\; -\;\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1+\frac{\frac{b_{t+1}}{(1+r_t)}+E_t\sum_{i=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB})}{p_t}) \tag{2.25}$$

Equation 2.25 is essentially a GE version of Proposition II from Modigliani-Miller(1958). The equity return is affected by the risk adjusted debt to equity ratio. Although firms' financing decisions do not impact the value of the firms, they do impact the return to equity holders in the presence of uncertainty.

As long as their is some potential that the financing constraints may bind then the wedge the equity return and investment return grows wider. For notational purposes define $\tilde{b_{t+1}} = \frac{b_{t+1}}{(1+r_t)} + E_t\sum_{t=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB})$ or as total short and long term debt positions. With occasionally binding financing constraints the return on equity is determined as follows:

$$E_t[R^e_{t+1}] - R^f_t = -\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1+\frac{\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})]}{p_t})$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t}. \tag{2.26}$$

or

$$E_t[R^e_{t+1}] = \;E_tR^i_{t+1} - \;\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}\frac{\left(\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})\right]\right)}{p_t}.$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t} + FP_t, \tag{2.27}$$

where $E_tR^i_{t+1} = E_t[mpk_{t+1}]$. Consistent with any frictionless GE asset pricing model with

leverage, the equity return is driven by the investment return $E_t R^i_{t+1}$ and the risk adjusted leverage term $\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]} \frac{\tilde{b_{t+1}}}{p_t}$. In addition to these standard factors, the occasionally binding constraints further increase the equity return as reflected in the remaining three terms.

3 Recursive Form and Numerical Solution Technique

The model is solved using numerical methods that work off the recursive set-up of this equilibrium. The state space is defined as k as well as the exogenous state ϵ that is driven by the aforementioned simple persistence rule. The endogenous state space is defined by the discrete set $Z = [k_L, k_H]$. Assume continuous, nonnegative equity pricing function and interest rate $p(k, \epsilon) : ExZ \rightarrow R+$ and $r(k, \epsilon) : ExZ \rightarrow R+$ that are taken as given by the firms and households. The bounds of $p(k, \epsilon)$ and $r(k, \epsilon)$ follow from the bounds of E and Z. Assuming that the short-term bond market clears, the following dynamic programming problem emerges:

$$S(k, \epsilon) = Max \frac{(c^{(1-\sigma)} - 1)}{1 - \sigma} + \beta ES(k', \epsilon') \tag{3.1}$$

subject to:

$$c = exp(\epsilon) f(k, \bar{l}) + (1 - \delta)k - k' \tag{3.2}$$

$$exp(\epsilon) f'(k, \bar{l})k + (1 - \delta)k - k' - \overline{RB} \geq 0 \ . \tag{3.3}$$

Given the concavity of the utility function and assumptions about the shock, a unique solution to the value function is determined. The pricing functions and the decisions rules that emerge once prices have converged constitute a competitive equilibrium for the model. The state space of capital spans the interval $[K_l, K_h]$ with NK discrete nodes.

The solution method uses aspects of Mendoza and Smith (2006), Heaton and Lucas (1996), and Krusell and Smith (1997). What makes the problem difficult to solve are the occasionally binding financing constraints. For this reason simple policy function iteration is not used. Instead, the algorithm is centered on value function iteration over a discretized grid in a similar manner to

Heaton and Lucas (1996) and Mendoza and Smith (2006). The downside of using value function iteration with a discrete grid is that the FOCs do not hold exactly. The larger the state space and finer the grid the less approximation error exists. The quantity moments do not change much as the grid gets finer. A fairly coarse grid can replicate reasonable moments. The price moments, on the other hand, are extremely variable when the grid is too coarse.

A discrete representation of state space for the households' problem is defined by (k, ϵ). This is done by calculating the steady state value of k and using that to center the grid. The outcome will yield decision rules $k'(k, \epsilon)$. The decision rules that solve (3.1) maximize the utility of domestic agents taking into account the economy's resource constraint, the financing constraint, the optimal rules determining wages, and the market-clearing condition of the bond market. Thus, the prices and allocations supported by the Bellman equation satisfy the following properties of the competitive equilibrium: (a) given wages, equity prices, and the risk free rate, c, b',and s' solve the constrained maximization problem of households (b) given the households MRS, div, k', and b' solves the maximization problem of firms, and (c) the market-clearing conditions for equity, goods and bonds hold.

3.1 Calibration and Functional Forms

For the numerical analysis, the following functional forms are used:

$$F(k_t, l_t) = k_t^{1-\alpha} l_t^{\alpha} \tag{3.4}$$

$$U(c_t) = \frac{[c_t^{1-\sigma} - 1]}{1-\sigma} \ . \tag{3.5}$$

Looking at the production function, α is the share of output allocated to capital. In equation (3.5), σ is the coefficient of risk aversion. Given these functional forms, the algorithm needs values for the following vector of parameters $[A, \alpha, \beta, \delta, \gamma, \overline{RB}, \epsilon_H, \epsilon_L]$. The first three parameters are set to be consistent with the RBC literature. A is simply set to one. Reasonable estimates of α, the capital share lie anywhere from 0.3 to 0.35, so its set to $\alpha = 0.32$. The quarterly rate of

time preference is $\beta = 0.991$. Depreciation rates often vary in the literature. Here, $\delta = 0.02$ on a quarterly basis. The coefficient of relative risk aversion varies from $2.0 - 10.0$. In terms of long term debt holdings, following Masulis (1988) the level of debt is chosen so that the debt-to capital ratio is 60%. The long term fixed interest rate is chosen to be 200 basis points above the steady state risk free rate, in line with historical premiums. The shocks are calibrated to the U.S. economy. The standard deviation and first-order auto-correlation of the Markov process are set to match those from a typical business cycle. This implies a standard deviation of 0.02 and, in the baseline case, persistence of $\theta = 0.95$.

4 Results

Occasionally binding financing constraints limit capital accumulation by firms. This is easiest to see by examining the limiting distribution of capital implied by the decision rules for capital as well as the productivity shocks. Figure 3 compares the long run distribution of capital in the basic RBC model to one where firms have the potential to be constrained given their target debt to equity position. The constraints alter the mean of capital as well as the standard deviation. While clearly the constraints have forced less capital accumulation, the distribution still is a standard bell rather than being truncated on the left. This suggests firms may not actually be constrained very often but rather adjust their capital stock so as to avoid being constrained in the future. For empirical studies this implies that ex-post one might not find a significant amount of truly constrained firms. However, that would not imply that the constraints are not impacting firms' decisions.

While the constraints have clearly altered the distribution of capital the next question is to examine how this translates into the moments of the aggregate prices. Table 1 compares various returns when financial frictions are added to a standard RBC model. In addition to adding the financial frictions Models II-V make two additional parameter changes to the basic RBC model. First, Model II explores having both technology shocks that impact the existing capital stock (disembodied) as well as investment specific technology shocks (embodied). Following Greenwood, Hercowitz, and Krusell (2000) investment specific technology shocks are added to the financial frictions model to examine the impact they have on both the quantity and price business cycle

moments. While Greenwood, Hercowitz, and Krusell (2000) have two separate types of capital goods, to keep the model parsimonious these shocks are added to investment directly in a similar manner to Fisher (2003). It is assumed that both the embodied and disembodied shocks are uncorrelated. Like the standard productivity shocks, these shocks are assumed to have the same simple persistence structure. Second, Model III varies the persistence of the productivity shock. Given data problems estimating aggregate total factor productivity and recent empirical evidence that plant and industry level productivity shock persistence seems to lie in the range of $.8 - .91$ quarterly, (Abraham and White (2006)) the impact of a lower persistence is tested. Model V includes both lower persistence (.825) and two types of shocks.

Comparing Models I and II on Table (1), we can see that adding financing constraints, dramatically increases both the equity premium and the investment premium particularly under greater risk aversion. In Models III & IV we can see investment specific shocks as well as lower persistence parameters on the shocks drive up excess returns even more. Model V is able to explain roughly a third of the equity premium. The last column on this table calculates the covariance of the investment return with the marginal rate of substitution. Looking at the basic RBC model, just as theory suggests, the equity premium is exactly equal to the negative of this covariance. As we add frictions, we see this covariance increases but not nearly to the degree of the equity premium. Financial frictions generate a large equity premium without forcing the covariance to be significantly high. The high premium can be attributed to the additional risk factors that are driving the premium. While the equity premium gets wider the risk free rate falls closer to historical levels, as seen in Model V, particularly for the higher levels of risk aversion.

While Jermann (1998) and Boldrin et al (2001) are also able to generate much higher equity returns in models with altered preferences and adjustment costs, they only do so, by accepting excessive volatility in the risk free rate. In contrast, as we see in Table 2 the standard deviation of the risk free rate increases but not excessively. The downside, is that like the standard RBC model, the standard deviation of the return to equity is pinned down by the risk free rate. There is little difference between the standard deviations of the two rates even with financing constraints. Therefore, the model underestimates the volatility of equity returns. Investment return volatility

is roughly eight times higher under with financial frictions but only if investment-specific shocks are added to the model.

To examine the ability of the model to replicate counter-cyclical variation in asset returns' volatility, Table (3) reports the conditional volatility of these returns. The asymmetry of the financial frictions is clearly reflected in the conditional standard deviations. In a recession, the volatilities of equity returns and interest rates are roughly 50% larger than in an economic boom, just below the empirical estimations of Schwert (1989).

Table (4) reports the correlation between output and the risk free rate. The data suggests that interest rates are counter-cyclical. In the standard RBC model, the interest rate is essentially acyclical. All five versions of this model, in contrast, deliver counter-cyclical interest rate behavior. While qualitatively the model can get interest rates in the correct direction with the business cycle, quantitatively, the correlation is much higher than we see in the data.

Having a production side to the economy forces us not only to try and match prices but aggregate quantity moments as well. Table 5 shows the impact of the financing constraints on the relative standard deviations of the model. In the standard RBC model, as the coefficient of risk aversion is increased, consumption becomes less volatile relative to output. More volatile investment means a more volatile capital stock and more volatile output. In the standard RBC model, with higher risk aversion households' consumption becomes less volatile and mean consumption rises. In the model with financing constraints, on the other hand, consumption becomes less volatile relative to output but the fall is not very dramatic and the impact on investment volatility much smaller. Because investment is limited, output does not rise as much and mean consumption falls. With financial frictions, if households are more risk averse they must pay for less volatile consumption with lower mean consumption. As table 5 indicates additional investment specific shocks do little to the relative standard deviation. In the model with constraints and lower persistence, investment is much more volatile and consumption must less volatile relative to output. Adding both lower persistence and investment-specific shocks as well as a higher level of risk aversion, the standard deviation of consumption relative to output is .68 and investment to output is 2.08 matching the data quite well.

Table 6 takes an expanded look at the business cycle moments assuming the coefficient of risk aversion is 10. Again, the table reports not only the model with financing constraints but also the model under two alternative parameterizations, lower shock persistence and an additional investment-specific tech shock. In all cases, financing constraints reduce output and investment volatility to better match the data. Altering persistence and adding investment-specific tech shocks is able to reduce the standard deviation of all three macro aggregates as seen in column one.

Adding frictions in all cases does not seem to help much in matching the consumption and investment correlations with GDP. In all four versions of the model shown, these correlations are significantly above those observed in the data, just as in the standard RBC model. This is likely due to the fact that labor is not a choice variable in the models. Looking at the last column, adding lower shock persistence reduces the first order autocorrelations of the macro aggregates to be in line with actual data.

To get at the dynamics of the model, Figure 4 and Figure 5 compare the forecast functions for the basic RBC model and one with financing constraints. The forecast functions are induced by a negative, one-standard deviation productivity shock at date 1, given the Markov process of productivity shocks and the decision rule for capital. These forecast functions are analogous to impulse-response functions conditional on starting at the mid-point of the limiting distribution and hitting them with the same negative productivity shock. The forecast functions have the advantage that they preserve all the non-linear aspects of the model's stochastic competitive equilibrium captured in the decision rules. The plots for the business cycle quantities (Figure 4) in many ways supports what was clear from the moments. Output rises in response to a positive productivity shock. While the investment plots have similar shapes in the frictionless model, investment is impeded by the possibility of being constrained and increases by less than half of what we see in the frictionless case. In the consumption plot, we see that in an economy without frictions the ability to transfer consumption from one period to the next by investing in firms enables households to have a smooth consumption profile. The frictions add the needed volatility to consumption. More volatile consumption and less volatile investment cause the capital stock to be much smoother, as evidenced in the final plot.

While the frictions dampen the quantity moments, they seem to amplify the price dynamics. Turning to the forecast functions for prices in Figure 5, we can see a dramatic change in the price of equity following a positive productivity shock relative to the frictionless case. Looking at the second plot, a main driver of this is the fact that the interest rate is highly counter-cyclical and thus the marginal rate of substitution very pro-cyclical. As the price of equity is expected to fall over time, the equity return slowly adjusts back to its steady state. The investment return on the other hand tends to rise dramatically as the productivity shock increases the return to investing. The lack of of investment flowing in keeps the return to investing higher for longer.

While discretizing the state space and iterating on the value function is less restrictive than solving the model using linear approximations, as discussed in Rouwenhorst (1995) an error is introduced in that the first condition may not hold exactly since the decision rules are only calculated on grid points. The error therefore is directly proportional to the coarseness of the grid. Table 7 reports the errors given the coarseness of the grid. A greater coefficient of relative risk aversion does introduce greater approximation error in that it introduces greater curvature to the value function.

5 Conclusions

A simple GE asset pricing model where firms face a limit on their sources of financing produces asset price dynamics that are consistent with stylized facts. The financing frictions generate a wedge between the households' marginal rate of substitution and the firms' stochastic discount factor that allows the risk free rate to be low while simultaneously the equity return to be high. In this manner, the model is able to explain more than a third of the observed equity premium. In addition, the constraints force the risk free rate and the volatility of returns to vary counter-cyclically, key features of the macro data. Theoretically, the presence of these occasionally binding financing constraints forces equity returns to be driven by several factors. Expected equity returns no longer depend simply on the covariance between the return and MRS but also on the relative replacement value to market value. There are two avenues to further pursue with this line of research. First, it seems important to examine how firms' behavior may differ if firms can accrue retained earnings to keep themselves from becoming constrained. Second, there may be some

interesting insights into the recent declines in the equity premium by introducing government bonds into this model. Since the financing constraints limit investment, consumers would be better able to smooth consumption if the government issued debt rather than raised taxes to finance its spending (no Ricardian Equivalence). In this manner, the increase in deficit financing by the government may partially explain the simultaneous decline in the equity premium over the last twenty years.

6 References

[1] Abraham, A., White,K., (2007)."The Dynamics of Plant-level Productivity in U.S. Manufacturing," *Unpublished Working Paper*.

[2] Aiyagari, R.,Gertler,M., (1999)."Overreaction" of Asset Prices in General Equilibrium," *The Review of Economic Dynamics* 2:3-35.

[3] Backus, David K.,Gregory, Allan K., Zin, Stanley E., (1989), "Risk Premiums in the Term Structure: Evidence from Artificial Economies," *Journal of Monetary Economies* 24: 371-400.

[4] Baker,M., Greenwood, R., Wurgler, J. (2002). "The Maturity of Debt Issues and Predictable Variation in Bond Returns" *The Journal of Financial Economics*70(2): 261-291.

[5] Beaubrun-Diant, K., Tripier,F. (2005). "Asset returns and business cycle models with investment adjustment costs," *Economic Letters* 86:141-146.

[6] Bernanke, B. , Gertler, M., Gilchrist, S. (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework," Handbook of Macroeconomics, edition 1, volume 1, chapter 21, 1341-1393.

[7] Boldrin, M., Christiano, L., Fisher J.D.M. (2001). "Habit Persistence, Asset Returns and the Business Cycle," *American Economic Review* 91:149-166.

[8] Campbell, John Y., (1999). "Asset prices, consumption, and the business cycle,", Handbook of Macroeconomics, edition 1, volume 1, chapter 19, 1231-1303.

[9] Carlstrom,C.T. and T.S. Fuerst, (1997). "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Framework," *The American Economic Review* 87:893-910.

[10] Cochrane, John H. (1991)."Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *The Journal of Finance* 46(1):209-237.

[11] Cochrane, John H. (2008)."Financial Markets and The Real Economy," in *Handbook of the Equity Risk Premium*: 237-330, Elsevier BV, forthcoming.

[12] Fazzari, S., G. Hubbard, and B. Peterson (1988). "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity.*

[13] Fisher, J.D.M.(2003). "Technology Shocks Matter," FRB Chicago Working Paper 2002-14.

[14] Gomes, J, A. Yaron and L. Zhang, (2003). "Asset Prices and the Business Cycles with Costly External Finance," *Review of Economic Dynamics and Control* 6(4), 767-788.

[15] Gomes, J., Yaron, A.,Zhang, L., (2006). "Asset Pricing Implications of Firms' Financing Constraints," *Review of Financial Studies* 19(4), 1321-1356.

[16] Greenwood, Jeremy, Z. Hercowitz, and P. Krusell (2000). "The Role of Investment-Specific Technological Change in the Business Cycle," *European Economic Review* 44: 91-115.

[17] Gross, David B. (1994). "The Investment and Financing Decision of Liquidity Constrained Firms," *Unpublished Manuscript MIT*.

[18] Heaton, J., Lucas, D. (1996). "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104 (3), 443-487.

[19] Jaffee, Dwight M., Russell T. (1976). "Imperfect Information, Uncertainty and Credit Rationing," *Quarterly Journal of Economics* 90:651-66.

[20] Jermann, U.,(1998). "Asset Pricing in Production Economies ," *Journal of Monetary Economics* 41:257-275.

[21] King, Robert and Sergio Rebelo, (1999). "Resuscitating Real Business Cycles,", Handbook of Macroeconomics, edition 1, volume 2, chapter 14,927-1007.

[22] Kiyotaki, N., Moore,J. (1997). "Credit Cycles," *Journal of Political Economy* 105(2):211-248.

[23] Krusell, P., Smith, A. (1997), "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1(02), 387-422.

[24] Masulis, Ronald W.(1988), "The Debt/Equity Choice," *The Institutional Investor Series in Finance* Ballinger Publishing, Cambridge MA.

[25] Mehra, R., Prescott, E. (1985),"The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15:145-161.

[26] Mendoza, E., Smith, K. (2006) "Quantitative Implications of a debt-deflation theory of Sudden Stops and asset prices," *Journal of International Economics* 70, 82-114.

[27] Modigliani, F, Miller, M. (1958) "The Cost of Capital, Corporation Finance and the Theory of Investment," *THe American Economic Review* 48(3):261-297.

[28] Rouwenhorst, Geert (1995). "Asset Pricing Implications of Equilibrium Business Cycle Models"in *Frontiers of Business Cycle Research* Cooley: 294-330.

[29] Schwert, G. William (1989). "Why Does Stock market Volatility Change Over Time" *Journal of Finance* 44(5), 1115-1153.

7 Appendix

.1 Investment Demand

The investment function results from working with the firms' foc with respect to capital (2.11) and defining $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$.

$$1 = E[\theta_{t+1}(1+\eta_{t+1})mpk_{t+1}] \tag{.1}$$

$$1 = E[\theta_{t+1}]E[(1+\eta_{t+1})mpk_{t+1}] + cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1}) \tag{.2}$$

$$1 = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})mpk_{t+1}] + cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1}) \tag{.3}$$

$$1 - cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1}) = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})mpk_{t+1}] \tag{.4}$$

From the dividend constraint $mpk_{t+1} = \frac{div_{t+1}+k_{t+2}-\frac{b_{t+2}}{(1+r_{t+1})}+b_{t+1}+\overline{RB}}{k_{t+1}}$. Substituting this into the equation and defining $\tilde{div}_{t+1} = div_{t+1} + k_{t+2} - \frac{b_{t+2}}{(1+r_{t+1})} + b_{t+1} + \overline{RB}$ we get the following:

$$k_{t+1}(1 - cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1})) = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})\tilde{div}_{t+1}] \tag{.5}$$

Substituting the equation of motion for capital into the equation we get:

$$i_t = \frac{1}{(1+r_t)}\frac{E_t\left[q_{t+1}\left(\tilde{div}_{t+1}\right)\right]}{[1 - cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1})]} - (1-\delta)k_t \tag{.6}$$

.2 The Price of Equity

The equilibrium equity price is determined by initially starting with the households FOC with respect to equity shares as represented by 2.9 where $\theta_{t+i} = \beta^i(\frac{u'(c_t+i)}{u'(c_t)})$.

$$p_t = E_t\left[\theta_{t+1} \ \ (p_{t+1} + div_{t+1})\right] \tag{.7}$$

Using the definition of dividends and the the assumption that firms' production function exhibit constant returns to scale the following equation emerges where $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$

$$p_t = E_t\left[\theta_{t+1} \ \ (mpk_{t+1}k_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - b_{t+1} - \overline{RB} + p_{t+1})\right] \tag{.8}$$

Which can be rearranged as follows:

$$p_t = E_t\left[\theta_{t+1} \ \ mpk_{t+1}\right]k_{t+1} + E_t\left[\theta_{t+1} \ \ (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - \overline{RB})\right] - \frac{b_{t+1}}{(1+r_t)} \tag{.9}$$

From the firms' FOC $E_t\left[\theta_{t+1} \ \ mpk_{t+1}\right] = 1 + \eta_t - E_t\left[\theta_{t+1} \ \ \eta_{t+1} \ \ mpk_{t+1}\right]$. Substitute this in to the previous equation.

$$p_t + \frac{b_{t+1}}{(1+r_t)} = \left(1 + \eta_t - E_t\left[\theta_{t+1} \ \ \eta_{t+1} \ \ mpk_{t+1}\right]\right)k_{t+1} + E_t\left[\theta_{t+1} \ \ (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - \overline{RB})\right] \tag{.10}$$

or

$$p_t + \frac{b_{t+1}}{(1+r_t)} - k_{t+1} = \eta_t k_{t+1} - E_t\left[\theta_{t+1} \ \ \eta_{t+1} \ \ mpk_{t+1}\right]k_{t+1} + E_t\left[\theta_{t+1} \ \ (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})})\right] - E_t(\theta_{t+1})\overline{RB} \tag{.11}$$

Iterating forward we get the following:

$$p_t + \frac{b_{t+1}}{(1+r_t)} + E_t \sum_{t=1}^{\infty} \left(\prod_{j=1}^{i-1} \theta_j \overline{RB} \right) = k_{t+1} + \eta_t k_{t+1} + E_t \left[\sum_{t=1}^{\infty} \theta_{t+i} \eta_{t+i} (k_{t+i+1} - mpk_{t+i} k_{t+i}) \right] \quad (.12)$$

or

$$p_t = k_{t+1} + \eta_t k_{t+1} + E_t \left[\sum_{t=1}^{\infty} \theta_{t+i} \eta_{t+i} (k_{t+i+1} - mpk_{t+i} k_{t+i}) \right] - \frac{b_{t+1}}{(1+r_t)} - E_t \sum_{t=1}^{\infty} \left(\prod_{j=1}^{i-1} \theta_j \overline{RB} \right) \quad (.13)$$

.3 Return to Equity

Re-arranging equation 2.9 we get the following equation for the return to equity:

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad \frac{(p_{t+1}+div_{t+1})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.14}$$

Applying the definition of dividends in 2.3 and assuming constant returns to scale technology:

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad \frac{(mpk_{t+1}k_{t+1}-k_{t+2}+\frac{b_{t+2}}{(1+r_{t+1})}+p_{t+1})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.15}$$

or

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad mpk_{t+1}\right]}{E[\theta_{t+1}]}\frac{k_{t+1}}{p_t} - \frac{cov\left[\theta_{t+1}, \quad \frac{(p_{t+1}+\frac{b_{t+2}}{(1+r_{t+1})}-k_{t+2})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.16}$$

Applying equation 2.20:

$$E_t[R^e_{t+1}] - R^f_t = -\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1 + \frac{\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})]}{p_t})$$

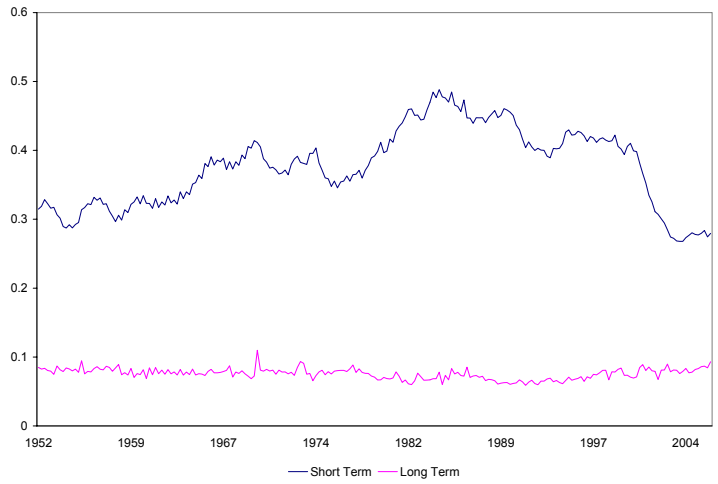
$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t}]. \tag{.17}$$

Applying equation 2.23:

$$E_t[R^e_{t+1}] = \quad E_t R^i_{t+1} - \quad \frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}\frac{\left(\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})\right]\right)}{p_t}$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t} + FP_t. \tag{.18}$$

Figure 1: New Issues of Short Term and Long Term Non-Farm, Non-Financial Corporate Debt



Note: Taken from Flow of Funds Data, following calculations of Baker, Greenwood, and Wurgler (2002). Long term debt is calculated as the sum of municipal bonds, corporate bonds, and mortgages. Short term debt is the sum of commercial paper, bank loans (nec), and "other". New issues of short term debt is simply short term debt for that period. New issues for long term debt is the change in long term debt outstanding plus 0.1 of lagged long term debt to get at the roll-overs. Both new issue series are divided by total debt to remove the growth component.

Table 1: **Asset Returns**

Model	$E(r^f)$	$E(r^e - r^f)$	$E(r^i - r^f)$	$cov(MRS, r^i)$
I. **Standard RBC model**				
$\gamma = 2$	3.63	0.0004	0.0004	-0.0004
$\gamma = 10$	3.58	0.003	0.002	-0.002
II. **Financing Constraints**				
$\gamma = 2$	3.61	0.05	0.57	-0.0006
$\gamma = 10$	3.13	0.12	1.18	-0.003
III. **Financing Constraints and Investment Shocks**				
$\gamma = 2$	3.61	0.05	0.60	-0.0008
$\gamma = 10$	3.16	0.18	1.12	-0.006
IV. **Financing Constraints and Low Persistence**				
$\gamma = 2$	3.57	0.14	0.61	-0.002
$\gamma = 10$	2.64	1.21	1.76	-0.007
V. **Financing Constraints, Investment Shocks, and Low Persistence**				
$\gamma = 2$	3.57	0.15	0.66	-0.002
$\gamma = 10$	2.61	1.34	1.83	-0.03
Data	1.96	4.74	-	-

r^f,r^e and r^i is the risk free rate,return to equity and return on investment all annualized. MRS refers to the marginal rate of substitution. The data is from Campbell (1999) using the long sample 1891-1994. γ refers to the coefficient of risk aversion applied to the power utility function in the model.

Figure 2: Firms' Investment Demand is Kinked Due to the Financing Constraints

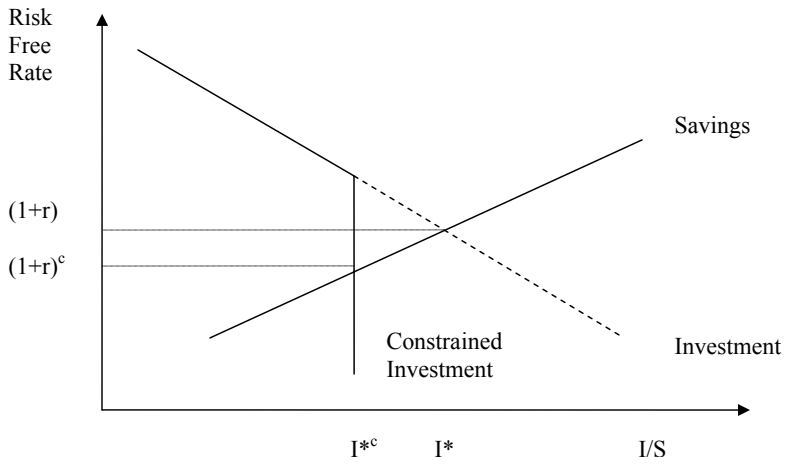


Figure 3: Long-Run Distribution of Capital

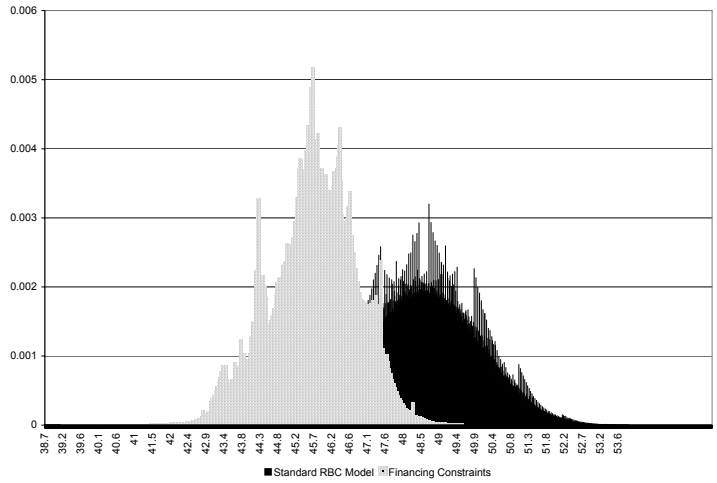


Table 2: **Asset Return Volatility**

	standard deviation (r^f)	standard deviation (r^e)	standard deviation (r^i)
Standard RBC model			
$\gamma = 2$	0.77	0.77	0.23
$\gamma = 10$	2.61	2.61	0.44
Financing Constraints			
$\gamma = 2$	0.62	0.62	0.24
$\gamma = 10$	3.05	2.97	0.26
Financing Constraints and Investment Shocks			
$\gamma = 2$	0.66	0.66	0.64
$\gamma = 10$	3.16	3.14	0.66
Financing Constraints and Low Persistence			
$\gamma = 2$	2.25	2.25	0.21
$\gamma = 10$	8.64	8.56	0.24
Financing Constraints, Investment Shocks, and Low Persistence			
$\gamma = 2$	2.26	2.26	1.61
$\gamma = 10$	8.85	8.83	1.62
Data	8.92	18.63	9.37

The data are from Campbell (1999) using the long sample 1891-1994 and Cochrane (1991). γ refers to the coefficient of risk aversion applied to the power utility function in the model.

Table 3: **Asset Return Volatility and the Business Cycle**

	Recession	Boom
Standard RBC model		
STD (r^f)	1.52	1.41
STD (r^i)	0.44	0.44
STD (r^e)	1.52	1.41
Financing Constraints and Low Persistence		
STD (r^f)	1.78	0.99
STD (r^i)	0.17	0.17
STD (r^e)	1.77	0.93

Table 4: **Interest Rates and the Business Cycle**

Correlation with Output	**Interest Rate**
Standard RBC model	-0.08
Financing Constraints	-0.82
Financing Constraints and Investment Shocks	-0.73
Financing Constraints and Low Persistence	-0.91
Financing Constraints, Investment Shocks, and Low Persistence	-0.88
Data	-0.35

The coefficient of risk aversion applied to the power utility function in the model is set to 10 and autocorrelation of shock 0.95 and 0.8 for low persistence. The data is taken from King and Rebelo (1996).

Table 5: **Business Cycle Properties**

	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_I}{\sigma_Y}$
Standard RBC model		
$\gamma = 2$	0.70	2.21
$\gamma = 10$	0.55	2.47
Financing Constraints		
$\gamma = 2$	0.84	1.52
$\gamma = 10$	0.80	1.66
Financing Constraints and Investment Shocks		
$\gamma = 2$	0.84	1.52
$\gamma = 10$	0.79	1.70
Financing Constraints and Lower Shock Persistence		
$\gamma = 2$	0.84	1.52
$\gamma = 10$	0.67	2.10
Financing Constraints, Investment Shocks and Lower Shock Persistence		
$\gamma = 2$	0.84	1.52
$\gamma = 10$	0.68	2.08
Data	0.74	2.93

This table reports the relative standard deviation of consumption (σ_C) and investment (σ_I) to output (σ_Y). The data is taken from King and Rebelo (1999.) γ refers to the coefficient of risk aversion applied to the power utility function in the model.

Table 6: **Long Run Business Cycle Moments**

Variable	standard deviation	standard deviation relative to GDP	correlation with GDP	first-order autocorrelation
Standard RBC model				
GDP	3.49	1.00	1.00	0.98
consumption	1.92	0.55	0.96	0.99
investment	8.62	2.47	0.98	0.97
Financing Constraints				
GDP	2.60	1.00	1.00	0.97
consumption	2.08	0.80	0.99	0.97
investment	4.30	1.66	0.99	0.97
Financing Constraints and Investment Shocks				
GDP	2.72	1.00	1.00	0.97
consumption	2.14	0.79	0.99	0.97
investment	4.62	1.70	0.99	0.97
Financing Constraints and Low Persistence				
GDP	2.28	1.00	1.00	0.87
consumption	1.53	0.67	0.99	0.88
investment	4.78	2.10	0.99	0.86
Financing Constraints, Investment Shock and Low Persistence				
GDP	2.32	1.00	1.00	0.87
consumption	1.58	0.68	0.99	0.88
investment	4.82	2.08	0.99	0.86
Data				
GDP	1.81	1.00	1.00	0.84
consumption	1.35	0.74	0.88	0.80
investment	5.30	2.93	0.80	0.87

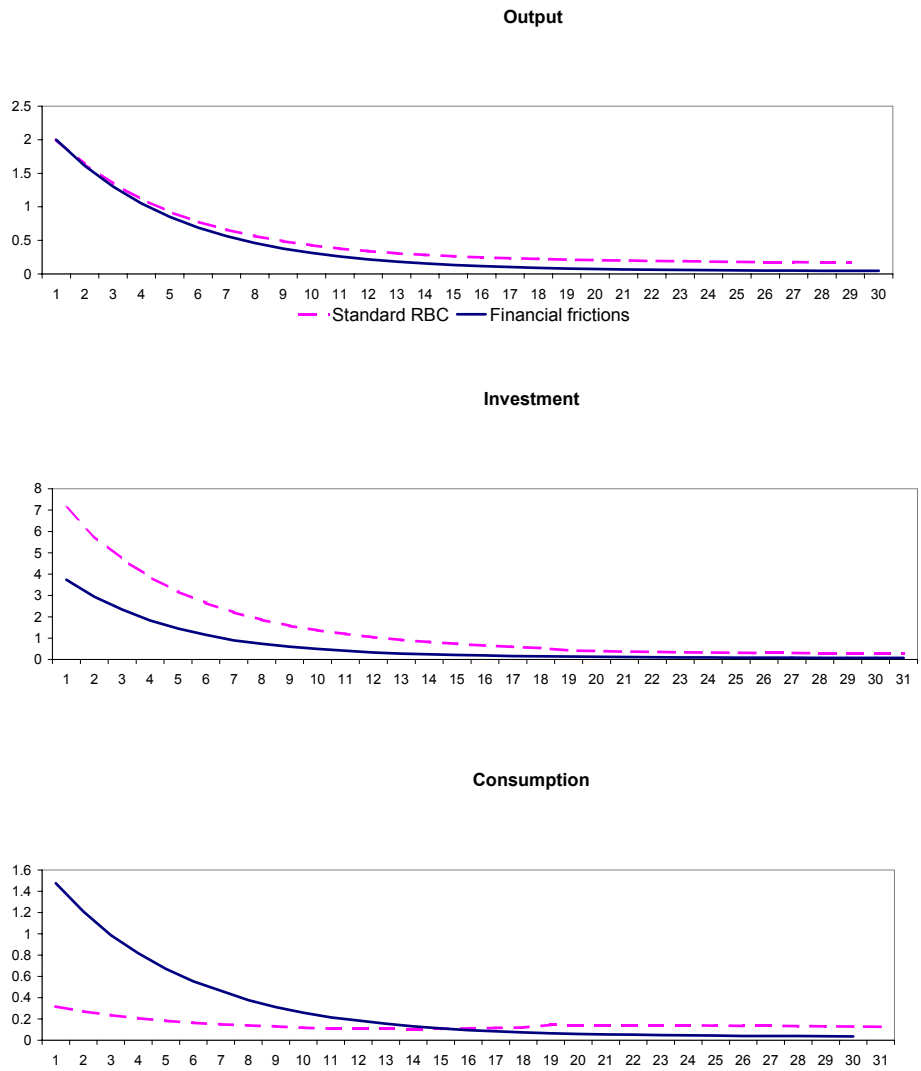
Standard deviations are in percentage terms. The coefficient of risk aversion applied to the power utility function in the model is set to 10. The data was taken from King and Rebelo (1999).

Table 7: **Approximation Errors in Euler Equation**

	$\gamma = 2$	$\gamma = 10$
Mean	1.0000	1.0000
Std	0.0010	0.0039
Minimum	0.9970	0.9861
Maximum	1.0027	1.0140

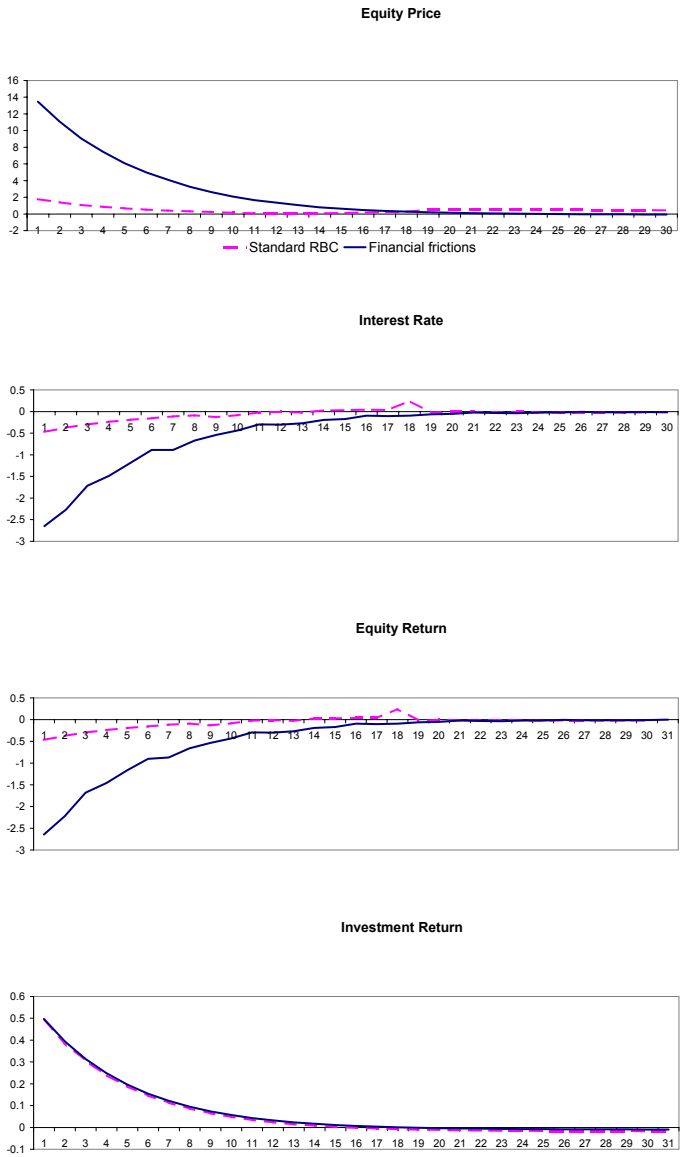
This table reports the Euler equation error. If the state space was continuous the ratio would be unity.

Figure 4: Forecasting Functions for Business Cycle Quantities



Note: Measures the response to a one standard deviation positive productivity shock. Measured as the percent deviation from long run means.

Figure 5: Forecasting Functions for Business Cycle Prices



Note: Measures the response to a one standard deviation positive productivity shock. Measured as the percent deviation from long run means.