Can Financing Constraints Explain the Asset Pricing Puzzles in Production Economies? *

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Abstract

General Equilibrium asset pricing models have a difficult time simultaneously delivering a sizable equity premium, a low and counter-cyclical real risk free rate, as well as cyclical variation in return volatility. To explain these stylized facts, this paper introduces occasionally binding financing constraints that impede producers' ability to invest in an otherwise standard real business cycle model. These financing constraints increase the marginal cost of investing without altering the marginal rate of substitution directly, generating a sizable equity premium as well as matching other standard business cycle quantity and price moments. The financial frictions drive a wedge between the marginal rate of substitution and firms' internal stochastic discount factors so that the shadow value of capital is no longer tied to the average price of capital serving to increase asset price volatility. The model delivers higher and more volatile asset returns during recessions as well as a sizeable equity premium.

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1 Introduction

There are several asset pricing stylized facts that prove difficult to capture in a General Equilibrium (GE) setting. The most notorious is that equity returns are high while the risk free rate is simultaneously low. Campbell (1999) estimates the ex-post premium to be just under seven percent and the risk free rate roughly two percent. While the large equity premium and low risk free rate are well documented and have been extensively explored in the literature, less emphasis has been placed on the counter-cyclical variation in return volatility (Schwert (1989)), short term real interest rates (King and Rebelo (1999)), and the return to equity (Fama and French (1989)).

Counter-cyclical bond and equity returns and return volatility hinge upon investment being constrained. In the extreme, if no investment occurred, the interest rate would be highly counter-cyclical because the expected productivity shocks would completely drive the inter-temporal price of consumption. In the opposite extreme, with perfectly elastic investment the interest rate would be acyclical as households could perfectly offset changes in their expected income stream through investing. To produce counter-cyclical real short term interest rates in an RBC model, investment must be restricted to keep agents from trying to excessively smooth consumption and force the inter-temporal price of consumption to respond to expected productivity shocks. An asymmetric restriction on investment can also explain why return volatility tends to vary counter-cyclically. If consumption is volatile the inter-temporal price of consumption will also be volatile. Schwert (1989) shows a significant increase in volatility for both equity returns and short term interest rates during recessions.

Delivering a sizable equity premium also depends on investment being restricted. Forward looking firms want to invest when investment returns are expected to be high next period. Assuming diminishing marginal returns to capital, greater investment delivers lower ex-post returns. The key to consistently high returns to capital then is to limit capital accumulation. In a GE framework, investment is naturally limited because the interest rate is not constant.[1]. Higher demand for capital must mean higher savings, achieved through lower consumption today. Households only

[1] Since returns tend to vary with the business cycle it makes sense to examine business cycle models to understand asset price dynamics (see Cochrane (2008) for further discussion).

accept lower consumption if the interest rates increase, which slows capital accumulation. While frictionless GE models are therefore able to deliver high returns to capital, these returns are realized through higher interest rates. Even if one increases the elasticity of investment demand by introducing adjustment costs or increases the inter-temporal elasticity of substitution by altering preferences, these changes impact investment returns by altering the inter-temporal price of consumption directly.

The model presented here explains both the cyclical nature of returns and the equity premium by exploring the dynamic effects on producers' decisions if firms face a lower bound on their sources of financing in an otherwise standard real business cycle (RBC) model.[2] Rather than focusing on the financing choice and the costs that may drive that choice, this paper simply supposes there is a limit to these financing options. In the economy, this restriction on financing is evident in stable debt to equity ratios, dividends being bounded by zero, and the fact that equity issuance accounts for less than 5% of total financing. Knowing an upper-bound on financing exists, firms must manage their internal resources so as not to become constrained.

The model works in a similar manner to Gomes, Yaron, and Zhang (2006), where investment is impeded not just by capital adjustment costs but also financing costs. Gomes et al (2006) examine the asset pricing implications from endogenously determined debt constraints found in the financial accelerator literature [3]. Like the financial accelerator literature, they are able to get nice hump shape quantity dynamics, however, due to risk-neutral entrepreneurs and permanently binding constraints the frictions have little impact on mean returns. In addition, they find that if one forces agency costs to be pro-cyclical (as bond spreads seem to indicate) the equity premium would actually be negative. In contrast to Gomes et al (2006), the financial frictions in this model are specified as an occasionally binding constraint rather than a cost function. Under this alternative set-up, the marginal rate of substitution is not affected directly which allows the risk free rate to remain low while the return to investment and equity are driven higher.

Unlike other dynamic GE models with non-trivial production sectors (Jermann (1998) and

[2] As Rouwenhorst (1995) shows, standard RBC models with standard preferences do an abysmal job at capturing asset price dynamics.

[3] Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and Carlstrom and Fuerst (1997).

Boldrin, Christiano, and Fisher (2001)), the model is able to deliver interesting asset price dynamics without necessarily altering preferences.[4] Jermann (1998) modifies preferences to account for habit formation, forcing preferences for consumption to be sticky/persistent and by adding adjustment costs to investing forcing firms to desire sticky/persistent investment. With sticky desired consumption and investment, Jermann (1998) matches quite well the observed mean returns of assets, however he gets the unfortunate by product of very volatile interest rates which is not a feature of the data. In a similar vain, Boldrin, Christiano, and Fisher (2001) show how the combination of preferences for habit formation and adjustment costs to production help to alter both the consumers and producers optimality conditions and therefore match mean returns. Instead of simply including quadratic adjustment costs, they add frictions that limit capital mobility between sectors. In a similar manner to Jermann (1998), they are able to match mean returns but at a cost of introducing increased volatility in those returns. While altering preferences is not necessary to delivering interesting asset price dynamics, the impact of the occasionally binding financing constraints are amplified substantially when habit formation is introduced.

In the model presented here, firms face a lower bound on financing for two reasons. First, long term debt is fixed. Second, short term debt cannot exceed the short term obligations to equity and debt holders. Firms are assumed to hold long term debt to satisfy a target debt to equity ratio. Trade-off theory suggests, the long term target debt-equity ratio is based upon varying costs and benefits of issuing debt which are only in part dependent on phase of the business cycle. Benefits of holding debt include tax advantages and reduced agency costs. Costs of holding debt include increased risk of financial distress and increased monitoring/contracting costs associated with higher debt levels. The simplest way to model the trade-off theory is to include in a two period model the tax advantages of issuing debt and the notion that financial distress can depend on the level of leverage. Because firms are taxed on earnings after interest deductions, there is a tax benefit to shareholders if firms increase leverage. These advantages must be weighed against the additional costs of financial distress. If the firm takes on too much leverage, there may be no payoff

[4] Occasionally binding financial frictions have also been used in partial equilibrium settings (Heaton and Lucas (1996))to try and generate greater asset price volatility. Not only have these partial equilibrium models been ineffective at quantitatively capturing the asset price dynamics, conceptually they do not link the price of risk to macro factors.

to the equity holders being the residual claimant. In this type of set-up, firms end up choosing leverage so that the marginal tax advantage of debt is just equal to the marginal leverage related costs. Following Frank and Goyal (2008), a simple model of this trade-off theory is explained in Appendix 7.1.

Since many of the factors that drive the target debt to equity ratio depend on non-state contingent characteristics, in the model long term debt is assumed to be independent of the business cycle and therefore constant at the business cycle frequency. Figure (1) shows the justification for this assumption. Using the Flow of Funds (FOF) data, the graph plots the variation in long and short term new issues. Because the FOF data reports outstanding debt levels rather than new issues specifically, the long term new issues have to be backed out of the level data as the note on Figure (1) explains. While long term debt outstanding makes up the majority of long term debt, new issues of long term debt as a percent of all credit is quite small and does not seem to show a strong business cycle pattern. In fact the correlation with GDP growth is nearly zero. As indicated on Figure (1), the contemporaneous and lagged correlation between long term debt and GDP is 0.03 and 0.15 respectively.

Firms can issue short term debt in the model, however they face a liquidity constraint that limits the amount of funds borrowed to less than the short term obligations to equity and debt holders.[5] If firms' liquidity constraints are currently binding or expected to bind in the future, the marginal cost of investing increases, limiting investment. The liquidity constraint causes the borrowing costs for firms to differ from the return on savings to the households. As a result, the shadow value of installed capital does not equal the average price of capital. While the return to investment drives the return to equity, these two returns are only equivalent if constraints are never expected to bind. Like an adjustment cost model, the introduction of the financing constraints provide a channel through which investment may be limited other than through the interest rate. Unlike an adjustment cost model, a model with occasionally binding constraints impedes optimal investment

[5] If the short term debt decision were not modeled explicitly, this constraint would simplify to the restriction that dividends could not be negative. While other models such as Gross (1994) and Gomes et al (2006) have also introduced either a non-negativity constraint or a lower bound on dividends, the difference in this model is that given firms' long term debt payments the presence of the constraint has a much larger impact on firms' optimal capital decisions.

without having a secondary impact on consumption through a cost term in the resource constraint. Investment is hindered, the marginal rate of transformation altered, but consumption and thus the interest rate is not overly volatile.

Occasionally binding financing constraints restrict investment in an asymmetric manner. The constraints have a higher probability of binding when firms are expecting a positive productivity shock next period but their capital stock is currently low, which typically occurs when the economy switches from a bad to a good state. This leads to a pro-cyclical return to investment. Investment returns tend to be high during booms when the marginal product of capital is high and investment is limited by financing constraints. In contrast, interest rates and equity returns tend to be counter-cyclical. Because investment is impeded, productivity shocks drive up the inter-temporal price of consumption. The higher interest rates during downturns, in turn, drive up the return to equity. The fact that investment is restricted mainly during booms also delivers asymmetry in the volatility of returns across the business cycle. During downturns investment is not impeded and the model behaves in a similar fashion to a standard business cycle model. In contrast during an expansion, investment is greatly limited decreasing the volatility of investment and therefore the return to investment as well as consumption, the interest rate and equity return. In this manner the financial frictions force the volatility of asset returns to vary in a counter-cyclical fashion.

2 The Structure of the Model

The model is set up as decentralized dynamic stochastic GE problem.[6] Firms possess the production technology and make optimal investment decisions. However, in certain states of nature, capital market imperfections force the supply of financing to be perfectly inelastic. Households work and save by purchasing claims on the value of firms, in the form of stocks and bonds.

[6] In many ways this model can be interpreted as a GE version of Aiyagari & Gertler (1999). They show how margin requirements for investors and portfolio costs for savers (consumers) drive equity prices above their fundamental value, generating excess volatility to equity returns over the exogenous risk free rate. By fixing the risk free rate, however, they are unable to explain why the return on the two savings vehicles behave differently.

2.1 Households

This closed economy is characterized by a large number of identical infinitely-lived households. The households choose consumption, equity holdings in firms, and bonds respectively to maximize their lifetime utility:

$$\mathbf{Max}\ \ \mathbf{E_o}\sum_{\mathbf{t=0}}^{\infty}\beta^{\mathbf{t}}\ \mathbf{u}[\mathbf{c_t}] \tag{2.1}$$

.

For the numerical results three different preference specifications are used. The first, most basic, is time separable utility where the households simply get utility from consumption. Since households do not receive utility from leisure, they choose to work their full endowment of time. Second, habit formation is added in a similar manner to Jermann (1998) and Boldrin, Christiano, and Fisher (2001). Under this specification, habit formation is dependent upon the households own past consumption, $U(c_t - \zeta c_{t-1})$. Habit persistence can potentially help in understanding asset price movements because this specification magnifies the effect on utility from small changes in current consumption. Third, a labor choice is added to the model. Following Greenwood et al. (1988) (GHH), households derive utility from a composite commodity $c - G(l)$, where $G(l)$ is a concave, continuously differentiable function that measures disutility of labor. The GHH composite good neutralizes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor supply depend only on the latter. Introducing a household labor decision, forces the model to explain key business cycle facts concerning labor. In addition, including a labor supply function allows the households to smooth their consumption through changes in labor. Being able to adjusts labor to smooth consumption could potentially minimize the impacts of the financial frictions in the model.

Households make their decisions based on the following period budget constraint:

$$\frac{b_{t+1}}{(1+r_t)} + p_t(s_{t+1} - s_t) + c_t \leq w_t l_t + \overline{RB} \ + b_t + div_t s_t \ \ , \tag{2.2}$$

where s_t represent the purchase of stocks between households and b_t represents the borrowing or lending by households. They earn income by working for w_t and receiving dividends on their equity holdings. They are able to transfer income through time by lending or buying stocks and bonds to smooth their consumption. Households take the price of the stock, p_t, and the return on the risk free bond, $(1 + r_t)$, as given.

2.2 Firms

A large number of identical firms produce an identical good using both labor (l_t) and capital (k_t). Labor and capital are combined using constant returns to scale technology $exp(\epsilon_t)F(k_t, l_t)$. This production function is subject to productivity shocks. Firms must pay labor at a market rate of w_t as well as make investments (i_t).

Productivity shocks follow a symmetric Markov process, exhibiting simple persistence. This specification minimizes the size of the exogenous state space without restricting the variance and first-order autocorrelation of the shocks. The shocks take a high or low value, $E(e_H, e_L)$. Symmetry implies that $e_L = -e_H$, and that the long-run probabilities of each state satisfy $\Pi(e_L) = \Pi(e_H) = 1/2$. Transition probabilities follow the simple persistence rule (see Backus, Gregory and Zin (1989)). Under these assumptions, the shocks have zero mean, their variance is $(e_H)^2$. Firms pay out dividends according to the following budget constraint:

$$div_t = \ exp(\epsilon_t)F(k_t, l_t) - w_t l_t - i_t - \overline{RB} \ - b_t + \frac{b_{t+1}}{(1+r_t)}. \tag{2.3}$$

Firms receive income from producing $F(k_t, l_t)$ and pay workers $w_t l_t$ as well as invest i_t in order to alter the amount of capital used in production next period.

Varying from a standard RBC model, it is assumed that firms hold a constant amount of long term debt on which it must pay interest each period $\overline{RB}$ as well as short term (one period) bonds. This implies that changes to the debt to equity ratio for the representative firm are due entirely to changes in short term financing. As the trade-off theory suggests, the long term target debt-equity ratio is based upon varying costs and benefits of issuing debt which are in large part unrelated to the phase of the business cycle. Since many of the factors that drive the target debt to equity ratio depend on non-state contingent characteristics, long term debt is assumed to be constant. Masulis (1988) shows that historically debt to book value ranges from .53-.75 for all non-farm non-financial corporations. One feature of the data is that these target debt to equity ratios do vary tremendously by industry and firm size. Therefore, a heterogenous firm model would need to explain why the optimal capital structure varies across firms. In contrast, with a representative firm, it is less restrictive to assume firms have constant long term debt obligations.

While long term debt is assumed to be fixed in the model, total debt to equity does vary due to changes in short term debt. Firms may take on short term debt (one period) to cover any cyclical financing needs. The total short term debt is captured in b_{t+1}. $(1 + r_t)$ is the risk free rate determined by the households.

Capital evolves according to the following standard equation of motion:

$$k_{t+1} = i_t + (1 - \delta)k_t. \tag{2.4}$$

As evident in (2.4) the firms do not face any direct costs to altering their capital stock. There is a large literature showing that adjustment costs to capital (Jermann (1998) and Boldrin, Christiano, and Fisher (2001)) as well as adjustment costs to altering investment directly (Beaubrun-Diant and Tripier (2005)) may be important for explaining investment dynamics. By leaving out adjustment costs in the baseline model, the financing constraints are forced to be the primary determinant of the investment dynamics. The model therefore can explore whether financing constraints themselves may eliminate the need for additional adjustment costs to slow down investment. In the numerical results, a capital adjustment costs function is added to the baseline model to isolate the additional benefit of including them. The costs to adjusting the capital stock depends on net investment and is added using the following function $\phi/2(k_{t+1} - k_t)^2$. This term gets subtracted from the right hand side of (2.3).

A key component of the model is that firms face limits on external financing. Since the goal is to understand the impact on financing constraints on asset prices rather than on explaining financing patterns over the business cycle, a simple way to model this is through a constraint on dividends and short term borrowing. In choosing optimal capital to maximize its value, firms face the following non-negativity constraint:

$$div_t + b_t \geq \frac{b_{t+1}}{(1 + r_t)}. \tag{2.5}$$

Equation (2.5) effectively limits short term debt positions to be less than current obligations to equity and debt holders. By rearranging (2.5), this constraint can be interpreted as the ratio of short term debt to current obligations must be less than one:

$$1 \geq \frac{b_{t+1}}{(1 + r_t)(div_t + b_t)}. \tag{2.6}$$

This type of constraint is consistent with the trade-off theory of firms' capital structure. If firms have some target debt to equity ratio then it is likely they would want to limit their new debt obligations to be less then their current obligations to all claimants. This would work to keep the firm in the neighborhood of the target debt-to-equity ratio. If short-term debt is not explicitly modeled (so that households borrow/lend to each other and purchase shares in the firms), this constraint reduces to a simple more traditional constraint that dividends have a lower bound, possibly zero (Gomes, Yaron and Zhang (2006)). Allowing negative dividends would be analogous to firms being able to issue new equity shares. While certainly secondary equity offerings do occur, empirical evidence suggests firms do not choose to offer new equity very often. Theoretically, one could argue issuing equity sends a bad signal concerning firms' investment prospects. As discussed in Gross (1994) and Frank and Goyal (2008) equity issues make up less than 5% of external financing.[7]

Combining (2.3), (2.4), and (2.5) the financing constraint can also be seen as a limitation on the use of internal resources of firms. In a similar manner to Gross (1994), the constraint can be interpreted as a non-negativity constraint on firms' cash flows:

$$exp(\epsilon_t)F(k_t, l_t) - w_t l_t - k_{t+1} + (1-\delta)k_t - \overline{RB} \ \geq 0. \tag{2.7}$$

The firms' objective is to maximize the discounted value of cash flows. The internally generated cash flow gets paid out in the form of dividends as follows:

$$\mathbf{E_t} \sum_{i=1}^{\infty} \theta_{t+i}[div_{t+i}]. \tag{2.8}$$

Since the households own the firms, adjusted dividends will be discounted at the intertemporal marginal rate of substitution. So that $\theta_{t+i} = \beta^i(\frac{u'(c_t+i)}{u'(c_t)})$, which firms take as given. Firms choose i_t, b_{t+1}, div_t to maximize adjusted dividends, satisfying their budget constraint and the financing constraints.

2.3 Households' behavior

The first order condition with respect to equity reveals the equation for the price of equity as follows:

[7]Given the limits on financing, it may be interesting to think about how firms manage their holdings of cash balances in order to avoid hitting their constraint. Currently, firms must pay out all positive cash flow in the form of dividends to households.

$$p_t = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \ (p_{t+1} + div_{t+1}) \right] \ . \tag{2.9}$$

Since it is assumed that firms cannot issue equity, the number of shares is normalized to one. Given these assumption, if (2.9) is iterated forward, the value of the firms next period is simply the discounted dividends which corresponds to the firms' objective function (equation (2.8)).

The first order condition (FOC) for bonds provides the following Euler equation:

$$1 = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right] \ (1 + r_t) \ . \tag{2.10}$$

While these equations may appear standard, there are two things to note. First, unlike a GE model with capital or investment adjustment costs the resource constraint which determines aggregate consumption and thus the marginal rate of substitution, is not affected by any costs, but is simply the residual of output less investment. Second, due to the limit on sources of financing a wedge emerges between the firms' stochastic discount factor and the households' marginal rate of substitution, determined by (2.10).

2.4 Firms Behavior

When firms choose capital, dividends, and bonds next period taking the discount rate on dividends as given, the following first order conditions emerge.[8] η_t and η_{t+1} are the nonnegative multipliers associated with the current and future financing constraint respectively:

$$1 + \eta_t = E_t \left[\theta_{t+1} (1 + \eta_{t+1}) \left(exp(\epsilon_{t+1}) F_{k_{t+1}} + (1 - \delta) \right) \right] \tag{2.11}$$

[8] The FOCs also include the three constraints and the corresponding Kuhn Tucker conditions.

$$1 = E_t\left[\theta_{t+1}\right]\left(1 + r_t\right). \tag{2.12}$$

The right hand side of (2.11) is the benefit of investing one unit today. Tomorrow, that unit provides firms with increased output, captured in the marginal product of capital. This benefit is then multiplied by the shadow value of capital tomorrow, $(1 + \eta_{t+1})$. The left hand side represents the cost of investing using internal resources. If firms are investing then dividend payments are decreased. If the financing constraints are currently binding the marginal cost of financing rises above one as η_t would be non-zero.

The marginal cost of investing using debt is determined by (2.12). Looking at (2.12), the financing costs using debt depends on the endogenous risk free rate, r_t, and the marginal rate of substitution, θ_{t+1}, which firms take as given. Taking equations (2.11) and (2.12) and combining the following equation emerges:

$$E_t\left[\theta_{t+1}\right]\left(1 + r_t\right) + \eta_t = E_t\left[\theta_{t+1}(1 + \eta_{t+1})\left(exp(\epsilon_{t+1})F_{k_{t+1}} + (1 - \delta)\right)\right]. \tag{2.13}$$

Equation (2.13) suggests that the marginal cost of investing using internal resources or debt is the same. Because both the amount of dividends paid out and short term debt obligations made are important for the financing constraint, the firm is constrained equally using either form of financing.

The cost of investing today differs from one if the financing constraint binds. If the financing constraint is not currently binding and is never expected to bind then the model collapses to the frictionless model where the shadow value of capital is always equal to one. Just like a capital adjustment cost model (Jermann (1998)), the financing constraint causes the cost of investing to be highest during the initial phase of an economic expansion, because that is when the occasionally binding constraints become relevant. To avoid being constrained, investment is delayed causing high returns to already installed capital.

Occasionally binding financing constraints impact the investment return but do not impact the marginal product of capital itself or the households' marginal rate of substitution directly. In an adjustment cost model, investment returns, which drive equity returns, have a first order dependence on adjustment costs. As firms invest, there is more capital next period, which lowers the cost of investing. These costs also directly impact households' marginal rate of substitution. Through the resource constraint, an increase in adjustment costs impede investment but are wasted resources and cannot be consumed, which reduces consumption today, causing the interest rate to rise. Adjustment costs in and of themselves can generate high investment returns but have the unfortunate by product of driving down the marginal rate of substitution and thus increasing the interest rate.

From (2.11) and (2.12) it is clear how these financing constraints impact the stochastic discount factor of the firms. Firms discount the return on investment not only by the households' marginal rate of substitution but also by next period's shadow value of capital which in turn depends on whether the constraints will bind next period. Therefore, the return on investment next period is driven by current and future financing constraints. In this manner, the cost of financing depends on the state of the economy and varies over the business cycle. Changes in endogenous quantities of the capital stock, the shadow value of capital next period, and the households' marginal rate of substitution affect the likelihood that the constraints binds and the marginal cost of investing.

2.5 Competitive Equilibrium

Given the exogenous stochastic process for the productivity shocks and initial states k_t, b_t, a competitive equilibrium is defined by sequences of state-contingent prices $w_t, p_t, \theta_t, 1 + r_t$ and allocations $k_{t+1}, b_{t+1}, c_t, i_t$ such that: (a) firms maximize the expected discounted dividends subject to CRS technology, the law of motion for capital, and the financing constraint; (b) households choose b_{t+1}, c_t, s_t to maximize expected discounted utility subject to their budget constraint; (c) the following markets clear.

the goods market,

$$c_t + i_t = F(k_t, l_t) \tag{2.14}$$

the bond market,

$$b_t = b_t^d \tag{2.15}$$

and the equity market,

$$s_t = s_t^d = 1. \tag{2.16}$$

2.6 Asset Prices and Returns

There are three key differences between this model and the standard RBC model that help match asset price dynamics. First, the investment demand function is kinked in some states of nature. Second, the price of equity depends on the probability of the constraints binding and not simply on the level of the desired capital stock. Last, the shadow value of capital does not equal the average value of capital generating more volatility in the capital stock.

2.6.1 The Risk Free Rate

The financing constraints generate a kink in investment demand in a similar manner to Fazzari, Hubbard, and Petersen (1988). Combining the FOC for the firms and the definition of dividend an investment demand emerges [9]:

$$i_t = \frac{1}{(1+r_t)} \frac{E_t\left[(1+\eta_{t+1})\left(\tilde{div}_{t+1}\right)\right]}{[1 - cov(\theta_{t+1}, (1+\eta_{t+1})mpk_{t+1})]} - (1-\delta)k_t \tag{2.17}$$

where $\tilde{div}_{t+1} = div_{t+1} + k_{t+2} + (1+r_t)b_{t+1} + \overline{RB} - b_{t+2}$. and $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$.

If the financing constraint does not currently bind, investment is influenced positively by higher expected dividends, more depreciated capital stock, and the probability the financing constraint may bind tomorrow ($(1+\eta_{t+1})$). While the first two factors are standard, the third indicates that if firms are expecting to be constrained in the future they invest more today to relax the future

[9] The derivation of 2.17 is found in Appendix 7.2.

constraints. Looking at the denominator, investment depends negatively on the interest rate. The sensitivity of the investment to the interest rate, however, depends on the risk premium on investing, as captured by $[1-cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1})]$. A higher negative covariance between the stochastic discount factor and the return to investment lowers the amount of investment a firm is willing to take on at a given interest rate. If the financing constraint binds, investment demand becomes inelastic with respect to the interest rate. Firms simply invest what they can while satisfying the financing limitation:

$$i_t = exp(\epsilon_t)Fk_t, l_t - w_t l_t - \overline{RB}. \tag{2.18}$$

Due to this kink in the investment demand function, when the constraint binds the equilibrium interest rate and investment are less than they would be in a frictionless world, as seen on Figure 2.

On the other hand, the households' savings function is always upward sloping with respect to the risk free rate. Rearranging (2.10) we get the following standard savings function:

$$s_{t+1} = y_t - u_c^{(-1)}\left(E_t(\beta u_c(c_{t+1}))(1+r_t)\right) \tag{2.19}$$

where $y_t = div_t s_t + w_t l_t + b_t + \overline{RB}$.

As Figure 2 shows, the constraints generate a wedge between the rate firms are willing to pay to borrow funds and the price the households receive. Investment demand is inelastic with respect to the interest rate once the constraints bind. The marginal rate of substitution is higher than in the frictionless case, which translates into a lower risk free rate, $(1+r^c)$. [10]

[10]Even if the financing constraint does not currently bind, the potential for the constraints to bind in the future does impact the slope of investment demand as reflected in the partial derivative of the investment function with respect to the risk free rate.

2.6.2 The Price of Equity

With occasionally binding financing constraints, the price of equity is driven by the level of the capital stock (as in a standard RBC model) but also depends on the probability of being constrained. This implies two important factors in the determination of the equity price relative to the frictionless model. First, since the probability of becoming constrained is state-dependent, additional asset price volatility is generated. Second, the average value of capital no longer has a one to one relationship with the shadow value of capital, resulting in a more volatile marginal cost of investing.

Working with the households' FOC with respect to equity shares and the definition of dividends, one can link the price of equity to its endogenous factors, all of which are time varying at the business cycle frequency (see Appendix 7.3 for derivation):

$$p_t = k_{t+1} + \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty} \theta_{t+i}\eta_{t+i}(k_{t+i+1} - mpk_{t+i}k_{t+i})\right] - \frac{b_{t+1}}{(1+r_t)} - E_t\sum_{i=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB}). \quad (2.20)$$

If the financing constraint never binds then $\forall(t)\eta_t = 0$ and $p_t + \frac{b_{t+1}}{(1+r_t)} + E_t\sum_{t=1}^{\infty}(\prod_{j=1}^{i-1}\theta_j\overline{RB}) = k_{t+1}$ or in other words, the total market value the firms (equity and debt) is equal to the replacement costs. However, if the constraint binds today or has a positive probability of binding in the future, then the value the firms may differ substantially from the replacement value. Breaking the direct link between the capital stock and the market value of the firms is key for any model trying to capture observed asset price volatility. As Rouwenhorst (1995) points out, the capital stock is not particularly variable, therefore, any model which relies exclusively on the volatility in capital to explain asset price volatility will have a difficult time replicating stylized facts. Financing constraints introduce an additional source of volatility to equity prices. Because these are occasionally binding constraints, the shadow value on the constraint does not always bind. The constraints have a higher probability of binding when firms expect a positive productivity shock next period. Since the probability of becoming constrained depends on the stage in the business cycle, equity prices tend to move in a cyclical fashion.

2.6.3 Marginal q no longer equals average q

Another implication of (2.20) is that the shadow value of capital (marginal q) for firms does not equal the market value over the replacement value (average q). Under this scenario, investment should become more volatile, generating greater variance in the capital stock itself. If the contrary holds, there will be a smoothing impact on investment and the capital stock. To see the breakdown of the relationship of marginal to average q, define the total market value of firms (debt and equity claimants) as $V_t = p_t + \frac{b_{t+1}}{(1+r_t)} + E_t \sum_{i=1}^{\infty} (\prod_{j=1}^{i-1} \theta_j \overline{RB})$ then substitute into 2.20:

$$\frac{V_t}{k_{t+1}} = (1 + \eta_t) + \frac{E_t \left[\sum_{i=1}^{\infty} \theta_{t+i} \eta_{t+i} (k_{t+i+1} - mpk_{t+i} k_{t+i}) \right]}{k_{t+1}}. \tag{2.21}$$

The total value of firms over the replacement value will not only differ from unity but also from the shadow value of capital today, $(1 + \eta_t)$. The market value divided by the replacement value $\frac{V_t}{k_{t+1}}$ depends on the marginal value of capital tomorrow $(1 + \eta_{t+1})$ as well as the probability the financing constraint binds in future, captured in the second term. If the financing constraints are expected to bind today or any period of time in the future, the current market value of the firm rises above the replacement value. A unit of capital inside the form will be worth more than outside the firm.

2.6.4 The Return to Investment

Given that the capital stock next period is not necessarily the same as the market value of the firms, it follows that the return to investing by firms and the return to equity the households receive will differ. Deriving each return separately we can determine the factors driving each and understand the magnitude of the wedge between them. If the model is to succeed in replicating stylized facts, this wedge should not be large given Cochrane (1991) which shows similar dynamics between the two ex-post returns.

The financing constraints are able to limit investment by generating a financing premium and thus driving up the return to investment. Rearranging the firms' FOC with respect to capital equation (2.11) the following equation emerges for the determination of the expected return to investing:

$$E_t[mpk_{t+1}] = \quad (1+r_t) \quad - \quad \frac{cov(\theta_{t+1}, mpk_{t+1})}{E_t[\theta_{t+1}]} - \frac{cov(\theta_{t+1}, \eta_{t+1} mpk_{t+1})}{E[\theta_{t+1}]} \quad + \frac{\eta_t}{E[\theta_{t+1}]} - E[\eta_{t+1} mpk_{t+1}] \tag{2.22}$$

or

$$E_t[mpk_{t+1}] = \quad (1+r_t) \quad - \quad \frac{cov(\theta_{t+1}, mpk_{t+1})}{E[\theta_{t+1}]} \quad + \quad FP_t, \tag{2.23}$$

where $FP_t = -\frac{cov(\theta_{t+1}, \eta_{t+1} mpk_{t+1})}{E[\theta_{t+1}]} + \frac{\eta_t}{E[\theta_{t+1}]} - E[\eta_{t+1} mpk_{t+1}]$. In this case, the marginal product of capital represents the derivative of the production function with respect to next period's capital stock net depreciated capital, $mpk_{t+1} = exp(\epsilon_{t+1}) F_{k_{t+1}} + (1 - \delta)$. From the FP_t term, we can see that the financing premium is time varying and tends to increase when the constraints are more likely to bind. Given this, the financing premium is largest when productivity turns positive following a sequence of bad shocks. In other words, firms are most constrained as the economy flips from a bad state to a good state.

Given a positive probability the constraints will bind in the future, the financing premium increases the return to investing relative to standard frictionless case. The return is no longer simply driven by the negative covariance between the marginal rate of substitution and the marginal product of capital, but also by the relationship between the relative returns and the financing constraints. More specifically, we can decompose the financing premium into several factors. First, there exists a negative covariance between the marginal rate of substitution and the product of the return to investing and the financing constraint. Second, if the constraint binds today firms cannot invest and returns to investing are expected to be higher.

2.6.5 The Return to Equity

While the firms' returns to investing directly impact the return households get from owning the firms, the relationship is no longer one to one. With the potential to hold both short and long term debt and financing constraints binding in certain states of nature, the difference between a value of a dollar inside and outside the firms is reflected in respective returns to investing and owning a firm. Given the households FOC with respect to equity (2.11) and the definition of dividends (2.3), the following equation emerges:

$$E_t[R^e_{t+1}] \quad - \quad R^f_t \quad = \quad - \quad \frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]} \frac{k_{t+1}}{p_t} \quad - \quad \frac{cov[\theta_{t+1}, (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})})]}{p_t E[\theta_{t+1}]}, \qquad (2.24)$$

where $R^f_t = (1 + r_t)$ and $R^e_{t+1} = \frac{div_{t+1}+p_{t+1}}{p_t}$. Without debt or financial frictions, the market value of capital is equal to replacement value of capital (2.20) so $\frac{k_{t+1}}{p_t} = 1$. In this case (2.24) reduces to the frictionless case where the equity premium is simply driven by the fact the equity pays off well in states of nature where the households do not care much for that additional payoff, captured in a negative covariance between the MRS and the return to equity.

If firms have the option to hold debt but financing constraints are never binding then the equity premium is amplified by the amount of debt relative to the price of equity. Applying (2.20) without financial frictions, we get the following:

$$E_t[R^e_{t+1}] \quad - \quad R^f_t \quad = \quad - \quad \frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1 + \frac{\frac{b_{t+1}}{(1+r_t)} + E_t \sum_{i=1}^{\infty}(\prod_{j=1}^{i-1} \theta_j \overline{RB})}{p_t}) \qquad (2.25)$$

Equation 2.25 is essentially a GE version of Proposition II from Modigliani-Miller(1958). The equity return is affected by the risk adjusted debt to equity ratio. Although firms' financing decisions do not impact the value of the firms, they do impact the return to equity holders in the presence of uncertainty.

As long as their is some potential that the financing constraints may bind then the wedge the equity return and investment return grows wider. For notational purposes define $\tilde{b_{t+1}} = \frac{b_{t+1}}{(1+r_t)} + E_t \sum_{t=1}^{\infty}(\prod_{j=1}^{i-1} \theta_j \overline{RB})$ or as total short and long term debt positions. With occasionally binding financing constraints the return on equity is determined as follows (see Appendix 7.4 for derivation):

$$E_t[R^e_{t+1}] - R^f_t = -\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1 + \frac{\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t[\sum_{i=1}^{\infty} \theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})]}{p_t})$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t}. \tag{2.26}$$

or

$$E_t[R^e_{t+1}] = \quad E_t R^i_{t+1} - \quad \frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]} \frac{\left(\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty} \theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})\right]\right)}{p_t}.$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t} + FP_t, \tag{2.27}$$

where $E_t R^i_{t+1} = E_t[mpk_{t+1}]$. Consistent with any frictionless GE asset pricing model with leverage, the equity return is driven by the investment return $E_t R^i_{t+1}$ and the risk adjusted leverage term $\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]} \frac{\tilde{b_{t+1}}}{p_t}$. In addition to these standard factors, the occasionally binding constraints further increase the equity return as reflected in the remaining three terms.

3 Recursive Form and Numerical Solution Technique

The model is solved using numerical methods that work off the recursive set-up of this equilibrium. The state space is defined as k as well as the exogenous state ϵ that is driven by the aforementioned simple persistence rule. The endogenous state space is defined by the discrete set $Z = [k_L, k_H]$. Assume continuous, nonnegative equity pricing function and interest rate $p(k, \epsilon) : ExZ \rightarrow R+$ and $r(k, \epsilon) : ExZ \rightarrow R+$ that are taken as given by the firms and households. The bounds of $p(k, \epsilon)$ and $r(k, \epsilon)$ follow from the bounds of E and Z. Assuming that the short-term bond market clears, the following dynamic programming problem emerges:

$$S(k,\epsilon) = Max\frac{(c^{(1-\sigma)}-1)}{1-\sigma} + \beta ES(k',\epsilon') \tag{3.1}$$

subject to:

$$c = exp(\epsilon)f(k,\bar{l}) + (1-\delta)k - k' \tag{3.2}$$

$$exp(\epsilon)f'(k,\bar{l})k + (1-\delta)k - k' - \overline{RB} \geq 0\ . \tag{3.3}$$

Given the concavity of the utility function and assumptions about the shock, a unique solution to the value function is determined. The pricing functions and the decisions rules that emerge once prices have converged constitute a competitive equilibrium for the model. The state space of capital spans the interval $[K_l, K_h]$ with NK discrete nodes.

The solution method uses aspects of Mendoza and Smith (2006), Heaton and Lucas (1996), and Krusell and Smith (1997). What makes the problem difficult to solve are the occasionally binding financing constraints. For this reason simple policy function iteration is not used. Instead, the algorithm is centered on value function iteration over a discretized grid in a similar manner to Heaton and Lucas (1996) and Mendoza and Smith (2006). The downside of using value function iteration with a discrete grid is that the FOCs do not hold exactly. The larger the state space and finer the grid the less approximation error exists. The quantity moments do not change much as the grid gets finer. A fairly coarse grid can replicate reasonable moments. The price moments, on the other hand, are extremely variable when the grid is too coarse.

A discrete representation of state space for the households' problem is defined by (k,ϵ). This is done by calculating the steady state value of k and using that to center the grid. The outcome will yield decision rules $k'(k,\epsilon)$. The decision rules that solve (3.1) maximize the utility of domestic agents taking into account the economy's resource constraint, the financing constraint, the optimal rules determining wages, and the market-clearing condition of the bond market. Thus, the prices and allocations supported by the Bellman equation satisfy the following properties of the competitive equilibrium: (a) given wages, equity prices, and the risk free rate, c, b',and s' solve the constrained maximization problem of households (b) given the households MRS, div, k', and b' solves the maximization problem of firms, and (c) the market-clearing conditions for equity, goods and bonds hold.

3.1 Calibration and Functional Forms

For the numerical analysis of the baseline model, the following functional forms are used:

$$F(k_t, l_t) = Ak_t^{1-\alpha} l_t^{\alpha} \tag{3.4}$$

$$U(c_t) = \frac{[c_t^{1-\sigma} - 1]}{1 - \sigma} \ . \tag{3.5}$$

Looking at the production function, α is the share of output allocated to capital. In equation (3.5), σ is the coefficient of risk aversion. Given these functional forms, the algorithm needs values for the following vector of parameters $[A, \alpha, \beta, \delta, \gamma, \overline{RB}, \epsilon_H, \epsilon_L]$. The first three parameters are set to be consistent with the RBC literature. A is simply set to one. Reasonable estimates of α, the capital share lie anywhere from 0.3 to 0.35, so its set to $\alpha = 0.32$. The quarterly rate of time preference is $\beta = 0.991$. Depreciation rates often vary in the literature. Here, $\delta = 0.02$ on a quarterly basis. The coefficient of relative risk aversion varies is 5.0 as in Jermann (1998). In terms of long term debt holdings, Masulis (1988) shows that historically debt to book value ranges from .53-.75 for all non-farm non-financial corporations. Following Masulis (1988) the level of debt is chosen so that the debt-to capital ratio is in the middle of the range at 60%. Because the leverage ratio determines how likely the financing constraints are to bind, the results show both relatively high and low leverage states within this range. The long term fixed interest rate is chosen to be 200 basis points above the steady state risk free rate, in line with historical premiums. The shocks are calibrated to the U.S. economy. The standard deviation and first-order auto-correlation of the Markov process are set to match those from a typical business cycle. This implies a standard deviation of 0.01785 and, in the baseline case, persistence of $\theta = 0.95$.

The baseline model is modified to examine the role of habit formation, the labor/leisure choice, and adjustment costs in a model with financing constraints. To get at these alternative preferences and adjustment costs several more parameters need to calibrated. For habit formation, the degree to which past consumption impacts current utility needs to specified. This corresponds to the ζ parameter in the following function, $U(c_t - \zeta c_{t-1})$. Empirical studies do not offer much guidance in choosing this parameter. Therefore, like Jermann (1998)and Boldrin, Christiano, and Fisher (2001), ζ is best chosen to match the moments of the data. This is achieved by setting $\zeta = 0.3$, a lower degree of habit persistence than $\zeta = 0.82$ and $\zeta = 0.73$ calibrated in the previous studies. The labor versus leisure is modeled following Greenwood et al (1988). Households derive utility from a composite commodity $c_t - G(l_t)$, where $G(l) = \frac{l^\gamma}{\gamma}$. Lacking information on the wage elasticity of the labor supply, as in Mendoza and Smith (2006), the calibration assumes unitary elasticity so $\gamma = 2.0$. The last alteration is the addition of an adjustment costs function, $\phi/2(k_{t+1} - k_t)^2$. The adjustment cost parameter ϕ is calibrated so that average costs of adjustment are less than 0.1 percent of output (like Mendoza (1991)), consistent with widely agreed upon idea that adjustment costs are small but economically significant as in Summers (1981).

4 Results

Occasionally binding financing constraints alter asset price dynamics, by limiting firms' ability to accumulate capital. This is easiest to see by examining the limiting distribution of capital implied by the decision rules for capital as well as the productivity shocks. Figure 3 compares the long run distribution of capital in the basic RBC model to one where firms have the potential to be constrained given their target debt to equity position. The constraints alter the mean of capital as well as the standard deviation. While clearly the constraints have forced less capital accumulation, the distribution still is a standard bell rather than being truncated on the left. This suggests firms may not actually be constrained very often but rather adjust their capital stock so as to avoid being constrained in the future. For empirical studies this implies that ex-post one might not find a significant amount of truly constrained firms. However, that would not imply that the constraints are not impacting firms' decisions.

While the constraints have clearly altered the distribution of capital the next question is to examine how this translates into the moments of the aggregate prices. Table 1 compares various returns when financial frictions are added to a standard RBC model. In addition to adding the financial frictions, Models III-IX make additional parameter changes to the basic RBC model. Model III explores the impact of increasing long term debt relative to capital in the model. This is important for isolating the impact of leverage on the financing constraints. Model IV varies the persistence of the productivity shock. Given data problems estimating aggregate total factor productivity and recent empirical evidence that plant and industry level productivity shock persistence seems to lie in the range of $.8 - .91$ quarterly, (Abraham and White (2006)) the impact of a lower persistence is tested. Model V examines the interaction between adjustment costs to net investment and the financing constraints. In Models VI and VII, household preferences are altered to account for a labor decision and habit formation respectively. Model VIII includes both habit formation and capital adjustment costs to the basic model with financing constraints. Last, Model IX examines the combination of variable labor, habit formation and financing costs.

Comparing Models I and II on Table (1), we can see that adding financing constraints dramatically increases both the premium on equity and investment. Consistent with theory, from the last column on Table (1) it is evident that the increase in the premium for both investment and equity is due to other factors than simply an increase in the covariance between the marginal rate of substitution and the return on investment. The negative covariance rose from 0.0008 to 0.0012 explaining only a very small portion of the rise. While the equity premium, rose from 0.002 to 0.228, it is still far below the 4.74% evident in the data. Because leverage increases the likelihood that the financing constraints bind, in Model III the premium on investment increases substantially from 0.468 to 1.651, without a significant effect on the other asset returns. In Model IV, by reducing the persistence of technology shocks, investment becomes more volatile doubling the premium on equity and investment versus the simple model with financial frictions. Adjustment costs to altering capital do not appear to have much impact on asset returns. Part of this could be due to the fact that this type of adjustment costs does not alter the capital accumulation equation but simply alters dividends by imposing additional costs on the firms based on their choice of net investment. The introduction of a labor decision in Model VI lowers the interest rate and raises the premium on equity and investment, but not by much. Habit formation in Model VII, on the other hand, seems to a have a dramatic influence in the model with occasionally binding financing constraints, raising the equity premium to 4.287% and the investment premium to 3.091%. Financing constraints limit investment, making consumption less smooth while a preference for habit persistence magnifies small changes in consumption, ensuring that households really care about smooth consumption over time. In this way including habit formation and financing constraints does quite well at matching the asset return data. Adding both variable labor and habit persistence to a model with financing constraints, allows the model to match the mean interest rate as well as deliver a sizeable equity premium as seen in Model IX.

The last column on this table calculates the covariance of the investment return with the marginal rate of substitution. Looking at the basic RBC model, just as theory suggests, the equity premium is exactly equal to the negative of this covariance. As we examine alternations to the basic model with financing constraints, we see this covariance increases but not nearly to the degree of the equity premium. Financial frictions generate a large equity premium without forcing the covariance to be significantly high. The high premium can be attributed to the additional risk factors that are driving the premium.

While Jermann (1998) and Boldrin et al (2001) are also able to generate higher equity returns in models with altered preferences and adjustment cost functions, they only do so, by accepting excessive volatility in the risk free rate. In contrast, as we see in Table 2 when habit formation and adjustment costs are combined with financing constraints the standard deviation of the risk free rate increases but not excessively. Model VIII is able to match quite closely the data on the standard deviation of the interest rate and the investment return. The downside is that like the standard RBC model, the standard deviation of the return to equity is pinned down by the risk free rate. There is little difference between the standard deviations of the two rates even with financing constraints. Therefore, the model underestimates the volatility of equity returns.

A key difference between the model presented here and other GE production based asset pricing models is that the occasionally binding financing constraints force firms to behave quite differently during a boom when the constraints matter and in a recession when they do not. Firms are able to precautionary save on behalf of the consumers to avoid binding constraints during recessions. When the model is simulated without the financing constraint, during recessions short term debt obligations are greater than current obligations to debt and equity owners, suggesting that in those states of nature the financing constraint would bind. However, when you apply constraint (2.5), firms increase steady state dividends just enough to avoid having the constraint bind during recessions.[11] During booms, on the other hand, firms cannot avoid the binding financing constraints. The financial accelerator models, in contrast, have the same propagation mechanism during expansions and contractions and are therefore likely to deliver more symmetrical expansions and recessions.

The cyclical asymmetries in the binding financing constraints lead to asymmetries in the cyclical variation in mean asset price returns and variances. As Rouwenhorst (1995) shows, a standard RBC model is unable to deliver much variation in conditional returns or return volatility at different phases of the business cycle due to the fact that adjustments in the capital stock are too smooth. To examine the ability of this model to replicate cyclical variation in asset returns and return volatility, Table (3) reports the conditional mean and standard deviation of the interest rate, investment return , and equity return. These conditional moments are reported for a standard RBC model and a RBC with financing constraints. Consistent with the data, the model with financing constraints is able to capture the counter-cyclical behavior of interest rates and equity returns as well as the pro-cyclical movement in the investment return. The asymmetry of the financial frictions is clearly reflected in the conditional standard deviations as well. In a recession, the volatilities of equity returns and interest rates are roughly 50% larger than in an economic boom, just below the empirical estimations of Schwert (1989).

Adding convex adjustment costs to proxy for these financing constraints fails to deliver these asset pricing asymmetries that are important in the data. To show this, the moments of the standard RBC model with only convex adjustment costs was estimated. To make a clear parallel between the two models, the adjustment costs parameter is chosen so that the standard deviation of investment match the model with financial frictions. Two important results came from this comparison. First,

[11] This is a similar result to Mendoza and Smith (2006) where we find in a quantitative debt deflation model precautionary savings reduces the risk of margin constraint binding in the long run such that the model can only generate a 5% chance of an economic "crisis" occurring.

in order for investment to become volatile enough, the adjustment cost parameter must be so high that adjustment costs account for 1% of GDP, significantly higher than the 0.1% suggested by Mendoza (1991) and Summers (1981). Second, as indicated by the standard deviations, the model does not deliver the asymmetric moments for booms and recessions. The standard deviation of the interest rate and the equity return was 0.63 during a recession and 0.69 during a boom.

Table (4) reports the correlation between output and the risk free rate. The data suggests that interest rates are counter-cyclical. In the standard RBC model, the interest rate is essentially acyclical. All seven versions of the model with financing constraints, in contrast, deliver counter-cyclical interest rate behavior. While qualitatively the model produces the correct sign, quantitatively the correlation is much higher than we see in the data. The two models with variable labor, Model VI & IX, come closest to matching -0.35 in the data at -0.50 and -0.55 respectively.

Having a production side to the economy forces us not only to try and match prices but aggregate quantity moments as well. Table 5 shows the impact of the financing constraints on the long run business cycle moments of the model. In the standard RBC model, as the coefficient of risk aversion is increased, consumption becomes less volatile relative to output. More volatile investment means a more volatile capital stock and more volatile output. In the standard RBC model, with higher risk aversion households' consumption becomes less volatile and mean consumption rises. In the model with financing constraints, on the other hand, consumption becomes less volatile relative to output but the fall is not very dramatic and the impact on investment volatility much smaller. Because investment is limited, output does not rise as much and mean consumption falls. With financial frictions, if households are more risk averse they must pay for less volatile consumption with lower mean consumption.

As Table 5 shows adding financial frictions does not drastically alter the quantity moments relative to the standard RBC model. The table reports not only the model with financing constraints but also the model under alternative parameterizations. In all cases, financing constraints increase the relative standard deviation of consumption to GDP over the RBC model but underestimate the standard deviation of investment to GDP. Adding frictions in all cases does not seem to help much in matching the consumption and investment correlations with GDP. These correlations are significantly above those observed in the data, just as in the standard RBC model. Looking at the last column, adding lower shock persistence reduces the first order autocorrelations of the macro aggregates to be in line with actual data. The inclusion of a labor choice in the Model VI and Model IX enables us to see how well the model matches labor dynamics across the business cycle. Labor's standard deviation relative to GDP and the first order autocorrelation are underestimated relative to the data, while the correlation with GDP is overestimated.

To get at the dynamics of the model, Figure 4 and Figure 5 compare the forecast functions for the basic RBC model and one with financing constraints. The forecast functions are induced by a negative, one-standard deviation productivity shock at date 1, given the Markov process of productivity shocks and the decision rule for capital. These forecast functions are analogous to impulse-response functions conditional on starting at the mid-point of the limiting distribution and hitting them with the same negative productivity shock. The forecast functions have the advantage that they preserve all the non-linear aspects of the model's stochastic competitive equilibrium captured in the decision rules. The plots for the business cycle quantities (Figure 4) in many ways supports what was clear from the moments. Output rises in response to a positive productivity shock. While the investment plots have similar shapes in the frictionless model, investment is impeded by the possibility of being constrained and increases by less than half of what we see in the frictionless case. In the consumption plot, we see that in an economy without frictions the ability to transfer consumption from one period to the next by investing in firms enables households to have a smooth consumption profile. The frictions add the needed volatility to consumption. More volatile consumption and less volatile investment cause the capital stock to be much smoother. There are two key differences in the response of dividends in the standard RBC model and the one with frictions. First, in the model with financing constraints dividends tend to take longer to adjust after a positive productivity shock, as firms are forced to manage dividends in order to avoid the financing constraint. Second, the adjustment costs in the financial frictions model force firms to accumulate internal resources before they invest. Therefore, for two periods dividends actually show positive growth before they decline.

While the frictions dampen the quantity moments, they seem to amplify the price dynamics.

Turning to the forecast functions for prices in Figure 5, we can see a dramatic change in the price of equity following a positive productivity shock relative to the frictionless case. Looking at the second plot, a main driver of this is the fact that the interest rate is highly counter-cyclical and thus the marginal rate of substitution very pro-cyclical. As the price of equity is expected to fall over time, the equity return slowly adjusts back to its steady state. The investment return on the other hand tends to rise dramatically as the productivity shock increases the return to investing. The lack of investment flowing in keeps the return to investing higher for longer.

While theoretically and empirically the model seems to support the importance of occasionally binding financing constraints in explaining the quantity and price dynamics, it is important to explore how sensitive these results are to both the leverage assumption and the way the financing constraint is modeled. Table 6 shows asset returns under no leverage, low leverage, and high leverage scenarios in three different models. Using the standard RBC model, consistent with theory leverage increases the return to equity slightly but has no impact on the investment return or the interest rate. When financing constraints are imposed through a non-negativity constraint in firms' internal resources as explained in equation (2.5), leverage plays an important role. If firms have no leverage, the internal resources are managed so that the constraints do not impede investment behavior and the model reduces to the frictionless case. As leverage rises, firms are more and more likely to become constrained and therefore adjust their investment plans substantially to avoid the constraint, driving up the returns on equity and investment. The last three rows examine the returns from a model where an a non-negativity constraint on firms' internal resources exists in combination with a margin requirement on short term debt holdings. Like Mendoza and Smith (2006), the margin requirement limits debt (short term) to be a certain fraction the capital stock. Because firms can avoid this constraint entirely by using internal resources or allowing dividends to be negative, this type of constraint must be used in conjunction with a non-negativity constraint on dividends or else the model would simply reduce to the standard RBC case. The model with the margin requirement behaves in a similar manner to the model with only one constraint. Leverage is still key to getting the constraints to have an impact. The higher leverage the higher the returns to equity and investment rise above the interest rate. Table 6 seems to indicate that the results are not particularly sensitive to the way in which financing is constrained.

While discretizing the state space and iterating on the value function is less restrictive than solving the model using linear approximations, as discussed in Rouwenhorst (1995) an error is introduced in that the first condition may not hold exactly since the decision rules are only calculated on grid points. The error therefore is directly proportional to the coarseness of the grid. Table 7 reports the errors given the coarseness of the grid. A greater coefficient of relative risk aversion does introduce greater approximation error in that it introduces greater curvature to the value function.

5 Conclusions

A simple GE asset pricing model where firms face a limit on their sources of financing produces asset price dynamics that are consistent with stylized facts. The financing frictions generate a wedge between the households' marginal rate of substitution and the firms' stochastic discount factor that allows the risk free rate to be low while simultaneously the equity return to be high. In this manner, the model is able to explain well the observed equity and investment premium. An important feature of the model is that occasionally binding financing constraints can produce asymmetries across the business cycle. The constraints force the means and volatility of the asset returns to vary over the business cycle, consistent with the data. Both means and variances of the interest rates and equity returns vary counter-cyclically while for investment the returns are pro-cyclical but volatility is counter-cyclical, key features of the macro data. Theoretically, the presence of these occasionally binding financing constraints forces equity returns to be driven by several factors. Expected equity returns no longer depend simply on the covariance between the return and MRS but also on the relative replacement value to market value.

There are two avenues to further pursue with this line of research. First, it seems important to examine how firms' behavior may differ if firms can accrue retained earnings to keep themselves from becoming constrained. Second, there may be some interesting insights into the recent declines in the equity premium by introducing government bonds into this model. Since the financing constraints limit investment, consumers would be better able to smooth consumption if the government issued debt rather than raised taxes to finance its spending (no Ricardian Equivalence). In this manner, the increase in deficit financing by the government may partially explain the simultaneous decline in the equity premium over the last twenty years.

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7 Appendix

.1 A Simple Model of the Trade-Off Theory

This model is a two period perfect-foresight simplification of the trade-off model in Frank and Goyal (2008). For a representative firm define B_{t+1} as the pay-off in the second period to bondholders, τ_B as the tax paid by bondholders and r as the interest rate. The value of the firm to the bondholders follows below.

$$V_{B_t} = \frac{(1+\tau_B)}{(1+r)} B_{t+1} \tag{.1}$$

Define τ_S as the tax paid by equity owners, $\phi(\frac{1}{B_{t+1}})$ as the likelihood equity claimants get a pay-off in the second period, $X_{t+1} - B_{t+1}$ as the internal resources of the firm net debt payments, and τ_c as corporate taxes. The value of the firm to the shareholders is then:

$$V_{S_t} = \frac{(1+\tau_S)}{(1+r)} \phi(\frac{1}{B_{t+1}})[(X_{t+1} - B_{t+1})(1-\tau_c)]. \tag{.2}$$

The total value of the firm is then:

$$V_{B_t} + V_{S_t} = \frac{1}{(1+r)}[(1+\tau_B)B_{t+1} + (1+\tau_S)\phi(\frac{1}{B_{t+1}})(X_{t+1} - B_{t+1})(1-\tau_c)]. \tag{.3}$$

Choosing B_{t+1} in order to maximize the total value of the firm delivers the following marginal condition:

$$(1+\tau_B) - (1+\tau_S)(1-\tau_c)\phi(\frac{1}{B_{t+1}}) = (1+\tau_S)(1-\tau_c)\phi'(\frac{1}{B_{t+1}})(X_{t+1} - B_{t+1}). \tag{.4}$$

.2 Investment Demand

The investment function results from working with the firms' foc with respect to capital (2.11) and defining $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$.

$$1 = E[\theta_{t+1}(1+\eta_{t+1})mpk_{t+1}] \tag{.5}$$

$$1 = E[\theta_{t+1}]E[(1+\eta_{t+1})mpk_{t+1}] + cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1}) \tag{.6}$$

$$1 = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})mpk_{t+1}] + cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1}) \tag{.7}$$

$$1 - cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1}) = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})mpk_{t+1}] \tag{.8}$$

From the dividend constraint $mpk_{t+1} = \frac{div_{t+1}+k_{t+2}-\frac{b_{t+2}}{(1+r_{t+1})}+b_{t+1}+\overline{RB}}{k_{t+1}}$. Substituting this into the equation and defining $\tilde{div}_{t+1} = div_{t+1} + k_{t+2} - \frac{b_{t+2}}{(1+r_{t+1})} + b_{t+1} + \overline{RB}$ we get the following:

$$k_{t+1}(1 - cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1})) = \frac{1}{(1+r_t)}E[(1+\eta_{t+1})\tilde{div}_{t+1}] \tag{.9}$$

Substituting the equation of motion for capital into the equation we get:

$$i_t = \frac{1}{(1+r_t)}\frac{E_t\left[q_{t+1}\left(\tilde{div}_{t+1}\right)\right]}{[1 - cov(\theta_{t+1},(1+\eta_{t+1})mpk_{t+1})]} - (1-\delta)k_t \tag{.10}$$

.3 The Price of Equity

The equilibrium equity price is determined by initially starting with the households FOC with respect to equity shares as represented by 2.9 where $\theta_{t+i} = \beta^i(\frac{u'(c_t+i)}{u'(c_t)})$.

$$p_t = E_t\left[\theta_{t+1} \;\; (p_{t+1} + div_{t+1})\right] \tag{.11}$$

Using the definition of dividends and the assumption that firms' production function exhibit constant returns to scale the following equation emerges where $mpk_{t+1} = exp(\epsilon_{t+1})F_{k_{t+1}} + (1-\delta)$

$$p_t = E_t\left[\theta_{t+1} \;\; (mpk_{t+1}k_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - b_{t+1} - \overline{RB} + p_{t+1})\right] \tag{.12}$$

Which can be rearranged as follows:

$$p_t = E_t\left[\theta_{t+1} \;\; mpk_{t+1}\right]k_{t+1} + E_t\left[\theta_{t+1} \;\; (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - \overline{RB})\right] - \frac{b_{t+1}}{(1+r_t)} \tag{.13}$$

From the firms' FOC $E_t\left[\theta_{t+1} \;\; mpk_{t+1}\right] = 1 + \eta_t - E_t\left[\theta_{t+1} \;\; \eta_{t+1} \;\; mpk_{t+1}\right]$. Substitute this in to the previous equation.

$$p_t + \frac{b_{t+1}}{(1+r_t)} = \left(1 + \eta_t - E_t\left[\theta_{t+1} \;\; \eta_{t+1} \;\; mpk_{t+1}\right]\right)k_{t+1} + E_t\left[\theta_{t+1} \;\; (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})} - \overline{RB})\right] \tag{.14}$$

or

$$p_t + \frac{b_{t+1}}{(1+r_t)} - k_{t+1} = \eta_t k_{t+1} - E_t\left[\theta_{t+1} \;\; \eta_{t+1} \;\; mpk_{t+1}\right]k_{t+1} + E_t\left[\theta_{t+1} \;\; (p_{t+1} - k_{t+2} + \frac{b_{t+2}}{(1+r_{t+1})})\right] - E_t(\theta_{t+1})\overline{RB} \tag{.15}$$

Iterating forward we get the following:

$$p_t + \frac{b_{t+1}}{(1+r_t)} + E_t \sum_{t=1}^{\infty} \left(\prod_{j=1}^{i-1} \theta_j \overline{RB} \right) = k_{t+1} + \eta_t k_{t+1} + E_t \left[\sum_{t=1}^{\infty} \theta_{t+i} \eta_{t+i} (k_{t+i+1} - mpk_{t+i} k_{t+i}) \right] \quad (.16)$$

or

$$p_t = k_{t+1} + \eta_t k_{t+1} + E_t \left[\sum_{t=1}^{\infty} \theta_{t+i} \eta_{t+i} (k_{t+i+1} - mpk_{t+i} k_{t+i}) \right] - \frac{b_{t+1}}{(1+r_t)} - E_t \sum_{t=1}^{\infty} \left(\prod_{j=1}^{i-1} \theta_j \overline{RB} \right) \quad (.17)$$

.4 Return to Equity

Re-arranging equation 2.9 we get the following equation for the return to equity:

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad \frac{(p_{t+1}+div_{t+1})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.18}$$

Applying the definition of dividends in 2.3 and assuming constant returns to scale technology:

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad \frac{(mpk_{t+1}k_{t+1}-k_{t+2}+\frac{b_{t+2}}{(1+r_{t+1})}+p_{t+1})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.19}$$

or

$$E_t[R^e_{t+1}] = R^f_t - \frac{cov\left[\theta_{t+1}, \quad mpk_{t+1}\right]}{E[\theta_{t+1}]}\frac{k_{t+1}}{p_t} - \frac{cov\left[\theta_{t+1}, \quad \frac{(p_{t+1}+\frac{b_{t+2}}{(1+r_{t+1})}-k_{t+2})}{p_t}\right]}{E[\theta_{t+1}]} \quad . \tag{.20}$$

Applying equation 2.20:

$$E_t[R^e_{t+1}] - R^f_t = -\frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}(1 + \frac{\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})]}{p_t})$$

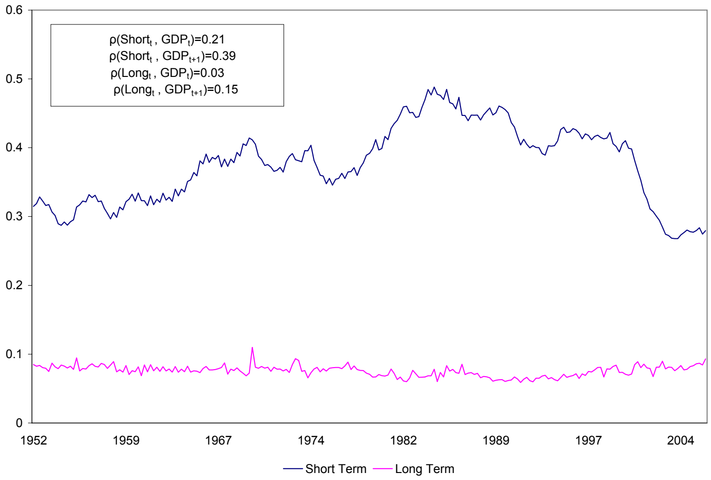
$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t}]. \tag{.21}$$

Applying equation 2.23:

$$E_t[R^e_{t+1}] = \quad E_t R^i_{t+1} - \quad \frac{cov[\theta_{t+1}, mpk_{t+1}]}{E[\theta_{t+1}]}\frac{\left(\tilde{b_{t+1}} - \eta_t k_{t+1} + E_t\left[\sum_{i=1}^{\infty}\theta_{t+i}\eta_{t+i}(mpk_{t+i}k_{t+i} - k_{t+i+1})\right]\right)}{p_t}$$

$$-\frac{cov[\theta_{t+1}, \eta_{t+1}k_{t+2}]}{E[\theta_{t+1}]p_t} + FP_t. \tag{.22}$$

Figure 1: New Issues of Short Term and Long Term Non-Farm, Non-Financial Corporate Debt



Note: Taken from Flow of Funds Data, following calculations of Baker, Greenwood, and Wurgler (2002). Long term debt is calculated as the sum of municipal bonds, corporate bonds, and mortgages. Short term debt is the sum of commercial paper, bank loans (nec), and "other". New issues of short term debt is simply short term debt for that period. New issues for long term debt is the change in long term debt outstanding plus 0.1 of lagged long term debt to get at the roll-overs. Both new issue series are divided by total debt to remove the growth component. To calculate the correlation of each series with the GDP, all series were first de-trended using an Hodrick-Prescott. The correlation was then calculated using percentage deviation from trend.

Table 1: **Asset Returns**

Model	$E(r^f)$	$E(r^e - r^f)$	$E(r^i - r^f)$	$cov(MRS, r^i)$
I. **Standard RBC model**	3.620	0.002	0.004	-0.0008
II. **Financing Constraints (FC)**	3.529	0.228	0.468	-0.0012
III. **FC and Increased Leverage**	3.529	0.211	1.651	-0.0013
IV. **FC and Lower Persistence**	3.337	0.460	0.734	-0.0034
V. **FC and Adjustment Costs**	3.525	0.222	0.376	-0.035
VI. **FC and Variable Labor**	3.460	0.311	0.928	-0.0024
VII. **FC and Habit Persistence**	0.900	4.287	3.091	-0.0061
VIII. **FC, Habit Pers. and Adj. Costs**	0.891	4.345	3.008	-0.179
IX. **FC, Habit Pers. and Labor**	1.640	3.657	2.898	-0.008
Data	1.96	4.74	-	-

r^f,r^e and r^i is the risk free rate,return to equity and return on investment all annualized. MRS refers to the marginal rate of substitution. The data is from Campbell (1999) using the long sample 1891-1994.

Figure 2: Firms' Investment Demand is Kinked Due to the Financing Constraints

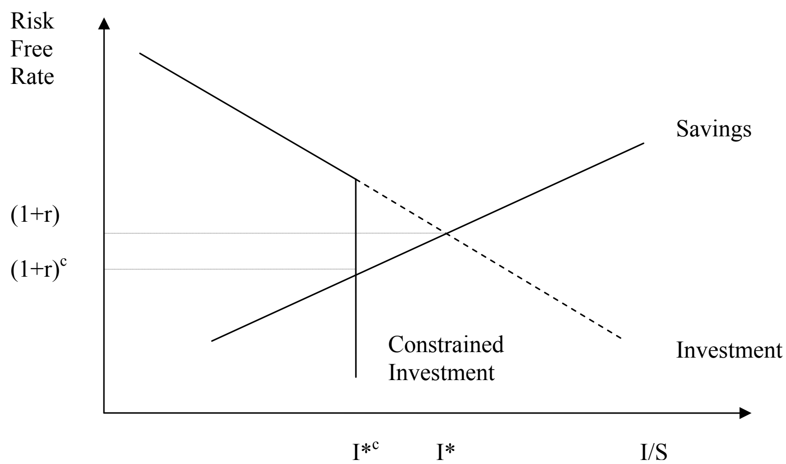


Figure 3: Long-Run Distribution of Capital

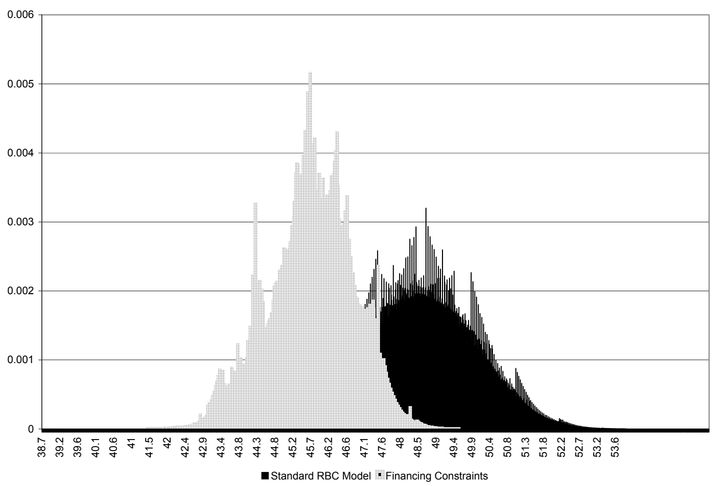


Table 2: **Asset Return Volatility**

	standard deviation (r^f)	standard deviation (r^e)	standard deviation (r^i)
I. **Standard RBC model**	1.341	1.340	0.285
II. **Financing Constraints (FC)**	1.528	1.539	0.205
III. **FC and Increased Leverage**	1.437	1.445	0.248
IV. **FC and Lower Persistence**	5.309	5.308	0.179
V. **FC and Adjustment Costs**	1.531	1.541	6.054
VI. **FC and Variable Labor**	2.226	2.236	0.385
VII. **FC and Habit Persistence**	7.692	7.530	0.206
VIII. **FC, Habit Pers. and Adj. Costs**	7.697	7.588	6.049
IX. **FC, Habit Pers. and Labor**	8.387	7.821	0.324
Data	8.92	18.63	9.37

The data are from Campbell (1999) using the long sample 1891-1994 and Cochrane (1991).

Table 3: **Asset Returns and the Business Cycle**

	Recession	Boom
Standard RBC model		
(r^f)	3.564	3.676
STD (r^f)	1.351	1.328
(r^i)	3.562	3.685
STD (r^i)	0.278	0.2773
(r^e)	3.565	3.678
STD (r^e)	1.350	1.328
Model with Financing Constraints		
(r^f)	5.936	0.967
STD (r^f)	1.702	0.924
(r^i)	3.886	4.189
STD (r^i)	0.122	0.127
(r^e)	6.257	0.124
STD (r^e)	1.699	0.941

Table 4: **Interest Rates and the Business Cycle**

Correlation with Output	**Interest Rate**
I. **Standard RBC model**	-0.07
II. **Financing Constraints (FC)**	-0.78
III. **FC and Increased Leverage**	-0.79
IV. **FC and Lower Persistence**	-0.85
V. **FC and Adjustment Costs**	-0.80
VI. **FC and Variable labor**	-0.50
VII. **FC and Habit Persistence**	-0.82
VIII. **FC, Habit Pers. and Adj. Costs**	-0.82
IX. **FC, Habit Pers. and Labor**	-0.55
Data	-0.35

The data is taken from King and Rebelo (1999).

Table 5: **Long Run Business Cycle Moments**

Variable	standard deviation	standard deviation relative to GDP	correlation with GDP	first-order autocorrelation
I. **Standard RBC model**				
GDP	2.76	1.00	1.00	0.98
consumption	1.71	0.62	0.95	0.99
investment	6.48	2.35	0.97	0.96
II. **Financing Constraints (FC)**				
GDP	2.22	1.00	1.00	0.97
consumption	1.88	0.85	0.99	0.97
investment	3.31	1.49	0.99	0.97
III. **FC and Increased Leverage**				
GDP	2.23	1.00	1.00	0.97
consumption	1.90	0.83	0.99	0.97
investment	3.76	1.63	0.99	0.97
IV. **FC and Lower Persistence**				
GDP	1.93	1.00	1.00	0.83
consumption	1.48	0.77	0.99	0.84
investment	3.39	1.75	0.99	0.82
V. **FC and Adjustment Costs**				
GDP	2.22	1.00	1.00	0.97
consumption	1.88	0.85	0.99	0.97
investment	3.30	1.49	0.99	0.97
VI. **FC and Variable labor**				
GDP	4.43	1.00	1.00	0.98
consumption	3.72	0.84	0.99	0.98
investment	6.80	1.53	0.99	0.98
labor	2.22	0.50	0.99	0.003
VII. **FC and Habit Persistence**				
GDP	2.22	1.00	1.00	0.97
consumption	1.88	0.85	0.99	0.97
investment	3.30	1.49	0.99	0.97
VIII. **FC, Habit Pers. and Adj. Costs**				
GDP	2.22	1.00	1.00	0.97
consumption	1.88	0.85	0.99	0.97
investment	3.30	1.49	0.99	0.97
IX. **FC, Habit Pers. and Labor**				
GDP	3.85	1.00	1.00	0.98
consumption	3.45	0.89	0.99	0.98
investment	5.35	1.39	0.99	0.974
labor	1.93	0.50	0.99	0.002
Data				
GDP	1.81	1.00	1.00	0.84
consumption	1.35	0.74	0.88	0.80
investment	5.30	2.93	0.80	0.87
labor	1.79	0.99	0.88	0.88

Standard deviations are in percentage terms. The data was taken from King and Rebelo (1999).

Table 6: **The Role of Leverage Under Various Types of Financing Constraints**

Model	$E(r^f)$	$E(r^e - r^f)$	$E(r^i - r^f)$
Standard RBC model			
no leverage	3.62	0.002	0.004
low leverage	3.62	0.002	0.004
high leverage	3.62	0.003	0.004
Occasionally Binding Financing Constraints (FC)			
no leverage	3.62	0.002	0.004
low leverage	3.55	0.10	0.36
high leverage	3.53	0.24	1.81
Occasionally Binding FC and Margin Requirement			
no leverage	3.62	0.002	0.004
low leverage	3.56	0.02	0.34
high leverage	3.66	0.15	1.48

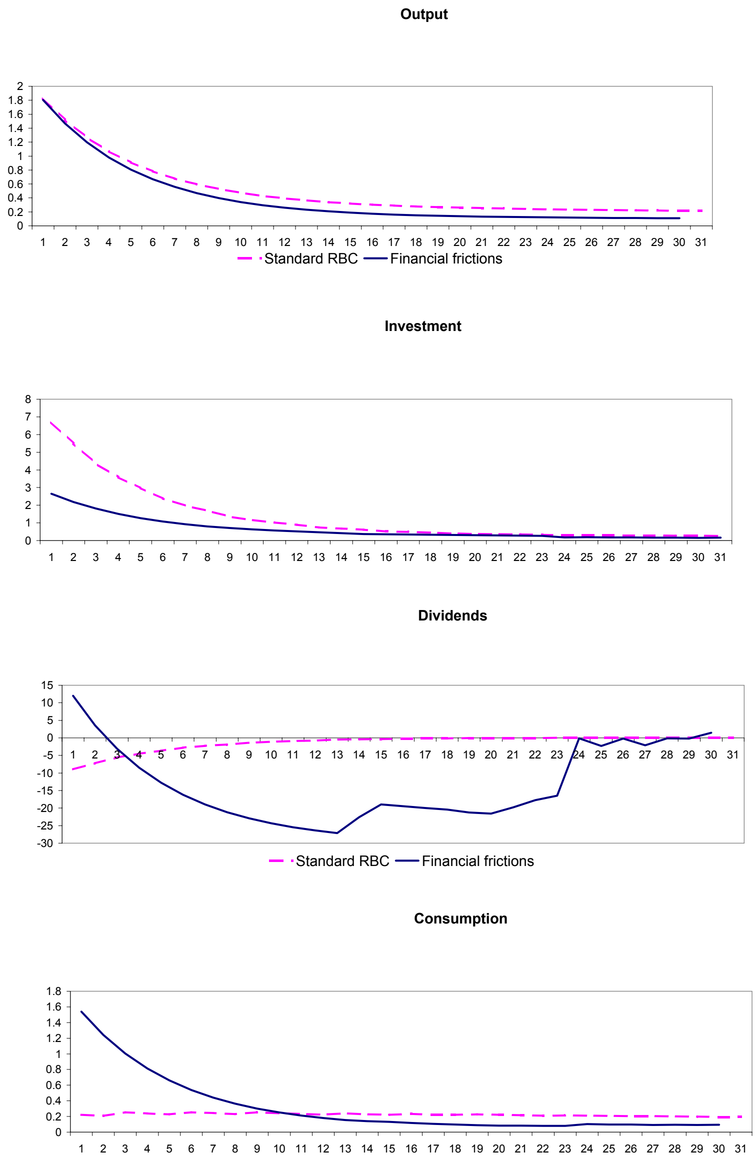
r^f,r^e and r^i is the risk free rate,return to equity and return on investment all annualized. MRS refers to the marginal rate of substitution.

Table 7: **Approximation Errors in Euler Equation**

	$\gamma = 2$	$\gamma = 10$
Mean	1.0000	1.0000
Std	0.0010	0.0039
Minimum	0.9970	0.9861
Maximum	1.0027	1.0140

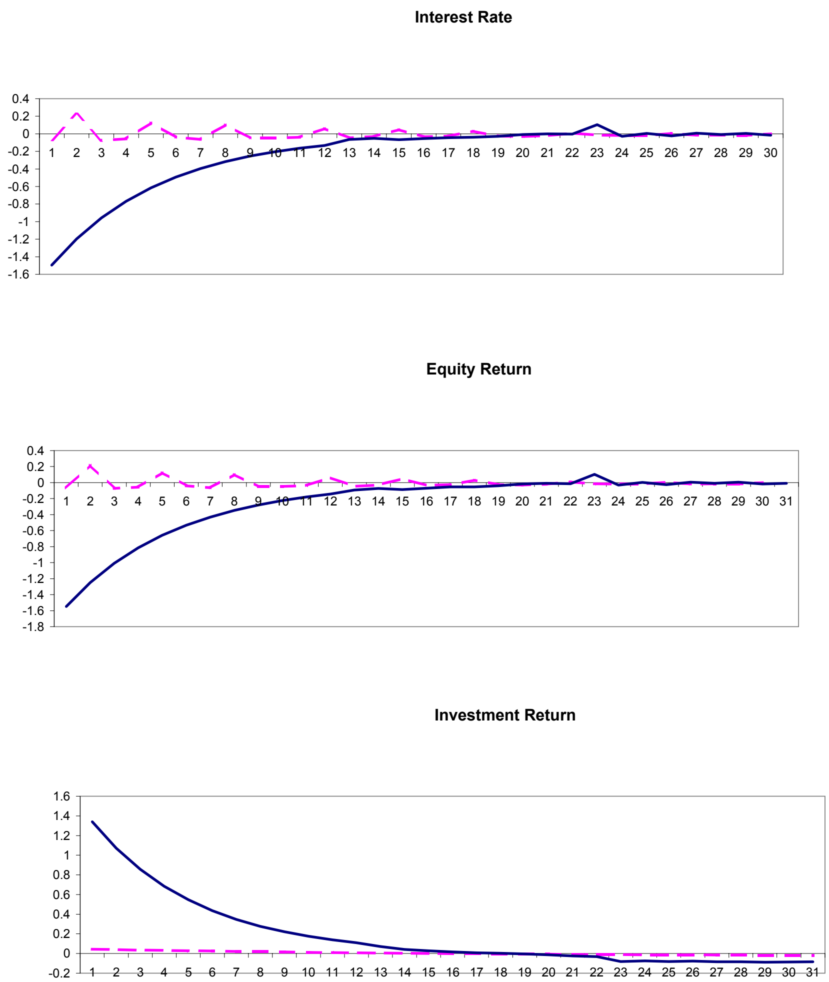
This table reports the Euler equation error. If the state space was continuous the ratio would be unity.

Figure 4: Forecasting Functions for Business Cycle Quantities



Note: Measures the response to a one standard deviation positive productivity shock. Measured as the percent deviation from long run means.

Figure 5: Forecasting Functions for Business Cycle Prices



Note: Measures the response to a one standard deviation positive productivity shock. Measured as the percent deviation from long run means.