PROBLEM SET #1 - ANSWER KEY

Part I - The Production Possibilities Frontier

Part a) Graph the production possibilities frontier described by:

```
> PPFa:=c^2+10*s^2=1000;
```

$$PPFa := c^2 + 10\,s^2 = 1000$$

The first step in graphing it is to solve the PPF for one of the variables:

```
> solsa:=solve(PPFa,s);
```

$$solsa := \frac{\sqrt{-10\,c^2 + 10000}}{10}, -\frac{\sqrt{-10\,c^2 + 10000}}{10}$$

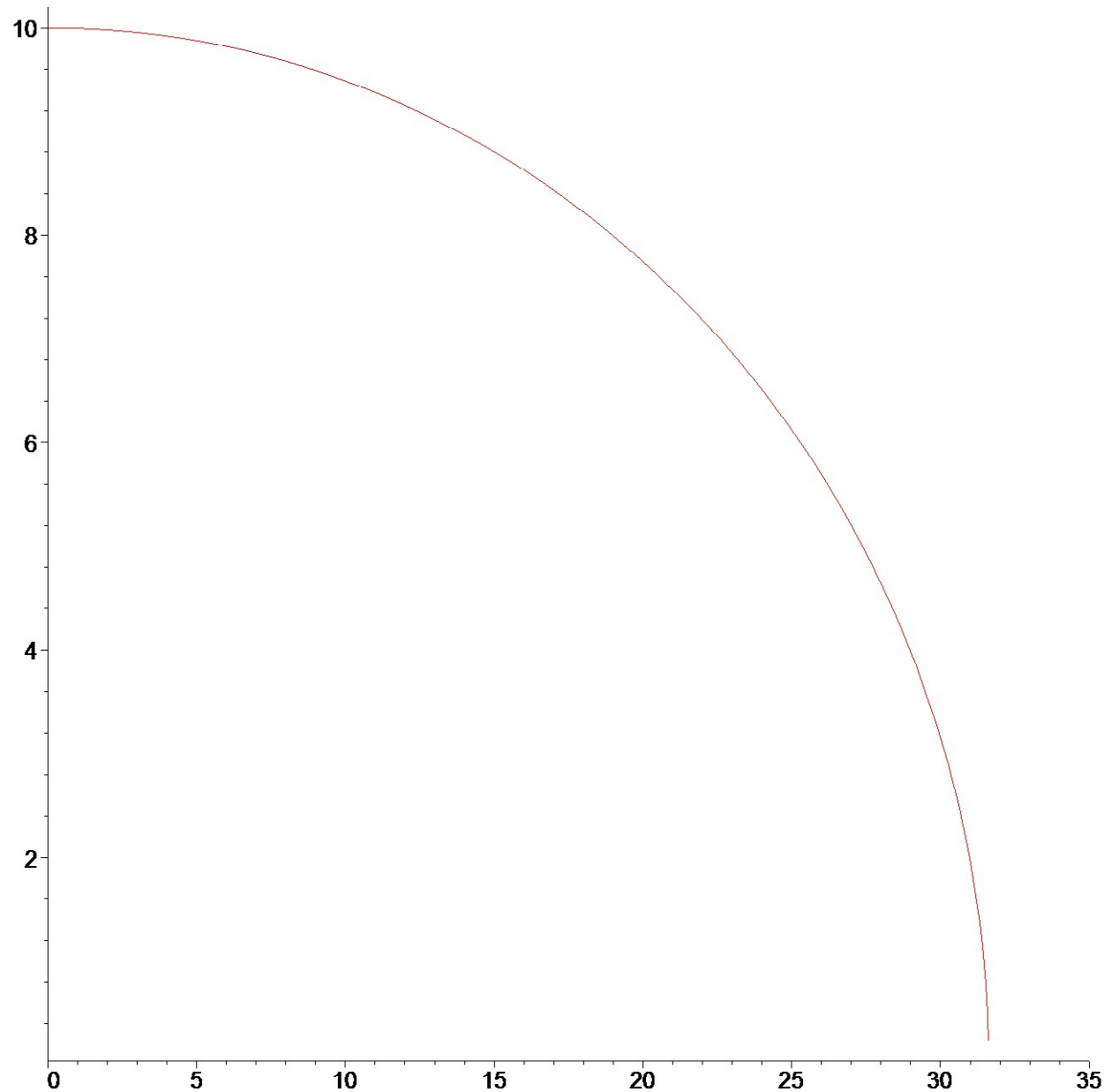
The first solution is the one we want, so we have:

```
> sedans:=solsa[1];
```

$$sedans := \frac{\sqrt{-10\,c^2 + 10000}}{10}$$

Which can be plotted as follows:

```
> plot(sedans,c=0..35);
```



Part b)

This part asks for a function expressing the number of sedans which can be produced as a function of the number of compacts produced. We have already done this en route to solving part a. The function is given by:

```
> sedans;
```

$$\frac{\sqrt{-10\,c^2 + 10000}}{10}$$

Part c)

The marginal rate of substitution is simply the derivative of the function with respect to one of the variables. Here, we take the derivative with respect to the number of compacts:

```
> MRS:=diff(sedans,c);
```

$$MRS := -\frac{c}{\sqrt{-10\,c^2 + 10000}}$$

Part d)

To find this out, simply substitute 20 into the formula given in part c). (Note the missprint on the problem set..."200" should be "20". We then have:

```
> MRS1:=evalf(simplify(subs(c=20,MRS)));
```

$$MRS1 := -0.2581988897$$

For you Maple fans out there, the command "evalf" gives a decimal instead of a fraction. This answer implies that, when 20 compacts are produced, the MRS is -.25, meaning that, by giving up 1/4 of a sedan, we can produce 1 compact car at this point.

Part e)

Here, I ask you to find the maximum revenues the firm can obtain, given the PPF. To do this, note that total revenues can be written as:

```
> TR:=50000*s+10000*c;
```

$$TR := 50000\,s + 10000\,c$$

which is the number of sedans multiplied by the price of sedans, and the number of cars multiplied by the price of cars.

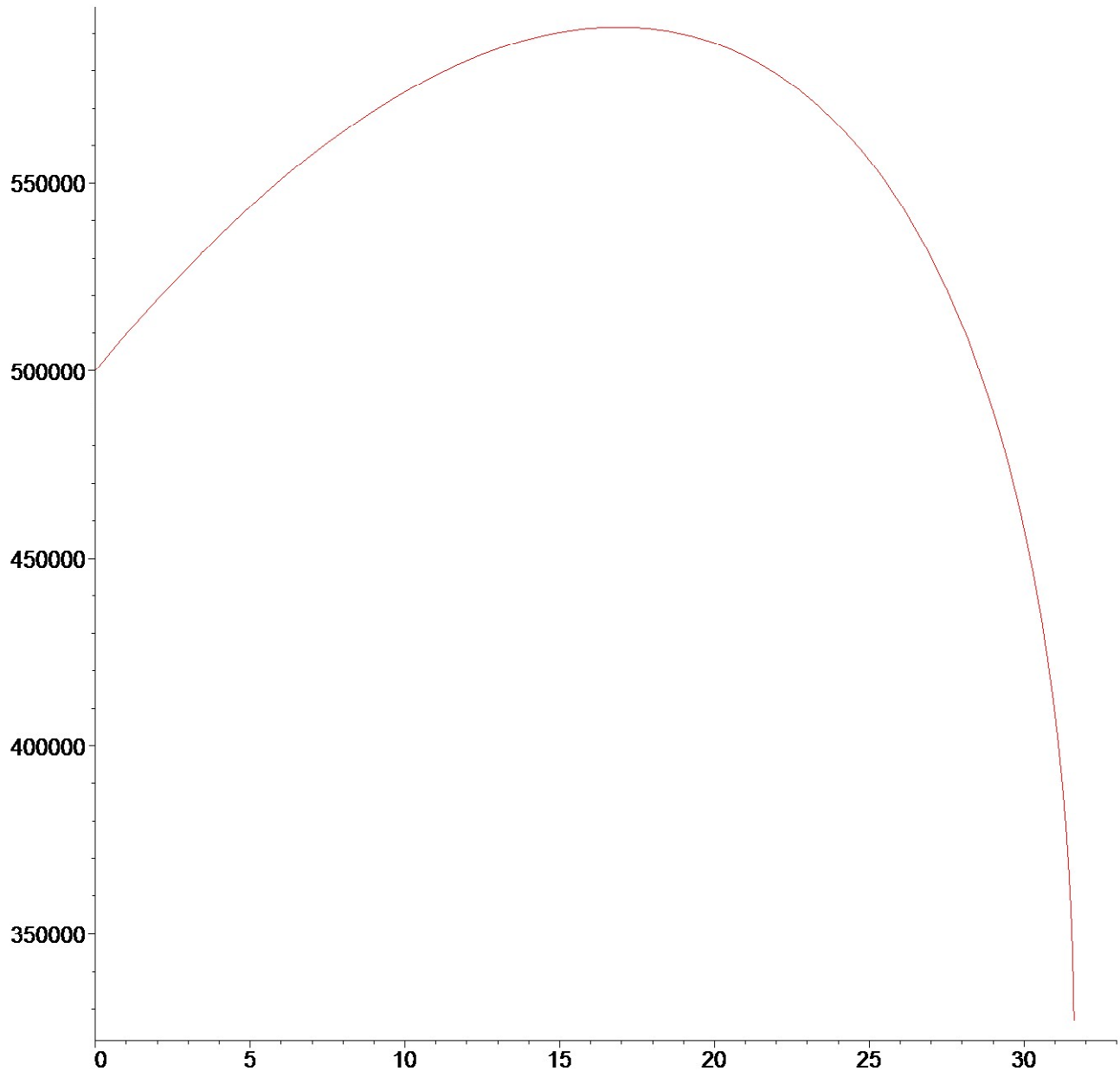
Let's substitute in our formula for the PPF, so we have:

```
> TR1:=subs(s=sedans,TR);
```

$$TR1 := 5000\sqrt{-10\,c^2 + 10000} + 10000\,c$$

To see how the problem really works, let's plot this function. This will tell us how total revenues change as we produce more compact cars. Note that production of more compacts implicitly means production of less sedans; this is what happens when we plug in the equation for the PPF!

```
> plot(TR1,c=0..33);
```



WE would like to find the "top of this hill," and this is a straightforward application of calculus. Let's take the derivative and set it equal to zero to find the flat point.

```
> derivative:=diff(TR1,c);
```

$$derivative := -\frac{50000\,c}{\sqrt{-10\,c^2 + 10000}} + 10000$$

We can now solve for the number of compacts by setting that result equal to zero (finding the point where the slope of the total revenue line is zero)

```
> compacts[max]:=evalf(solve(derivative=0,c));
```

$$compacts_{max} := 16.90308509$$

Of course, the number of sedans is easily obtained by plugging this back into our equation:

```
> sedans[max]:=subs(c=compacts[max],sedans);
```

$$sedans_{max} := 8.451542548$$

Part II - Graphing some PPFs

We have the following equations in this section:

```
> PPF1:=x^3+y^3=200;
```

$$PPF1 := x^3 + y^3 = 200$$

```
> PPF2:=x^2+2*y/(x+1)=100;
```

$$PPF2 := x^2 + \frac{2\,y}{x+1} = 100$$

```
> PPF3:=x+10*y=100;
```

$$PPF3 := x + 10\,y = 100$$

```
> PPF4:=x+(x+1)^2*y=100;
```

$$PPF4 := x + (x+1)^2\,y = 100$$

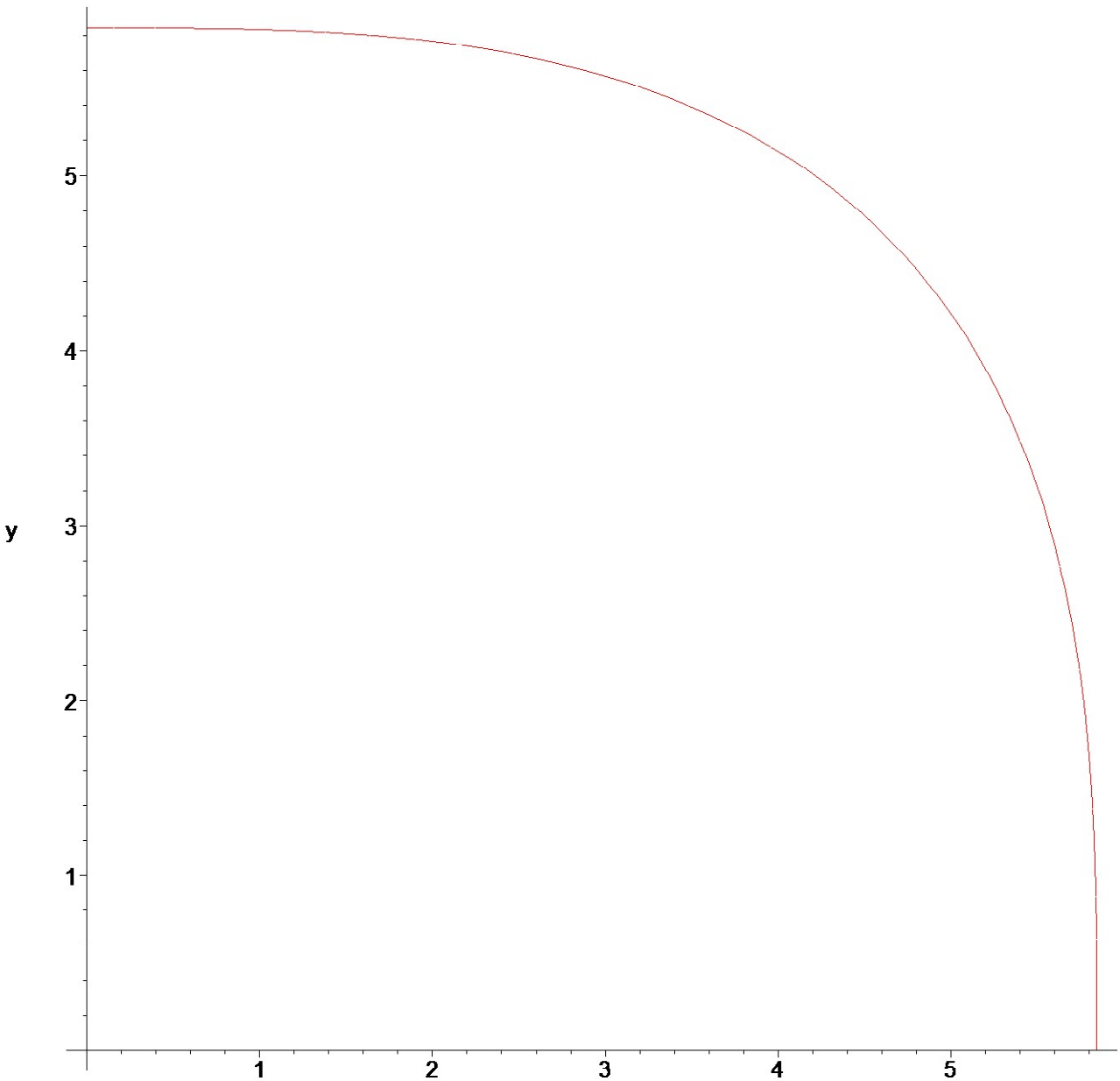
I'll first plot each of these...it is easiest here to bust out the implicitplot command. We must first start up the plots

```
> with(plots);
Warning, the name changecoords has been redefined
```

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

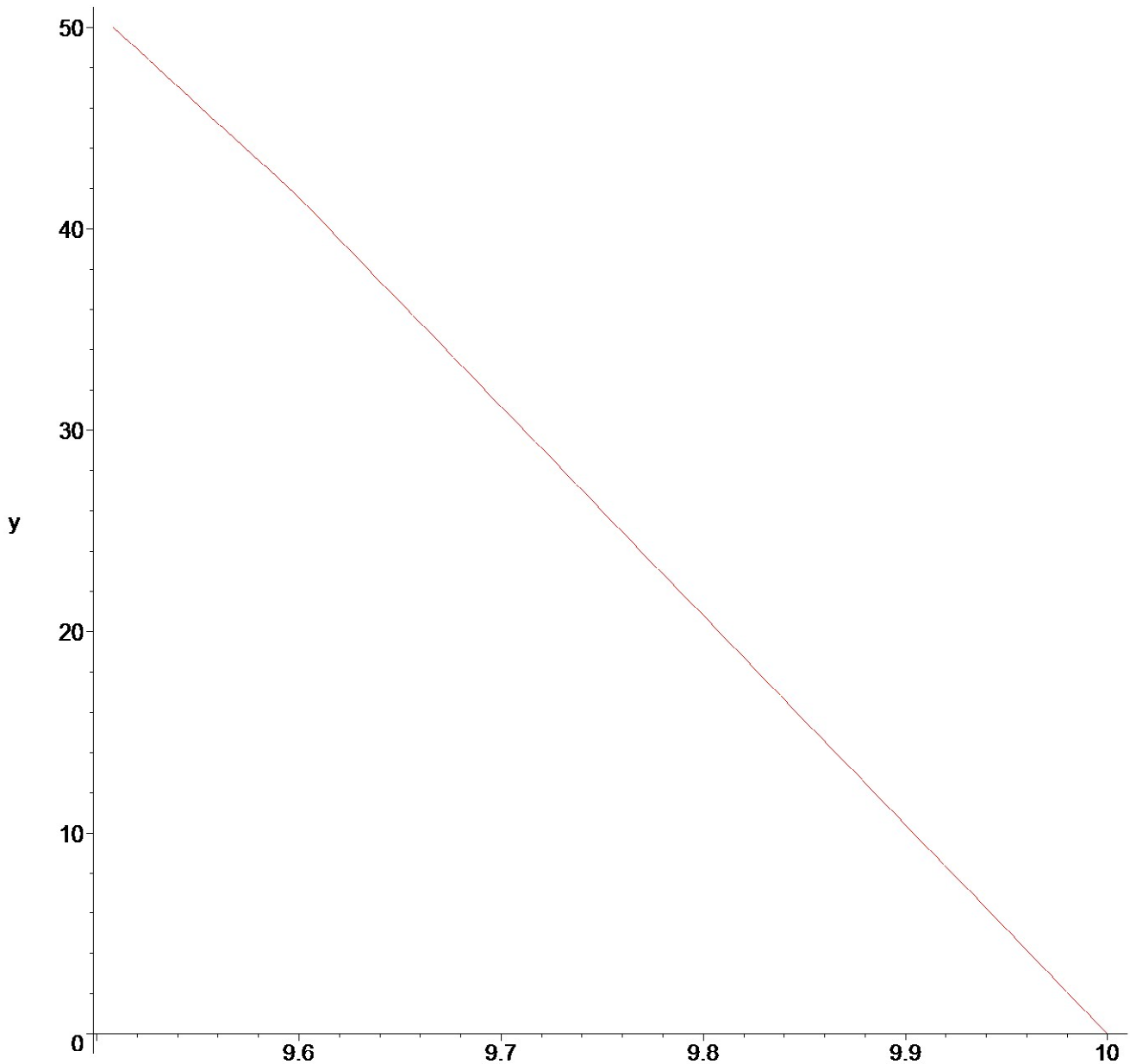
Part a) graph.

```
> implicitplot(PPF1,x=0..6,y=0..6);
```



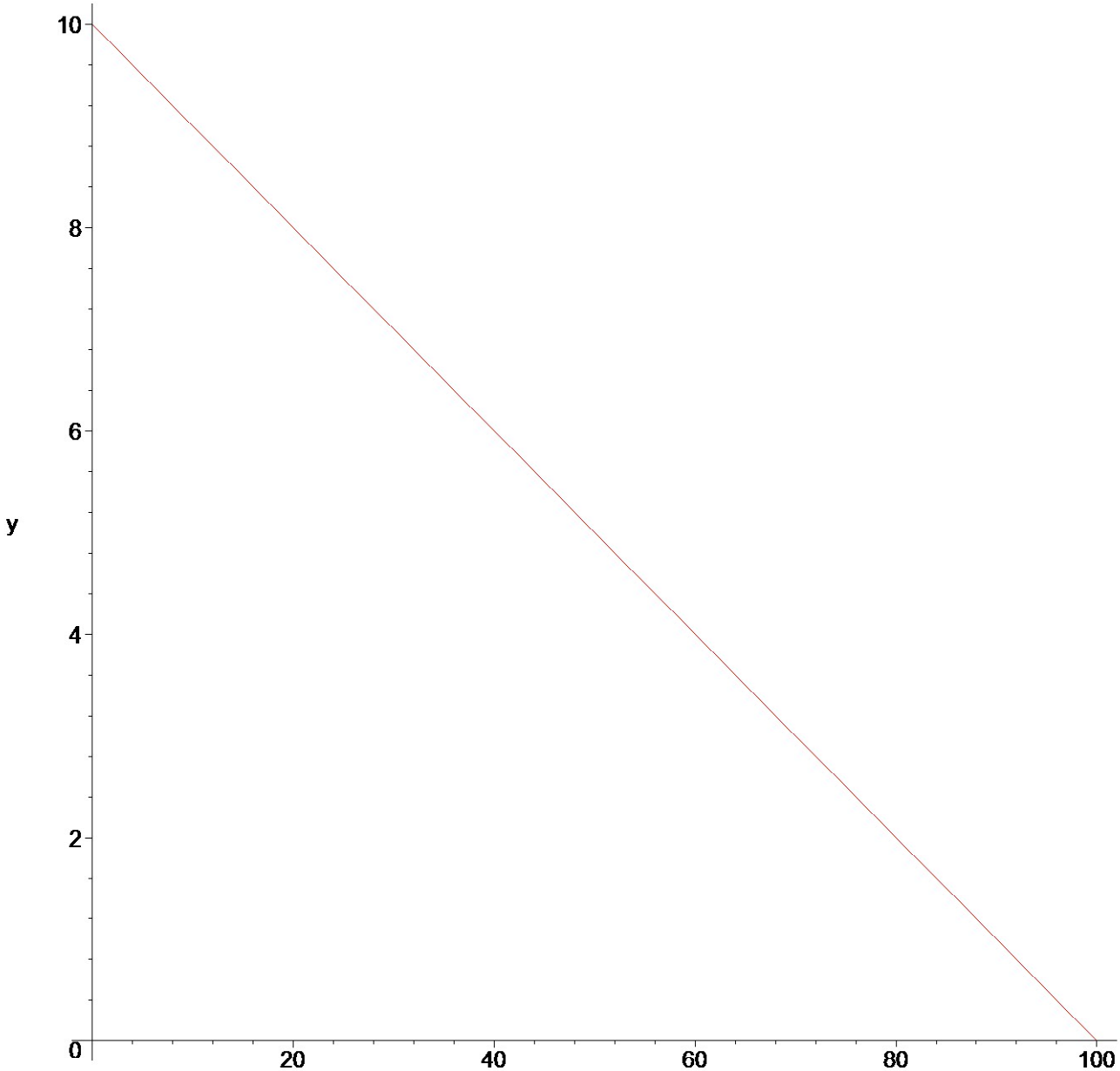
Part b) graph

```
> implicitplot(PPF2,x=0..10,y=0..50);
```



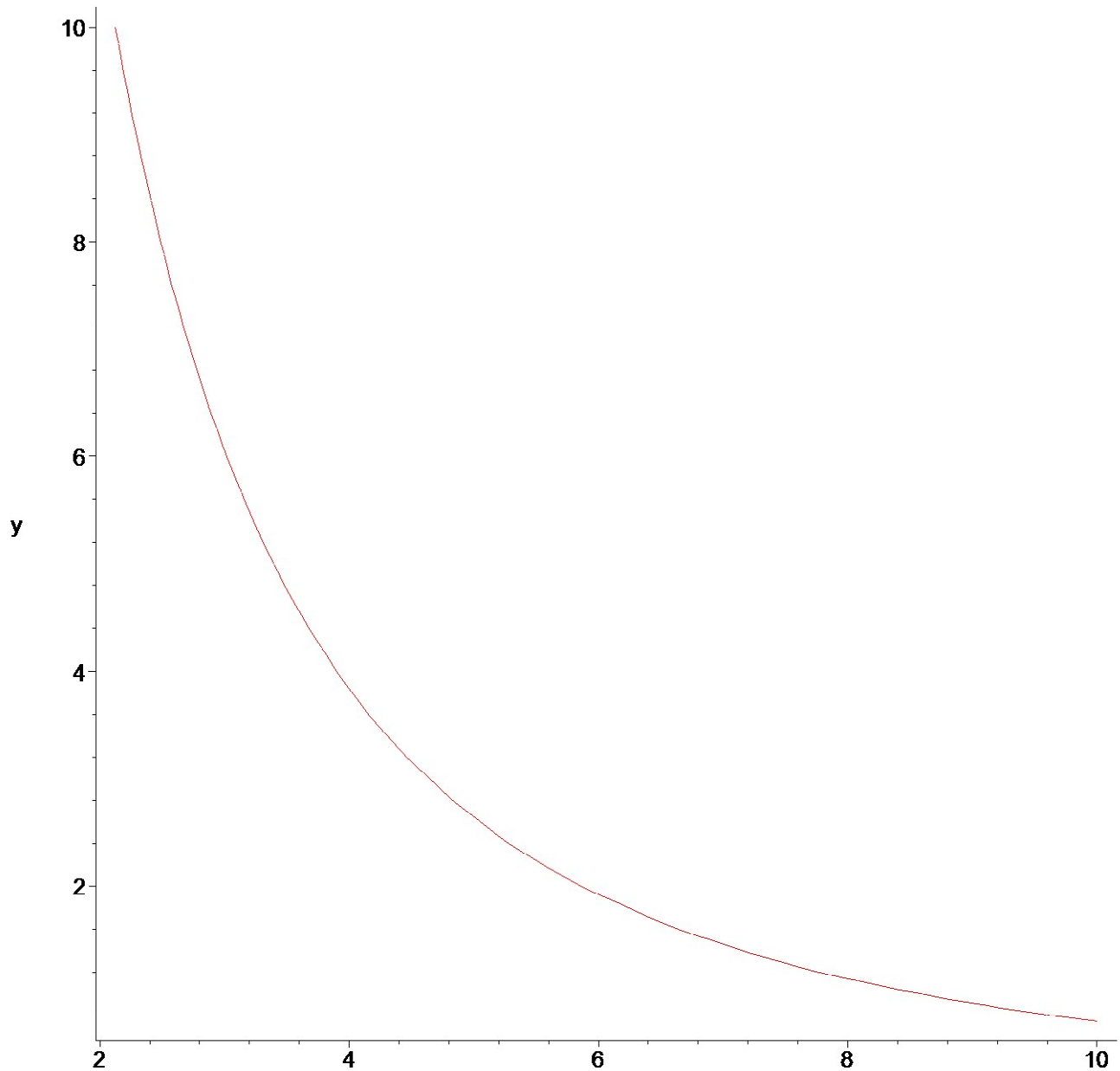
Part c) graph

```
> implicitplot(PPF3,x=0..100,y=0..10);
```



Part d) graph

```
> implicitplot(PPF4,x=0..10,y=0..10);
```



There are a couple of things unusual about some of the PPFS. Most obviously, the last PPF bows in the wrong direction. This means that the MRS is actually decreasing as one gets towards the axes, in contrast to the usual assumption. Further, PPF2 is actually upward sloping for part of its range.

Part III - Exchange and Comparative Advantage

Here, the French PPF is given by:

```
> PPFFrance:=w+5*b=100000;
```

$$PPFFrance := w + 5\,b = 100000$$

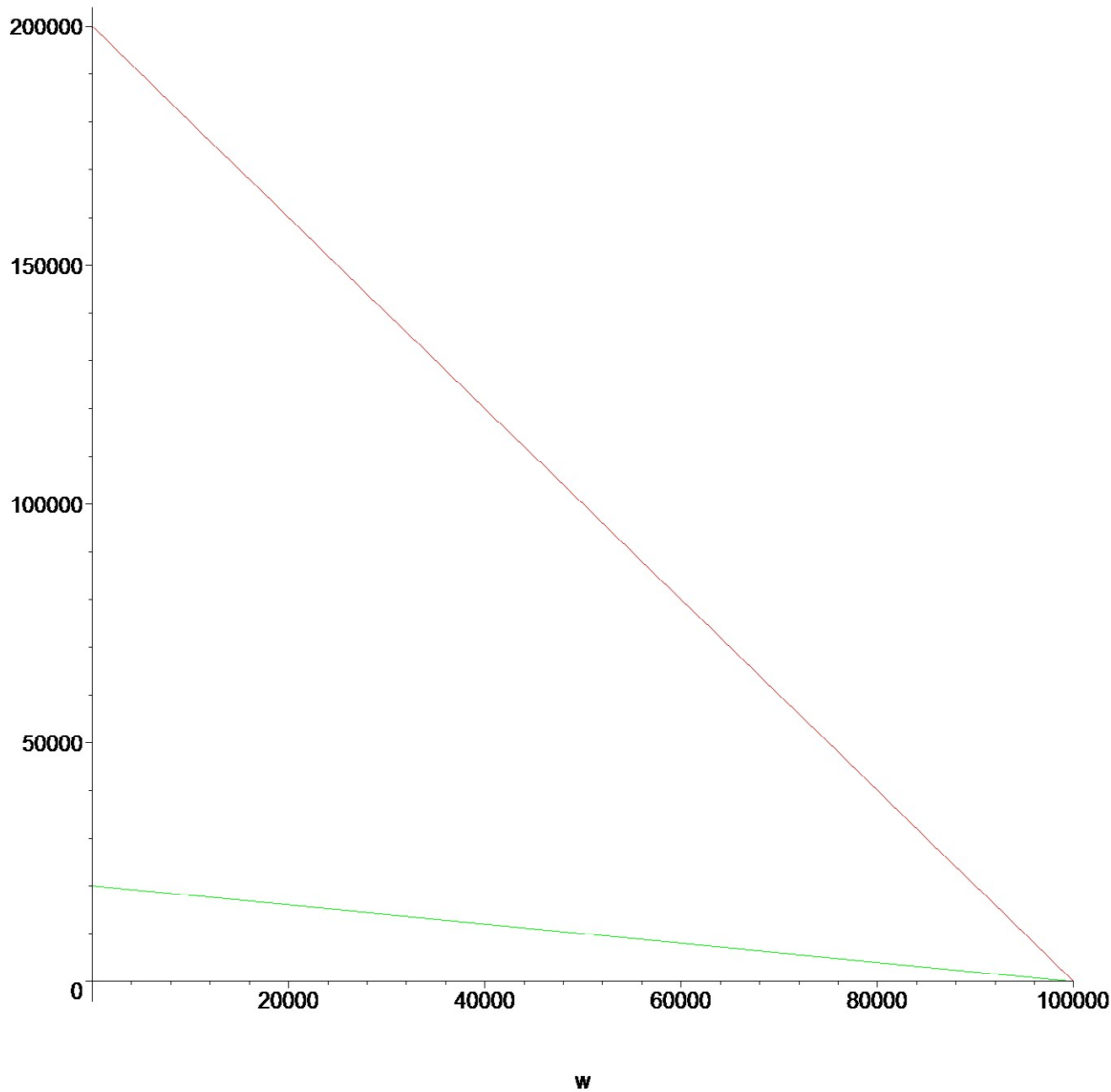
While the German PPF is:

```
> PPFGermany:=2*w+b=200000;
```

$$PPFGermany := 2\,w + b = 200000$$

In part a, I ask you to graph both of these on the same diagram. An alternative to the implicit plot routine in this case is given by the following command, which solves each of the functions for beer, and then plots each as a function of the amount of wine produced:

```
> plot({solve(PPFGermany,b),solve(PPFFrance,b)},w=0..100000);
```



Part b)

The amount of beer that Germany can produce as a function of wine is given by:

```
> solve(PPFGermany,b);
```

$$-2\,w + 200000$$

Implying that by producing a unit of wine, Germany gives up the opportunity to produce two units of beer. Of course, the other side of this coin is that Germany gives up 1/2 a unit of wine to produce a unit of beer. By the same token, in France we have:

```
> solve(PPFFrance,b);
```

$$-\frac{w}{5}+20000$$

Meaning that, in France, production of a unit of wine involves giving up a 1/5 a unit of beer. Alternatively, by producing one unit of beer, France gives up the opportunity to produce five units of wine.

These are the Marginal Rates of substitution in each country.

Part c)

Judging from the fact that Germany's PPF lies everywhere above Frances, Germany has an absolute advantage in production of both goods. However, France has a comparative advantage in wine production, because the opportunity costs are lower (note that this implies that Germany has a comparative advantage in production of beer).

Part d and e)

Any price of beer in terms of wine that lies between the countrys' opportunity costs will benefit both. For example, in Germany, the opportunity cost of producing a unit of wine is 2 beers. In France, the opportunity cost of producing a unit of wine is is 1/5 a unit of beer. Any price of exchange between these two will generate gains to both countries. For example, if a unit of beer trades for one unit of wine, Germany can produce just beer, and trade each beer for one wine. The New PPF for germany therefore is:

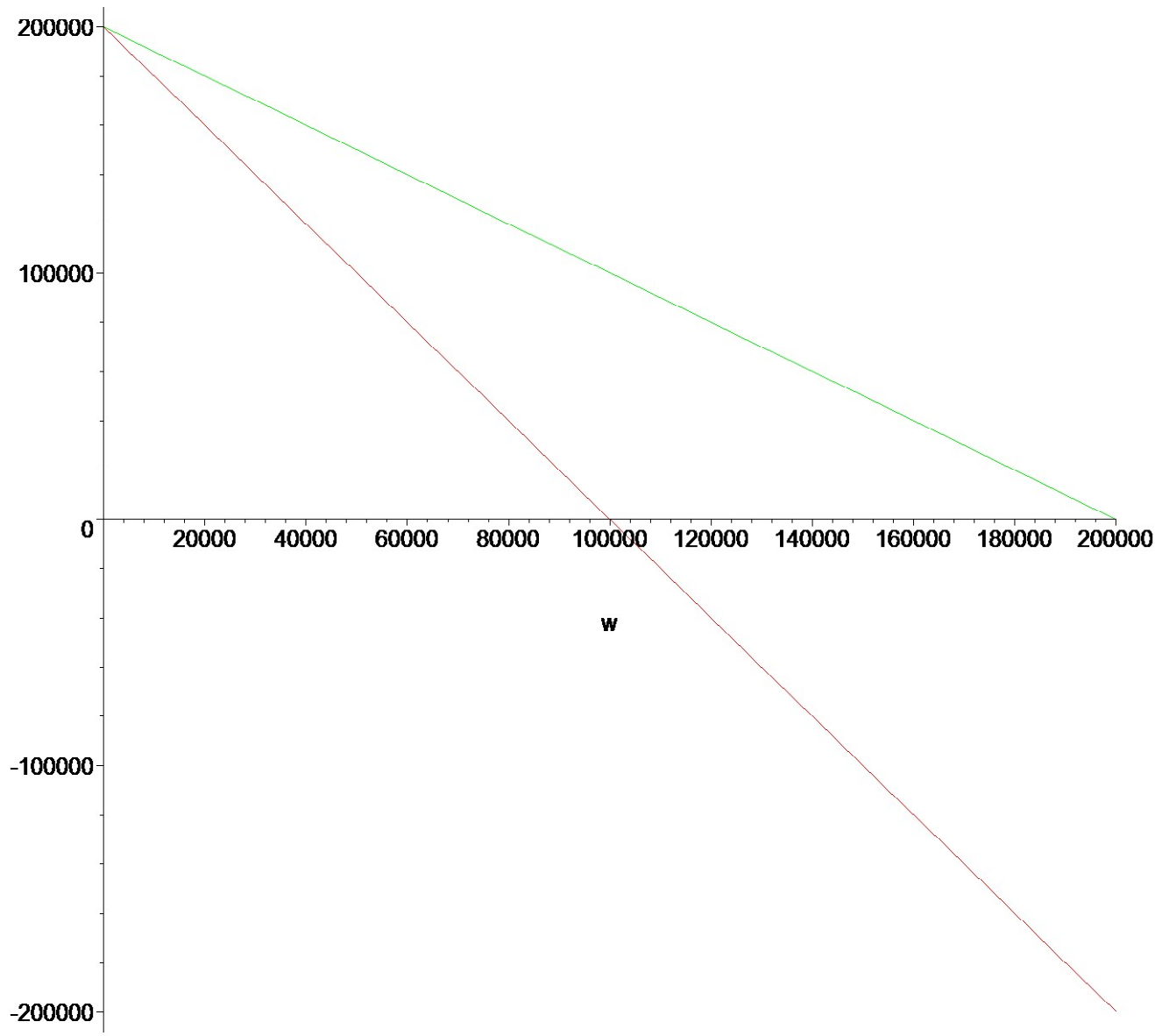
```
> PPF2Germany:=w+b=200000;
```

$$PPF2Germany := w + b = 200000$$

```
> PPF2France:=w+b=200000;
```

$$PPF2France := w + b = 200000$$

```
> plot({solve(PPFGermany,b),solve(PPF2Germany,b)},w=0..200000);
```

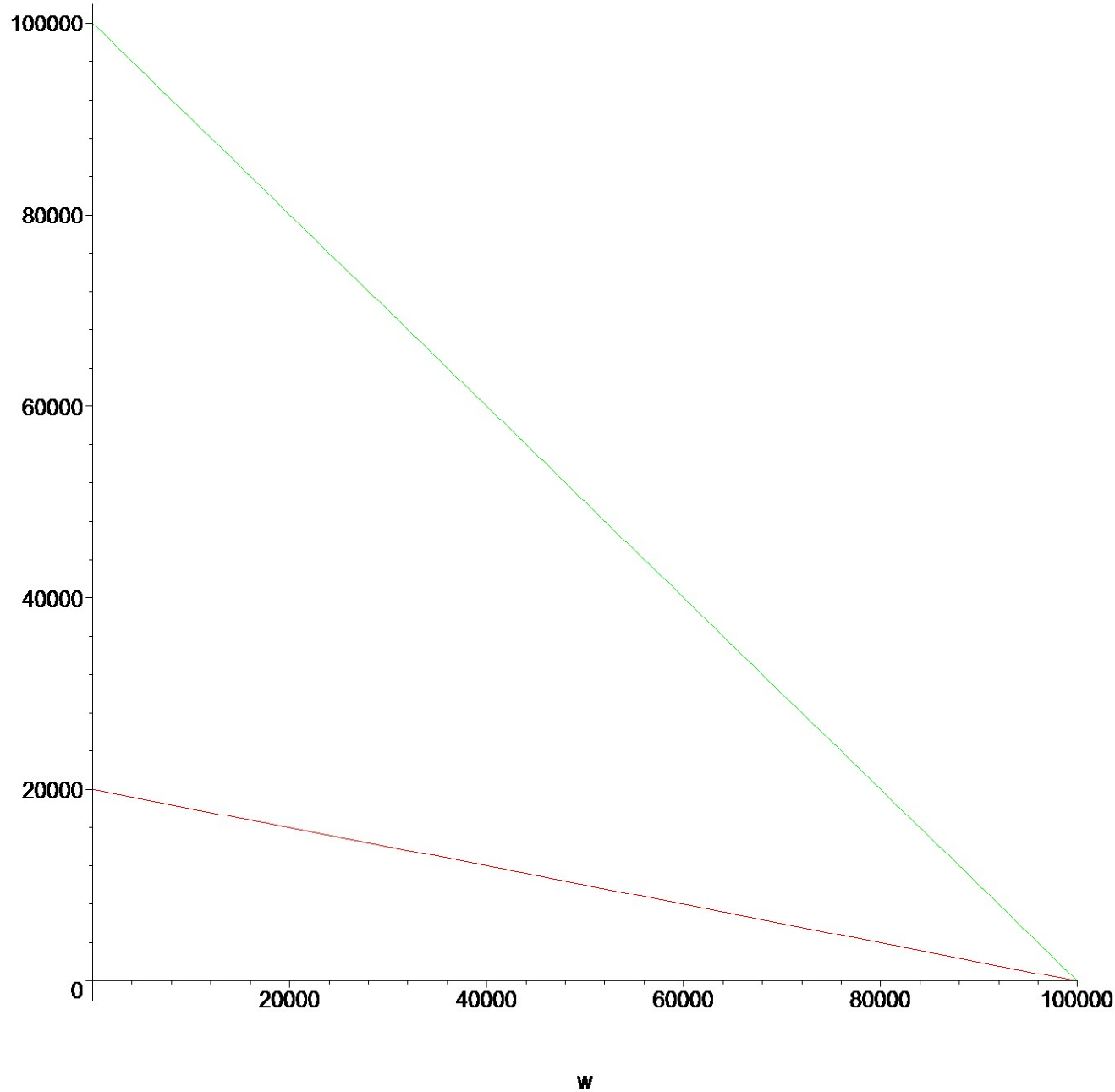


The New French PPF is given as follows:

```
> PPF2France:=w+b=100000;
```

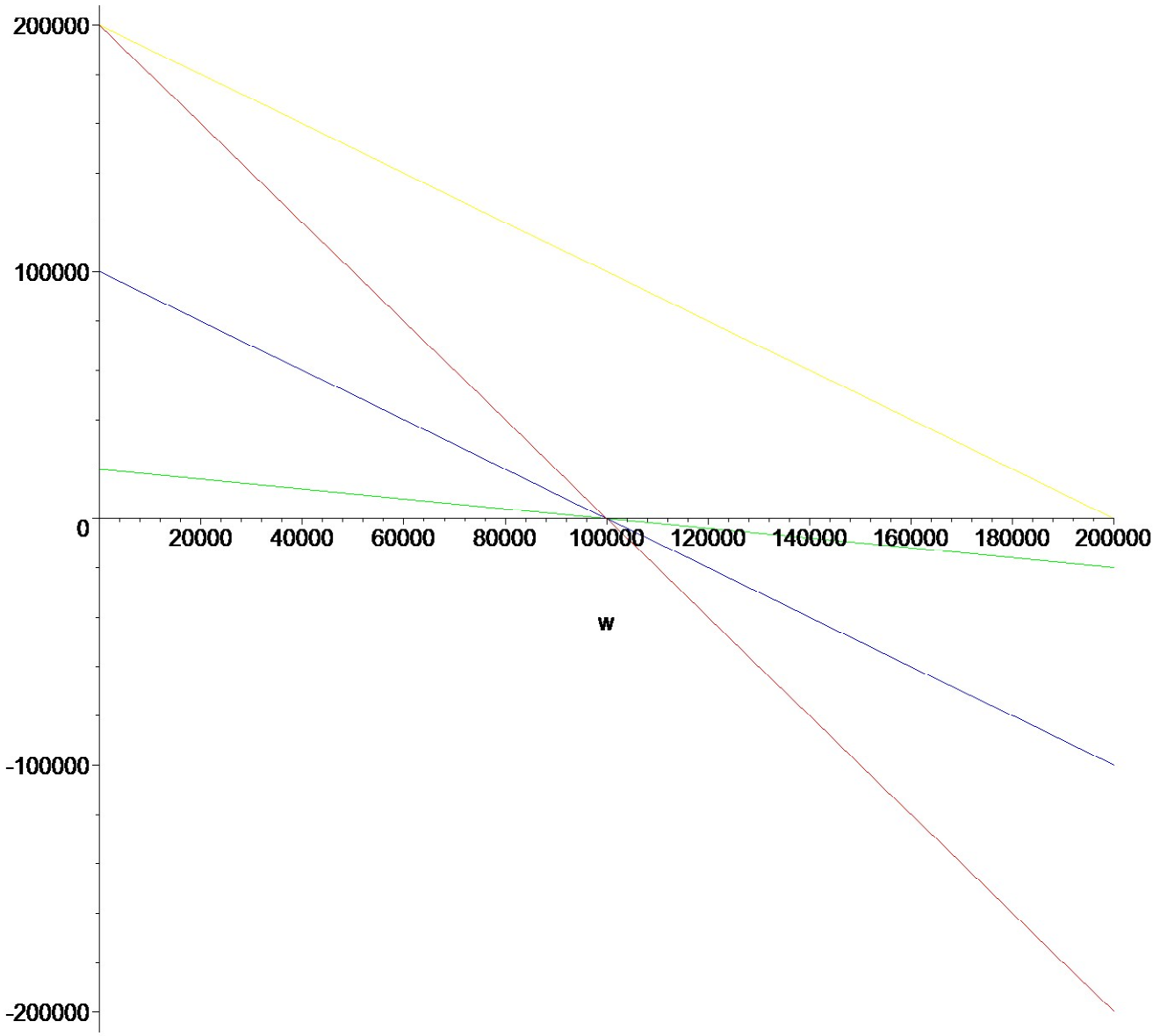
$$PPF2France := w + b = 100000$$

```
> plot({solve(PPFFrance,b),solve(PPF2France,b)},w=0..100000);
```



If we put them all on the same graph, we have:

```
> plot({solve(PPFFrance,b),solve(PPF2France,b),solve(PPFGermany,b),s
  olve(PPF2Germany,b)},w=0..200000);
```



Here, the French PPF is initially the yellow line, and pivots to the blue line. The German PPF is initially the red line, and pivots to the yellow line.

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