PROBLEM SET #1 - ANSWER KEY

Problem I.a.i. The first step is to solve the function at hand for Y, which gives:

```
> solve(x^2+y^2=20,y);
```

$$\sqrt{-x^2+20},\ -\sqrt{-x^2+20}$$

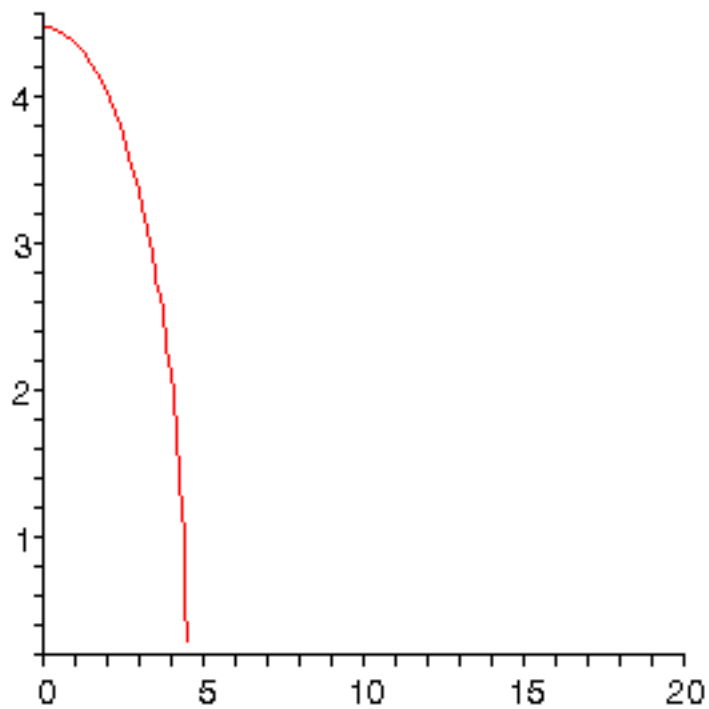
Taking the derivative of this with respect to X will give us the MRS (don't use the negative solution)

```
> MRS[i]:=diff(sqrt(-x^2+20),x);
```

$$MRS_i := -\frac{x}{\sqrt{-x^2+20}}$$

Note that the expression is simply "Y" from the first operation, so we could just as easily write this function as X/Y. The graph of this function looks like:

```
> plot(sqrt(-x^2+20),x=0..20);
```



Problem I.a.ii. Repeat the same steps as before: solve for Y, then differentiate the result with respect to X:

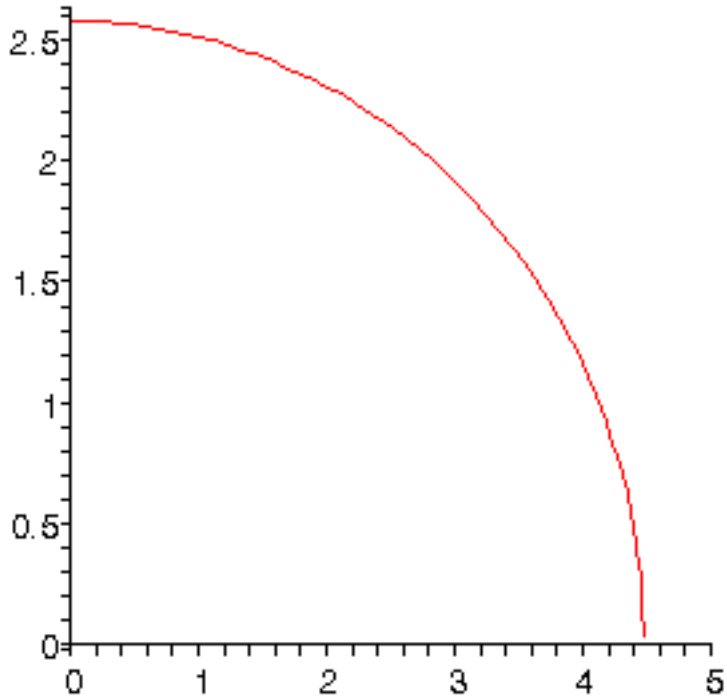
```
> solve(x^2+3*y^2=20,y);
```

$$\frac{1}{3}\sqrt{-3x^2+60},\ -\frac{1}{3}\sqrt{-3x^2+60}$$

```
> MRS[ii]:=diff(1/3*sqrt(-3*x^2+60),x);
```

$$MRS_{ii} := -\frac{x}{\sqrt{-3\,x^2 + 60}}$$

```
> plot(1/3*sqrt(-3*x^2+60),x=0..5);
```



Problem I.a.iii. Solve, differentiate, plot.

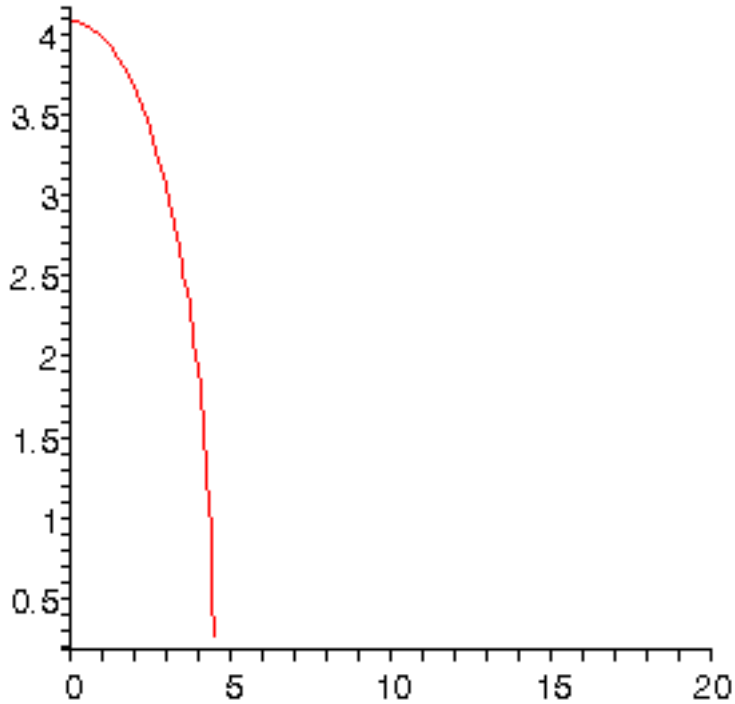
```
> solve(5*x^2+6*y^2=100,y);
```

$$\frac{1}{6}\sqrt{-30\,x^2 + 600},\ -\frac{1}{6}\sqrt{-30\,x^2 + 600}$$

```
> MRS[iii]:=diff(1/6*sqrt(-30*x^2+600),x);
```

$$MRS_{iii} := -\frac{5\,x}{\sqrt{-30\,x^2 + 600}}$$

```
> plot(1/6*sqrt(-30*x^2+600),x=0..20);
```



Problem I.a.iv.

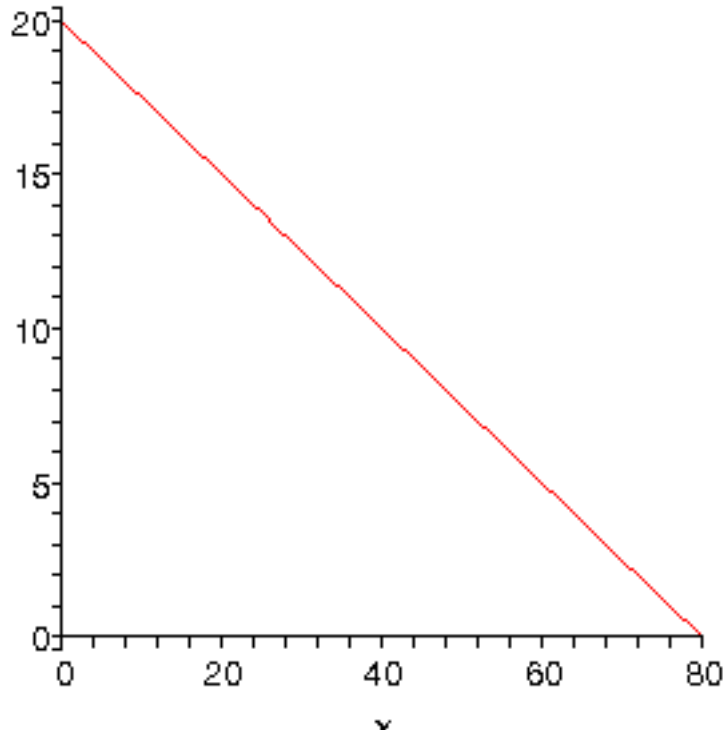
```
> solve(x+4*y=80,y);
```

$$-\frac{1}{4}x + 20$$

```
> MRS[iv]:=diff(-1/4*x+20,x);
```

$$MRS_{iv} := \frac{-1}{4}$$

```
> plot(-1/4*x+20,x=0..80);
```



Problem I.a.v.

```
> solve(x^3+y^3=20,y);
```

$$(20 - x^3)^{(1/3)}, -\frac{1}{2}(20 - x^3)^{(1/3)} - \frac{1}{2} I \sqrt{3} (20 - x^3)^{(1/3)}, -\frac{1}{2}(20 - x^3)^{(1/3)} + \frac{1}{2} I \sqrt{3} (20 - x^3)^{(1/3)}$$

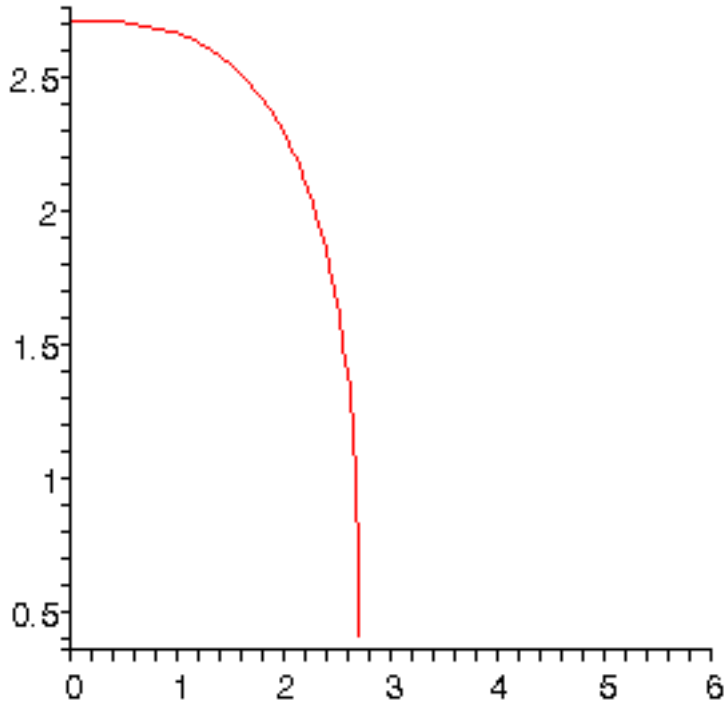
Take the first solution as the only one that really makes sense here!

```
> MRS[v]:=diff((20-x^3)^(1/3),x);
```

$$MRS_v := -\frac{x^2}{(20 - x^3)^{(2/3)}}$$

NOte that this is equivalent to x squared divided by y squared!

```
> plot((20-x^3)^(1/3),x=0..6);
```



I. b. Note that the PPF in v has a bigger bulge than it did before. This owes to the fact that the tradeoff has changed due to the different technological relationship. The MRS in this case is flatter over some portion, but then gets steeper quickly over some portion.

Problem II.a. Baker seeks to maximize profits, which can be written as:

```
> profits:=p[x]*x+p[y]*y;
```

$$profits := p_x\, x + p_y\, y$$

In this case, we know that the prices are given as 5 and 10 respectively, so we can write:

```
> profits1:=subs({p[x]=5,p[y]=10},profits);
```

$$profits1 := 5\, x + 10\, y$$

Baker's PPF is given as in I.a., and it is best to first solve the PPF for x, which gives:

```
> solve(x^2+y^2=20,y);
```

$$\sqrt{-x^2 + 20},\ -\sqrt{-x^2 + 20}$$

Substitute this into the profit1 function for y above (our simplified profit function) to give:

```
> subs(y=sqrt(-x^2+20),profits1);
```

$$5\, x + 10\, \sqrt{-x^2 + 20}$$

This function describes Baker's profits only as a function of x. To maximize this, we take the derivative and set it equal to zero. This gives:

```
> diff(5*x+10*sqrt(-x^2+20),x);
```

$$5 - \frac{10\, x}{\sqrt{-x^2 + 20}}$$

```
> x[iia]:=solve(5-10/(-x^2+20)^(1/2)*x=0,x);
```

$$x_{iia} := 2$$

So Baker will, given the prices, produce 2 units of X. We can easily find out how much of Y he will produce by substituting this into the PPF, which gives:

```
> y[iia]:=simplify(subs(x=2,sqrt(-x^2+20)));
```

$$y_{iia} := 4$$

So Baker also produces 4 units of Y.

II.b. If the price of good Y falls to 8, Bakers profits are:

```
> profits2:=subs({p[x]=5,p[y]=8},profits);
```

$$profits2 := 5\,x + 8\,y$$

Substituting in the PPF equation as before gives:

```
> subs(y=sqrt(-x^2+20),profits2);
```

$$5\,x + 8\,\sqrt{-x^2 + 20}$$

To maximize, again, take the derivative with respect to X and set the result equal to zero. This gives:

```
> diff(5*x+8*sqrt(-x^2+20),x);
```

$$5 - \frac{8\,x}{\sqrt{-x^2 + 20}}$$

```
> x[iib]:=evalf[5](simplify(solve(5-8/(-x^2+20)^(1/2)*x=0,x)));
```

$$x_{iib} := 2.3702$$

Not the neatest looking thing, but a result nonetheless! For Y, we get:

```
> y[iib]:=evalf[5](simplify(subs(x=10/89*sqrt(445),sqrt(-x^2+20))));
```

$$y_{iib} := 3.7925$$

II.c. Baker gets a new tractor, changing the PPF. Our first step is to solve the PPF for y, so we can use this in the profit function. THis gives

```
> solve(x^2+y^2=35,y);
Warning,  computation interrupted
```

Now, the profit function is:

```
> profits3:=5*x+8*y;
```

$$profits3 := 5\,x + 8\,y$$

So substituting in our new PPF for this gives:

```
> subs(y=sqrt(-x^2+35),profits3);
```

$$5\,x + 8\,\sqrt{-x^2 + 35}$$

Again, to maximize profits, take the derivative and set equal to zero. Performing this operation leaves us with:

```
> diff(5*x+8*sqrt(-x^2+35),x);
```

$$5 - \frac{8\,x}{\sqrt{-x^2 + 35}}$$

```
> x[iic]:=evalf[5](solve(5-8/(-x^2+35)^(1/2)*x));
```

$$x_{iic} := 3.1355$$

```
> y[iic]:=evalf[5](simplify(subs(x=5/89*sqrt(3115),sqrt(-x^2+35))));
```

$$y_{iic} := 5.0168$$

II.d. Here we have essentially the same problem, just with a different PPF. Again, the steps are: solve the PPF for y, substitute the result into the profit function, differentiate, and solve. We get:

```
> solve(x^2+y^2/2=20,y);
```

$$\sqrt{-2\,x^2 + 40},\ -\sqrt{-2\,x^2 + 40}$$

```
> profits4:=5*x+8*y;
```

$$profits4 := 5\,x + 8\,y$$

```
> subs(y=sqrt(-2*x^2+40),profits4);
```

$$5\,x + 8\sqrt{-2\,x^2 + 40}$$

```
> diff(5*x+8*sqrt(-2*x^2+40),x);
```

$$5 - \frac{16\,x}{\sqrt{-2\,x^2 + 40}}$$

```
> x[iid]:=evalf[5](solve(5-16/(-2*x^2+40)^(1/2)*x,x));
```

$$x_{iid} := 1.8078$$

```
> y[iid]:=evalf[5](simplify(subs(x=10/51*sqrt(85),sqrt(-2*x^2+40))));
```

$$y_{iid} := 5.7849$$

```
>
```

III. This problem is mathematically easy, but perhaps conceptually difficult. The PPFS are given by:

```
> PPFB:=10=1/3*A+1/4*M;
```

$$PPFB := 10 = \frac{1}{3}A + \frac{1}{4}M$$

```
> PPFA:=10=1/3*A+1/5*M;
```

$$PPFA := 10 = \frac{1}{3}A + \frac{1}{5}M$$

Let's first rearrange these so we can plot them:

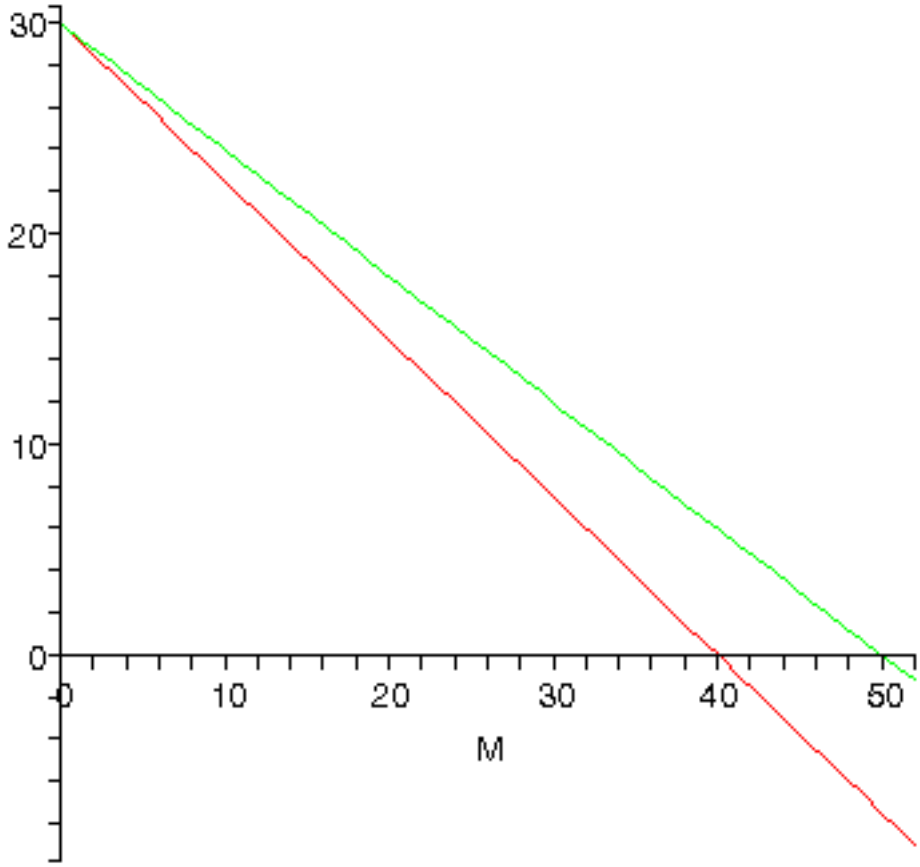
```
> AB:=solve(PPFB,A);
```

$$AB := 30 - \frac{3}{4}M$$

```
> AA:=solve(PPFA,A);
```

$$AA := 30 - \frac{3}{5} M$$

```
> plot({AB,AA},M=0..52);
```



Evidently, the green line is the PPF for Austria, PPFA, while the red line is the PPF for Bulgaria.

In this case, the marginal rate of transformation is simply the slope of the line; that is, in Bulgaria, production of one unit of M results in a loss of 3/4 of an Agricultural unit, while in Austria, production of one unit of M results in a loss of 3/5 of an agricultural good. Note that this implies that Austria should specialize in production of Manufacturing goods, because their opportunity costs are lower.

Alternatively, we could have written the equations as:

```
> MB:=solve(PPFB,M);
```

$$MB := 40 - \frac{4}{3} A$$

```
> MA:=solve(PPFA,M);
```

$$MA := 50 - \frac{5}{3} A$$

This reveals that the opportunity costs of production of A in Bulgaria are 4/3 of an M, while they are 5/3 of an M in Austria. Therefore, Bulgaria should specialize in production of A, as her opportunity costs are lower.

To answer the last part of the question: world trade needs to occur at trading prices somewhere between the two opportunity costs. It is clear that an A needs to trade for at least 4/3 of an M, and at most 5/3 of an M. Conversely, an M needs to trade for at least 3/5 of an A, and at most 3/4 of an A. Let's suppose that an M trades for 7/10 of an A, or that an A trades for 10/7 an M.

```
>
```

IV.a. The first step is simply to graph both of the PPFs. This is easily done by first solving each function for f:

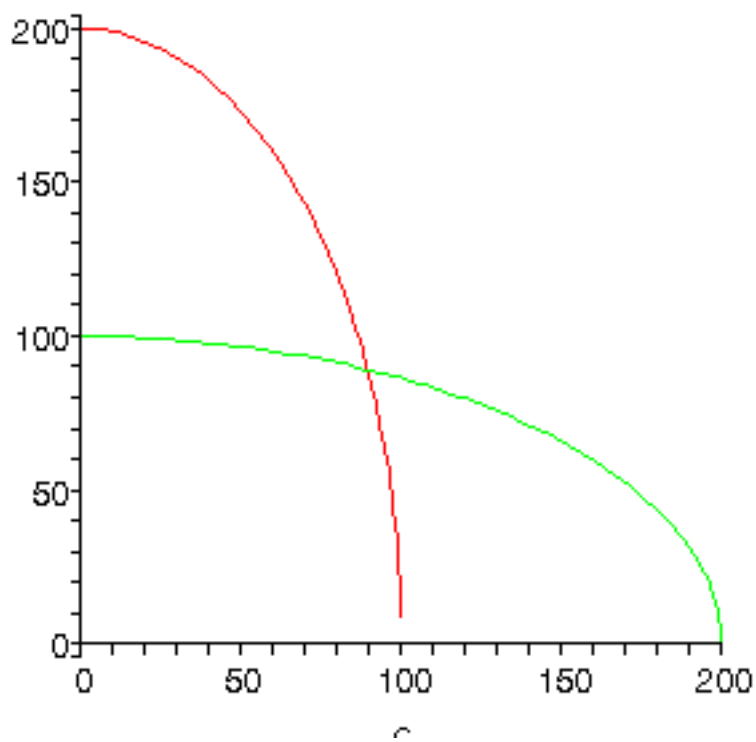
```
> solve(f^2/4+c^2=10000,f);
```

$$2\sqrt{-c^2 + 10000}, -2\sqrt{-c^2 + 10000}$$

```
> solve(f^2+c^2/4=10000,f);
```

$$\frac{1}{2}\sqrt{-c^2 + 40000}, -\frac{1}{2}\sqrt{-c^2 + 40000}$$

```
> plot({2*sqrt(-c^2+10000),1/2*sqrt(40000-c^2)},c=0..200);
```



These are the PPFs of you (green) and your shipmate (red). Apparently, your shipmate is more gifted in coconut production, while you are a more able producer of fish.

IIb. This is pretty straightforward...You can double your production of both tasks if you fish and your shipmate produces coconuts. As the amount of both goods produced has increased, the previous point cannot

be efficient

IIc. First, note that when you and your shipmate are both producing goods, you are both on your PPFs and are both allocating your resources to producing things you are relatively better at. Note, however, that it is possible to produce more. For example, instead of setting output at 200, you could set output of fish at 199, and produce almost 10 additional units of coconuts yourself! If you and your shipmate both did this, you'd have about 19 more units of coconuts. To see this, note that your output of coconuts is obtained by solving your PPF for the number of coconuts produced. This gives:

```
> solve(f^2/4+c^2=10000,c);
```

$$\frac{1}{2}\sqrt{-f^2+40000},\ -\frac{1}{2}\sqrt{-f^2+40000}$$

The amount of coconuts you can produce is zero if you produce 200 fish, but if you produce 199 fish, you can get:

```
> subs(f=199,1/2*sqrt(-f^2+40000));
```

$$\frac{1}{2}\sqrt{399}$$

```
> newccnts := evalf[5](1/2*sqrt(399));
```

$$newccnts := 9.9875$$

```
>
```

By right-clicking on this and using the "approximate" button, this can be seen to be roughly 10 new coconuts!

The easiest way to find a point where both of you are producing efficiently is to 1) fix the amount one person is producing, and then choose the other's output so that the MRS is equal. For example, suppose your shipmate produces 140 coconuts, which results in him producing 71.415 fish (substitute into the PPF to see this):

```
> evalf[5](subs(c=140,1/2*sqrt(40000-c^2)));
```

$$71.415$$

The expression for your shipmate's MRS at this point is obtained by differentiation and substituting in c=140:

```
> diff(1/2*sqrt(40000-c^2),c);
```

$$-\frac{c}{2\sqrt{-c^2+40000}}$$

```
> evalf[5](subs(c=140,-1/2/(40000-c^2)^(1/2)*c));
```

$$-.49011$$

So your shipmate's MRS is -.49011 when producing 140 coconuts and 71.415 fish. Your MRS is:

```
> diff(2*sqrt(-c^2+10000),c);
```

$$-\frac{2\,c}{\sqrt{-c^2+10000}}$$

If we can figure out a combination of goods so that your MRS is equal to your shipmates, we've found an efficient point. To do this, we must solve:

```
> eq1:=-2/(-c^2+10000)^(1/2)*c=-(.49011);
```

$$eq1 := -\frac{2\,c}{\sqrt{-c^2+10000}} = -.49011$$

Which will give us just such a number. Let's do this now.

```
> solve(eq1, c);
```

$$23.80126243$$

Which implies if you produce 23.8 coconuts, and using the PPF

```
> subs(c=23.8,2*sqrt(-c^2+10000));
```

$$194.2530309$$

194 fish, you are at an efficient point. You can check this by verifying that your PPF and your shipmates PPF have the same MRS at this point.

If both you and your shipmate are faced with a market, you each have seek to maximize revenues from the goods produced. Your and your shipmate's revenues can be written as:

```
> R[you]:=p[f]*f[you]+p[c]*c[you];
```

$$R_{you} := p_f f_{you} + p_c\, c_{you}$$

```
> R[sm]:=p[f]*f[sm]+p[c]*c[sm];
```

$$R_{sm} := p_f f_{sm} + p_c\, c_{sm}$$

```
>
```

In fact, you can do this problem at a general level and see that prices cause you two to equate your MRS's, which is our efficiency condition. For example, If we substitute into your and your shipmate's revenue function using the PPF, we get:

```
> R[you2]:=subs(f[you]=2*sqrt(-c[you]^2+10000),R[you]);
```

$$R_{you2} := 2\,p_f \sqrt{-c_{you}^{\,2}+10000} + p_c\, c_{you}$$

```
> R[sm]:=subs(f[sm]=1/2*sqrt(40000-c[sm]^2),R[sm]);
```

$$R_{sm} := \frac{1}{2} p_f \sqrt{40000 - c_{sm}^{\,2}} + p_c\, c_{sm}$$

To maximize revenue, take the derivative and set it equal to zero.Differentiating gives:

```
> diff(R[you2],c[you]);
```

$$-\frac{2\,p_f\, c_{you}}{\sqrt{-c_{you}^{\,2}+10000}} + p_c$$

```
> diff(R[sm],c[sm]);
```

$$-\frac{p_f\, c_{sm}}{2\sqrt{40000 - c_{sm}^{\,2}}} + p_c$$

setting equal to zero and solving for your output gives:

```
> c[y,o]:=solve(-2*p[f]/(-c[you]^2+10000)^(1/2)*c[you]+p[c]=0,c[you]);
```

$$c_{y,\,o} := \frac{100\,p_c}{\sqrt{p_c^{\,2} + 4\,p_f^{\,2}}}, -\frac{100\,p_c}{\sqrt{p_c^{\,2} + 4\,p_f^{\,2}}}$$

While for your shipmate you get:

```
> c[sm,o]:=solve(-1/2*p[f]/(40000-c[sm]^2)^(1/2)*c[sm]+p[c]=0,c[sm]);
```

$$c_{sm,\,o} := \frac{400\,p_c}{\sqrt{4\,p_c^{\,2} + p_f^{\,2}}}, -\frac{400\,p_c}{\sqrt{4\,p_c^{\,2} + p_f^{\,2}}}$$

To figure out coconuts Baker produces at the prices of 5 seashells for coconuts and 10 seashells for fish, substitute these into the (positive) solution to get:

```
> evalf[5](subs(p[c]=5,p[f]=10,100*p[c]/(p[c]^2+4*p[f]^2)^(1/2)));
```

$$24.255$$

Use the PPF to figure out the rest, which gives:

```
> subs(c=24.255,2*sqrt(-c^2+10000));
```

$$194.0277813$$

As the amount of fish you produce. A similar calculation can be done for your roommate (which I leave to you).Your MRS at this point is:

```
> MRS[you]:=-2*p[f]/(-c^2+10000)^(1/2)*c;
>
```

$$MRS_{you} := -\frac{2\,p_f\,c}{\sqrt{-c^2 + 10000}}$$

```
> evalf[5](subs(c=24.255,MRS[you]));
```

$$-0.50004\,p_f$$

```
>
>
```

You can easily verify that your roommate also has this RTS at this point, because markets are efficient. Or, you could simply note that both you and your roommate maximize revenue by setting your MRS's equal to the price ratio, which makes them equal to each other.

III.f. Baker could simply change the price of the goods if he wanted more of one, though this necessarily implies that he produces less of the other.