FE341: Microeconomics
Homework 4
Due October 18th

Do 3 of the following 4:

1. From the textbook page 253: Chapter 7 Questions for Review #5

NOTE: This question asks about the $MRTS_{KL}$!

To minimize cost, the manufacturer should use a combination of capital and labor so the rate at which he can trade capital for labor in his production process is the same as the rate at which he can trade capital for labor in external markets. The manufacturer would be better off if he increased his use of capital and decreased his use of labor, decreasing the $MRTS_{LK}$. He should continue this substitution until his MRTS equals the ratio of the rental rate to the wage rate. The MRTS in this case is equal to the MP_K/MP_L. As the manufacturer uses more K and less L the MP_K will decrease (diminishing returns to capital) and the MP_L will increase, both of which will decrease the MRTS until it is equal to the ratio of the input prices (rental rate on capital divided by the wage rate).

2. From the textbook page 253: Chapter 7 Questions for Review #6

The isocost line represents all possible combinations of labor and capital that may be purchased for a given total cost. The slope of the isocost line is the ratio of the input prices of labor and capital. If input prices are fixed, then the ratio of these prices is clearly fixed and the isocost line is straight. Only when the ratio or factor prices changes as the quantities of inputs change is the isocost line not straight.

3. From the textbook page 253: Chapter 7 Questions for Review #7

Marginal cost can be increasing while average variable cost is either increasing or decreasing. If marginal cost is less (greater) than average variable cost, then each additional unit is adding less (more) to total cost than previous units added to the total cost, which implies that the AVC declines (increases). Therefore, we need to know whether MC is greater than the AVC to determine whether AVC is increasing or decreasing.

4. From the textbook page 253: Chapter 7 Questions for Review #8

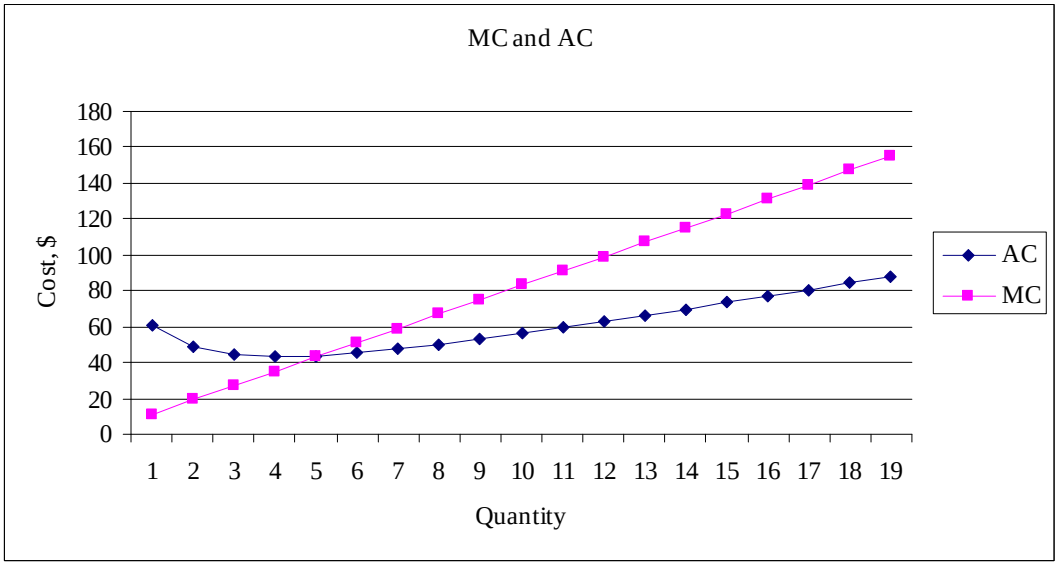
If the average variable cost is increasing (decreasing), then the last unit produced is adding more (less) to total variable cost than the previous units did, on average. Therefore, marginal cost is above (below) average variable cost. In fact, the point where MC exceeds AVC is also the point where AVC starts to rise.

Do 3 of the following 4:

5. The total cost function for a firm producing widgets is $TC = 4q^2 + 3q + 100$, where q is the number of widgets produced per week.
   a. Calculate and graph the average cost function, and calculate and graph the marginal cost function.

$$AC = \frac{TC}{Q} = \frac{4q^2 + 3q + 100}{q} = 4q + 3 + \frac{100}{q}$$

$$MC = \frac{dTC}{dQ} = 8q + 3$$



b. At what level of widget output does average cost reach a minimum, and what is the average cost at this level of output?

To find the cost-minimizing amount of output, take the derivative of average costs and set it equal to zero. This gives us the equation: $4 - \frac{100}{q^2}, q = 5$ so cost is minimized at a cost of 5. Substituting this into the average cost function gives AC 20+3+20=43.

c. Show that the marginal cost curve intersects the average cost curve at its minimum.

The easiest way to do this is to show that marginal cost and average cost are equal at the minimum level of average cost. Substituting q=20 into the marginal cost function gives 3(400)-1600+430=30, verifying the answer.

6. From the textbook page 254: Chapter 7 Exercises #8

a. K is fixed at 5, therefore the Q^{SR} = 25L (short run production function). Thus, for any level of output q, the number of labor teams hired is L = q/25. The total cost function is given by the sum of the costs of capital, labor and raw materials: TC = rK + wL +2000q TC = 50,000 + 5,000 (q/25) + 2000q = 50,000 + 2,200q
AC = TC/q = 50,000/q + 2,200
MC = dTC/dq = 2,200

b. To produce q = 250 engines, we need labor teams L = q/25 or L = 10. Average costs are 2,400 (plug into AC above).

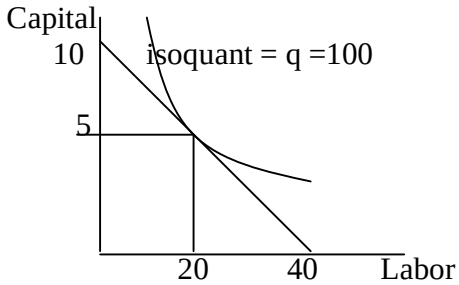
c. Now K is no longer fixed at 5, so q = 5KL. We need to find the combination of K and L that minimizes costs at any level of q. This is where the slope of the isoquant (MRTS) is equal to the slope of the isocost à MP_L/MP_K = w/r. The MP_L dq/dL = 5K and the MP_K = dq/dK = 5L while the ratio of the input prices (w/r) = 5000/10,000 = ½. Therefore, need K/L = ½.

7. From the textbook page 254: Chapter 7 Exercises #10

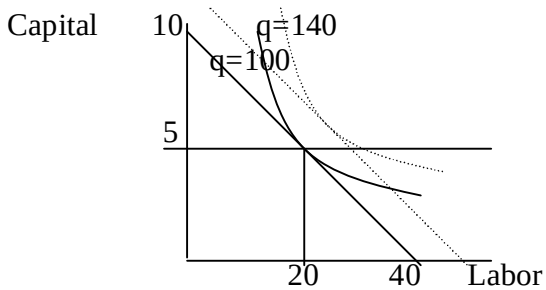
If the firm can produce one chair with either 4 hours of labor or 4 hours of capital machinery (or any combination), then the isoquant is a straight line (these are perfect substitutes à q = K + L) and has a slope of 1. The isocost line is TC = 30L + 15K and has a slope of 2 (labor on x-axis). The cost minimizing point is a corner solution; the firm will use only capital (the less expensive input) to produce the 1 chair.

8. From the textbook page 254: Chapter 7 Exercises #11

a. Isoqost = TC = 80K + 20L = 800 (plugging in L = 20 and K = 5 yields TC). The isoquant is convex, TC minimizing means the isoquant is tangent to the isocost.



b. L = 39.2. To solve for this, $q = 10K^{1/2}L^{1/2}$, and q = 140 and K = 5 à L = 39.2. The new TC = $1184 = 20*39.2 + 80*5. Because capital is fixed in the short run the firm can only move out to a new level of output by increasing labor (ending up at the point where the solid line at K = 5 hits the new dashed isoquant). This is not likely to be cost minimizing (you can see where cost minimization occurs where the dashed isocost intersects the dashed isoquant; at a higher level of K).



c. Minimize TC = 80K + 20L subject to $q = 10K^{1/2}L^{1/2} = 140$. 1st solve isocost for L = 196/K; 2nd plug into TC = 80K + 20(196/K) = 80K + 3920/K; 3rd differentiate TC with respect to K and set equal to 0: dTC/dK = 80 $3920/K^2 = 0$; 4th solve for K, K =7; 5th substitute back into isocost to find L = 196/7 = 28; this yields a TC = $1120.