The Road Not Taken - What Is The "Appropriate" Path to Development When Growth is Unbalanced?

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Abstract

This paper develops a model that endogenizes both directed technologies and demography. Potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to evaluate potentially divergent development paths. We find that unskilled-labor bias technological growth can induce higher fertility and lower education, exerting downward pressure on growth in per person income. Despite this, for most plausible developing country scenarios, unskilled intensive growth produces more per person income than skill intensive growth. This result is robust to a variety of growth modeling assumptions.

- *Keywords*: directed technical change, demography, human capital, fertility
- *JEL Codes*: O31, O33, J13, J24

1 Introduction

The last half century has seen great divergence in living standards among the countries of the developing world; while rich nations have maintained fairly consistent rates of growth (2 or 3% per annum), poorer nations have traversed widely different growth paths (between -1 and 7%). This paper suggests a possible source of this divergence by producing a model emphasizing the interdependence between directed technical change and demography. In this model, potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to analyze the macroeconomic effects of "unbalanced growth," where a country develops *either* labor-intensive techniques and expands the pool of unskilled labor, or skill-intensive techniques and expands the pool of human capital. Which path will lead to greater overall prosperity is the primary focus of this paper.

The model emphasizes how economic growth can often be an unbalanced process, where choices are made between alternative modes of production.[1] A farm can be maintained either with uneducated farmers wielding hand tools, or with farmers skilled in using agronomic instruments and automated machinery. A factory can be structured as an assembly line run mainly with unskilled workers supervised by a few skilled ones, or as a computer-controlled facility mainly run by skilled workers with a few unskilled janitors.[2] A road can be built using lots of manual labor physically laying down stone and brick by hand, or construction workers trained in using bulldozers and steamrollers. These examples highlight not only that technologies can be directed towards particular factors, but also that each country can take its own unique development path, producing similar things in very different ways (Owen et al. 2009).

This paper boils down all these considerations into a simple question - would greater aggregate wealth be generated with skilled-labor biased technological growth (the "skill-intensive path") or unskilled-labor biased technological growth (the "unskilled-intensive path")? The answer of course is that it depends. It depends on how *productive* or *abundant* skilled and unskilled labor are. And it depends on how technological changes can affect *future* supplies of skilled and unskilled labor.

By exploring the simultaneity between technological changes directed towards particular factors and the factors themselves, we can explore some of these issues. This approach constitutes a notable departure from the existing literature on technologies that augment specific factors or sectors. These works often either highlight the "inappropriateness" of growth in technologies that can be implemented by only a small portion of the economy, or they demonstrate the effects of biased technical growth on factor inequality. In the former case, Basu and Weil (1998),

[1] Ray (2010) suggests that the typical abstraction of balanced growth is both unrealistic and not particularly helpful for many important questions in development.

[2] This example comes from Caselli and Coleman (2006)

Zeira (1998), and Acemoglu and Zilibotti (2001) illustrate how technologies designed for capital-intensive (physical or human) societies that diffuse to capital-scarce regions are used ineffectually there, if at all. Other works counter by highlighting the indispensability of investment in general, and education in particular, for growth (Barro 2001, Ehrlich and Murphy 2007). In the latter case, works such as Katz and Murphy (1992), Acemoglu (1998), Kiley (1999), Xu (2001), and Acemoglu (2002) show that when factors are grossly substitutable, skill-biased technological growth will raise skill premia and thus factor inequality, while unskill-biased technological growth will lower them.

But these works do not take into account that factors can evolve, and will adjust to changing economic circumstances.[3] If factors do change in these models, they typically do so exogenously. But this partial equilibrium approach may mislead us, particulary when it comes to *long-run* growth (Acemoglu 2010). Allowing for the co-evolution of factors and technologies can alter our perspective of the "appropriate" technological path - that is, the path that generates more macroeconomic growth. Two new considerations emerge with this approach. The first is that factor-composition shifts from unbalanced growth can have different effects on subsequent technological progress. The other consideration is that different technological paths can produce different rates of population growth; long-term living standards are thus affected both through the numerator (effects on income) and the denominator (effects on capita).[4]

With simulations of the model, we discover a number of things. First, by raising the returns to education, skill-biased technological growth can induce higher education and (through quality-quantity tradeoffs in child-rearing) lower fertility; this provides an additional boost to per person income. This case shows that catering technologies towards the larger sectors or more abundant factors may produce dynamically harmful effects like education decreases.

However, we also find that increases in the overall workforce caused by unskilled intensive technological growth can generate faster technological progress (by raising the scale of the market for innovations, or by generating greater knowledge spillovers from a larger pool of skilled workers); this can actually induce more income growth can the alternative, skill-intensive path. Indeed as we will see, a falling population can have pernicious technological effects in the context of endogenous growth. Thus we see that to answer our titular question, the declines in education generated by unskilled-intensive growth must be weighted against its effects on subsequent technological progress.

This paper heavily borrows from Acemoglu's important work on directed technological change (Acemoglu 1998, 2002). But this work departs from that literature in two fundamental ways.

[3]Papers that do consider interactions between technology and human capital include Stokey 1988, Chari and Hopenhayn 1991, Grossman and Helpmann 1991, Young 1993, Redding 1996, Galor and Weil 2000, and Galor and Moav 2000. None however assess the appropriate path to development for an economy in the context of such simultaneity.

[4]Galor and Mountford (2006) stress changes in fertility in explaining divergent growth paths in history.

First, the literature relies almost exclusively on analyzing long-run *balanced* growth paths, while here we look solely on the *unbalanced* case (where technological growth occurs only in one sector of the economy), implicitly assuming that countries often face a choice in its overall growth direction as it transitions to balanced growth.[5] Second, as already mentioned, the literature almost always treats factors of production as exogenously determined, whereas here they are endogenous in the model.[6]

The rest of the paper is organized as follows. Section 2 motivates the paper by looking at some cross-country data. Section 3 presents the model in steps, first presenting a model of semi-endogenous biased technological growth, and then merging this with a simple theory of demography. This model then motivates our simulation experiments in section 4. Section 5 provides some concluding remarks.

2 Some Data

2.1 A Cross-Section of Factor-Specific Technologies

We begin by taking account of estimated factor-specific productivities of a cross-section of countries. Consider the following production function for country i:

$$Y = [(A_{l,i}L_i)^\sigma + (A_{h,i}H_i)^\sigma]^{1/\sigma} \tag{1}$$

where Y is aggregate GDP. Here we specify production as one with a constant elasticity of substitution between skilled and unskilled labor aggregates H_i and L_i (this elasticity being $1/(1-\sigma)$). $A_{l,i}$ and $A_{h,i}$ are the efficiency levels of unskilled and skilled labor in country i.[7]

If factors of production are paid their marginal products, the "skill-premium" can be written as:

$$\frac{w_{h,i}}{w_{l,i}} = \left(\frac{A_{h,i}}{A_{l,i}}\right)^\sigma \left(\frac{H_i}{L_i}\right)^{\sigma-1} \tag{2}$$

Caselli and Coleman (2006) note that one can study cross-country productivity differences using equations (1) and (2), for these represent two equations with two unknowns. That is,

[5] According to Temple (2003), "A balanced growth path is a special case which is likely to demand restrictive assumptions..it would be a mistake to assume that a good model of growth necessarily gives rise to a balanced growth path."

[6] Acemoglu 1998 relegates the possibility of endogenously determined human capital in the appendix to his paper, while he does not discuss the possibility either in Acemoglu 2002 or in the chapter on directed technical change in his growth textbook (Acemoglu 2008).

[7] This functional form resembles the production function used in section 3, where we endogenize technological growth; efficiency coefficients will proxy for the breadth and depth of factor-complementary machines.

given data on Y_i, L_i, H_i, and $\frac{w_i^h}{w_i^l}$, we can back out each country's implied pair of technological coefficients and compare them.[8]

Key to this exercise is our parameter choice for $\sigma \leq 1$.Careful empirical labor studies such as Autor et al. (1998) and Ciccone and Peri (2005) have found that the elasticity of factoral substitution between more and less skilled workers most likely lies between 1 and 2.5 (consistent with a value of σ between 0 and 0.6). Both for this exercise and the simulations in section 4, we choose a benchmark value of $\sigma = 0.5$ for a proxy elasticity parameter most applicable for a wide range of countries and for a wide variety of skilled and unskilled labor categories.[9]

Figure 1 depicts the relationships between relative technical skill-bias (A_h/A_l), relative skill-endowments (H/L), and income per capita across a broad array of countries. Immediately clear is the positive associations between technical skill-bias and skill endowment, and between technical skill-bias and income levels. These positive relationships hold whether we consider a skilled worker as someone with primary schooling, or someone with secondary schooling, or even someone with a college education. This was precisely one of the main points behind Caselli and Coleman's study. Not only do wealthy nations enjoy large pools of skilled labor, they also relatively higher levels of skill-biased technology.

But from these static pictures it is not clear which technological path would produce more output for any particular country *over time*. On the one hand, a country with a relative abundance of unskilled labor should greatly benefit by making them more productive. On the other hand, unskilled labor's *level* of productivity may already be fairly low; unskilled-bias technical change that induces a rise in L and a fall in H would then lower the relatively-more productive factor and raise the relatively-less productive factor.

We begin exploring these issues by allowing the factors of production to *respond* to biased technological changes, first in a comparative static experiment in section 2.2, and then in a fully specified general equilibrium model in section 3. This is an important feature to consider, given the common observation that developing economies often have too many factors allocated to low productivity work.[10]

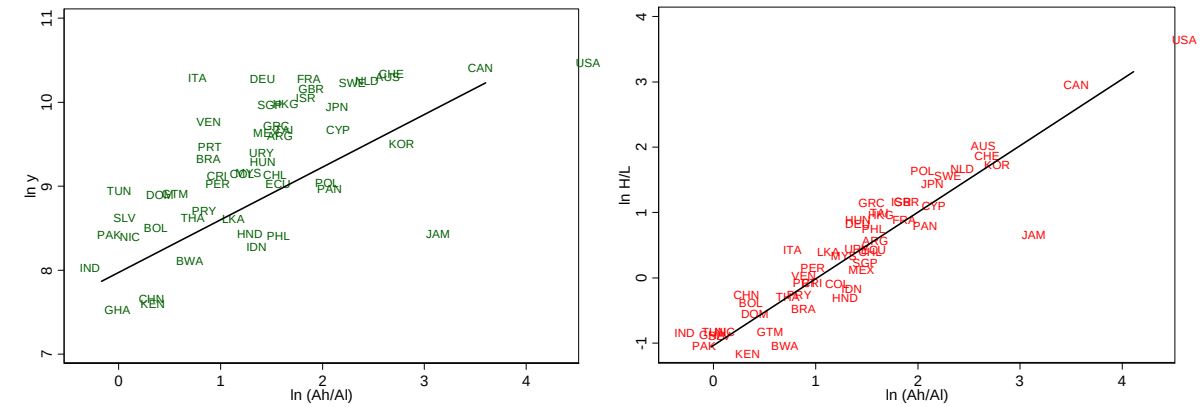
[8]The data is also from Caselli and Coleman (2006). Y is average GDP per capita for 1985-1990, taken from the Penn World Tables. Labor levels are constructed using the implied Mincerian coefficients from Bils and Klenow (2000). Wages for skilled and unskilled are constructed using Mincerian coefficients and the duration in years of the various schooling levels. See their paper for more details.

[9]Ciconne and Peri (2005) themselves estimate σ to be 0.5 when considering U.S. high school dropouts as unskilled labor and high school graduates as skilled labor (although their preferred measure is 0.33).

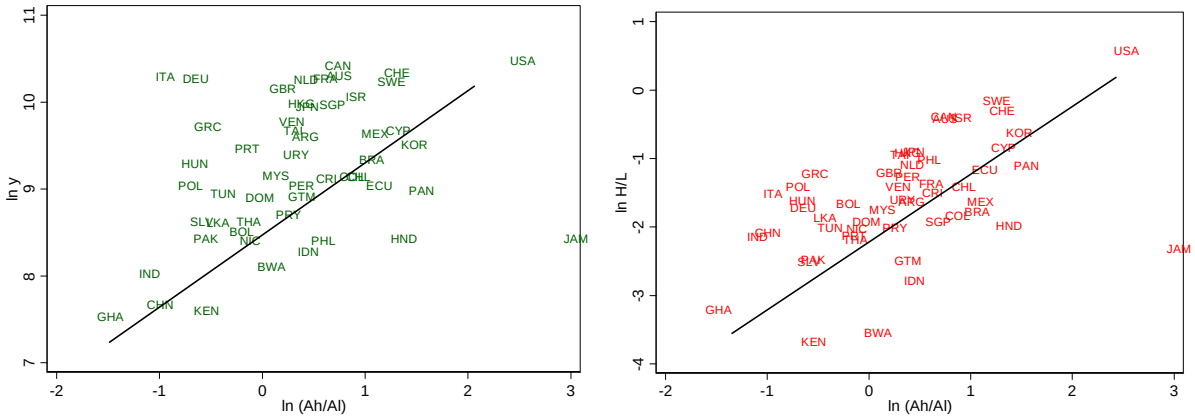
[10]See for example Graham and Temple (2003), Rustuccia (2004), Vollrath (2009), and Chanda and Dalgaard (forthcoming).

Figure 1: Relative Technologies versus Relative Factors and Output ($\sigma = 0.5$)

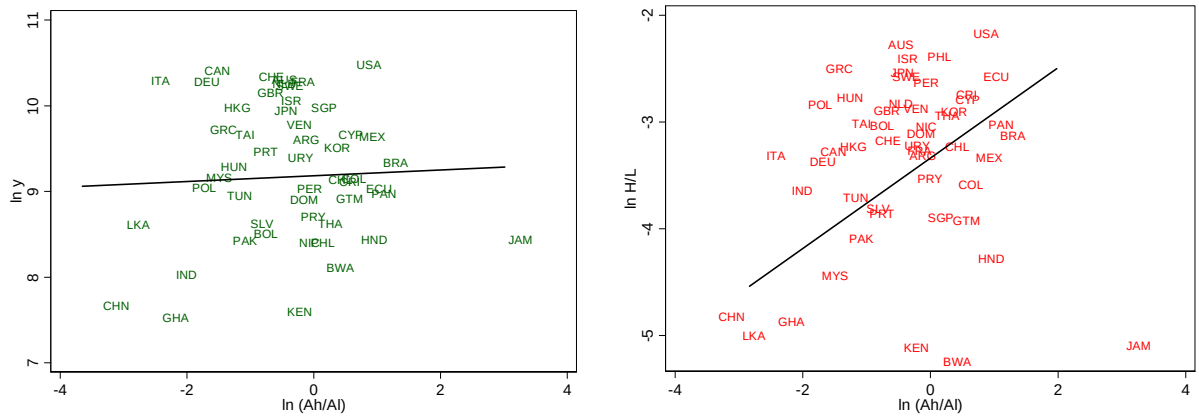
H = primary school or more completed



H = secondary school or more completed



H = college or more completed



2.2 Unbalanced Growth - A Comparative Static Experiment

Here we consider changes in total output, Y, that can occur when we have the factors of production respond to exogenous unbalanced technological growth. First, let us totally differentiate the production function given by (1):

$$dY = \left(\frac{\partial Y}{\partial A_l}\right) dA_l + \left(\frac{\partial Y}{\partial A_h}\right) dA_h + \left(\frac{\partial Y}{\partial L}\right) dL + \left(\frac{\partial Y}{\partial H}\right) dH \tag{3}$$

Both types of technologies and both types of factors have the potential to change. Let us assume that when technologies are *biased* towards factor L, it induces L to rise and H to fall (higher unskilled-intensive productivity makes some people become unskilled laborers instead of skilled ones). On the other hand, technological growth that is biased towards H induces L to fall and H to rise (higher skilled-intensive productivity makes some erstwhile unskilled laborers become skilled ones). That is, $dA_l > 0 \Rightarrow -dH = dL > 0$. And $dA_h > 0 \Rightarrow -dL = dH > 0$.[11] Let us consider two possibilities. The first is where $dA_l = 1$ and $dA_h = 0$. This is the case of unskilled-bias technological change (where the total change in output can be written as dY_{unsk}). The second case is where $dA_h = 1$ and $dA_l = 0$. This is the case of skilled-bias technological change (where the total change in output can be written as dY_{sk}).

When there is unskilled-biased technological change, the total change in income can be written as

$$dY_{unsk} =$$

$$\left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (L \cdot dA_l + A_l \cdot dL) + (A_h H)^{\sigma-1} (A_h \cdot (-dH))\right) \tag{4}$$

where dY_{unsk} is the total change in income with unskilled intensive growth. Note that here $dA_h = 0$ and the change to H is negative. On the other hand, when there is skilled-biased technological change, the total change in income can be written as

$$dY_{sk} =$$

$$\left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (A_l \cdot (-dL)) + (A_h H)^{\sigma-1} (A_h \cdot dA_h + (A_h dH))\right) \tag{5}$$

where dY_{sk} is the total change in income with skilled intensive growth. Note that here $dA_l = 0$ and the change to L is negative.

Does skilled labor-biased technological growth produce more output than unskilled labor-biased technological growth? If labor is strictly fixed, the answer is no. With Caselli and Coleman

[11] Note that only when $\sigma > 0$ can we consider A_l unskilled *biased* and A_h skilled *biased*. This is a reasonable assumption given previous estimates of σ. See Acemoglu 2002 for a fuller discussion.

(2006)'s categorization and calculations of L and H, $L > H$ even for wealthy nations. Since factors are grossly substitutable, technologies used by the more abundant factor will generate the greater aggregate gain.

However, the more responsive are factors to biased technological changes, the greater are the relative output gains from skill-biased technological change. This follows simply from the fact that H is inherently the more productive factor. This comes both from its relative scarcity (so its marginal productivity tends to be higher even if technologies are symmetrical) and from the higher productivity coefficient on H compared to the one for L. So if labor tends to readily switch from one type to the other with unbalanced technical progress, skill-intensive growth tends to produce more output.

Combining both observations, we see that each country has a *threshold level* of factoral responsiveness, whereby $dY_{sk} = dY_{unsk}$. Figure 2 plots $dY_{sk} - dY_{unsk}$ against the degree of factoral response for two illustrative countries, Argentina and Great Britain. If we consider H to be those with at least some secondary schooling, we can see that a one-unit change in A_h would require a 0.64 unit shift from L to H to produce more output than a similar change in A_l in Argentina, while it would require only a 0.35 unit shift from L to H in Britain.

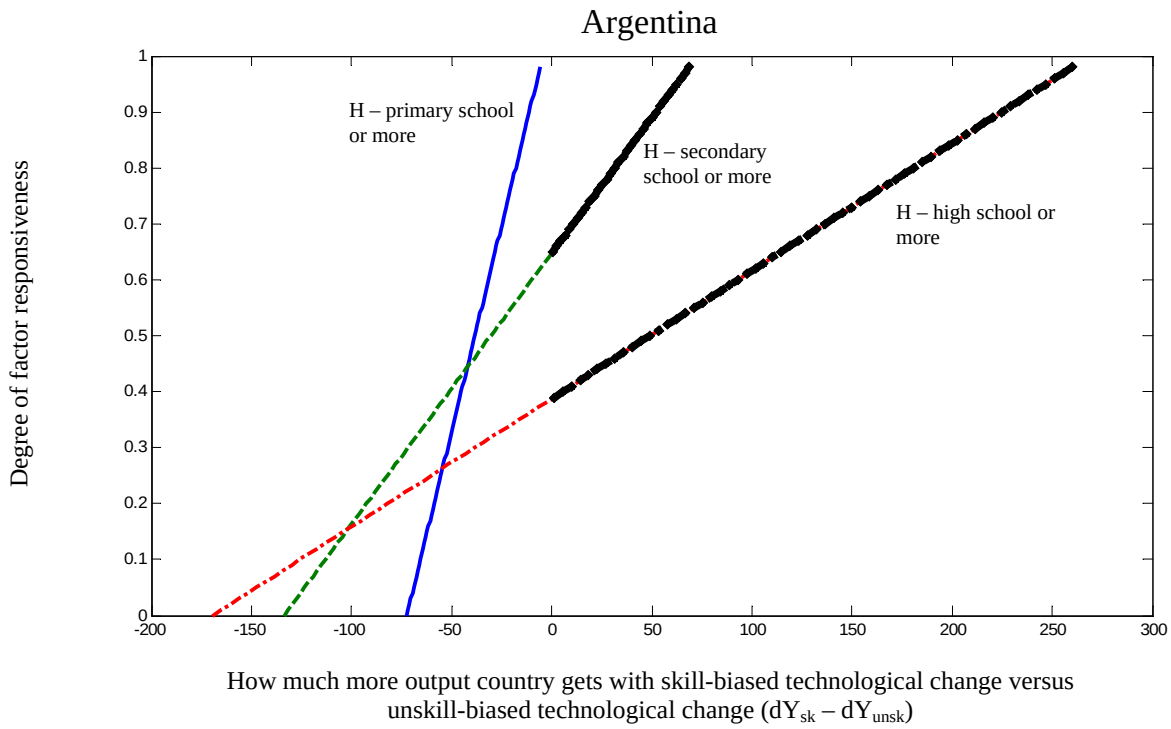
Thus we see that because countries have their own unique pairs of factor supplies and productivities, they will have different factoral response threshold levels. Figure 3 plots each country's threshold level of factor responsiveness (where $dY_{sk} = dY_{unsk}$) against its GDP per capita. We can see that the poorer the nation is on average, the greater will factors need to respond to technological changes for skill-intensive growth to be the superior path to development. Because poorer nations tend to have greater relative quantities of unskilled labor, and also tend to have relatively less productive skilled labor, factors need to respond with greater magnitude in order for skill-intensive technical growth to produce relatively more output.

However, as we compare the top and bottom scatterplots we can see that the more narrow is our definition of H, the *smaller* is the threshold factor responsiveness. This is simply because increases from the relatively more scarce factor produces greater benefits, for the marginal productivities of the more scarce factor tends to be larger. This in effect flips Acemoglu's discussion of so-called "market-size effects" on its head: if factors are allowed to respond to technological change, such change that augments the *less* abundant factor may produce *more* output in the longer run.

Yet to suggest from this partial equilibrium analysis that skilled-biased technological growth in the context of responsive factors generates faster growth would be premature. If we believe there exists a quality-quantity tradeoff in child-rearing (Becker and Lewis 1973; Becker and Barro 1988),[12] educational changes will also generate fertility changes. And education and fertility

[12]This is also stressed by Galor and Mountford (2006, 2008). In these papers population changes come from trade specialization patterns. Technological changes however can affect demographic patterns in very similar ways.

Figure 2: Skilled versus Unskilled Technological Growth When Factors Respond - Comparing Two Countries



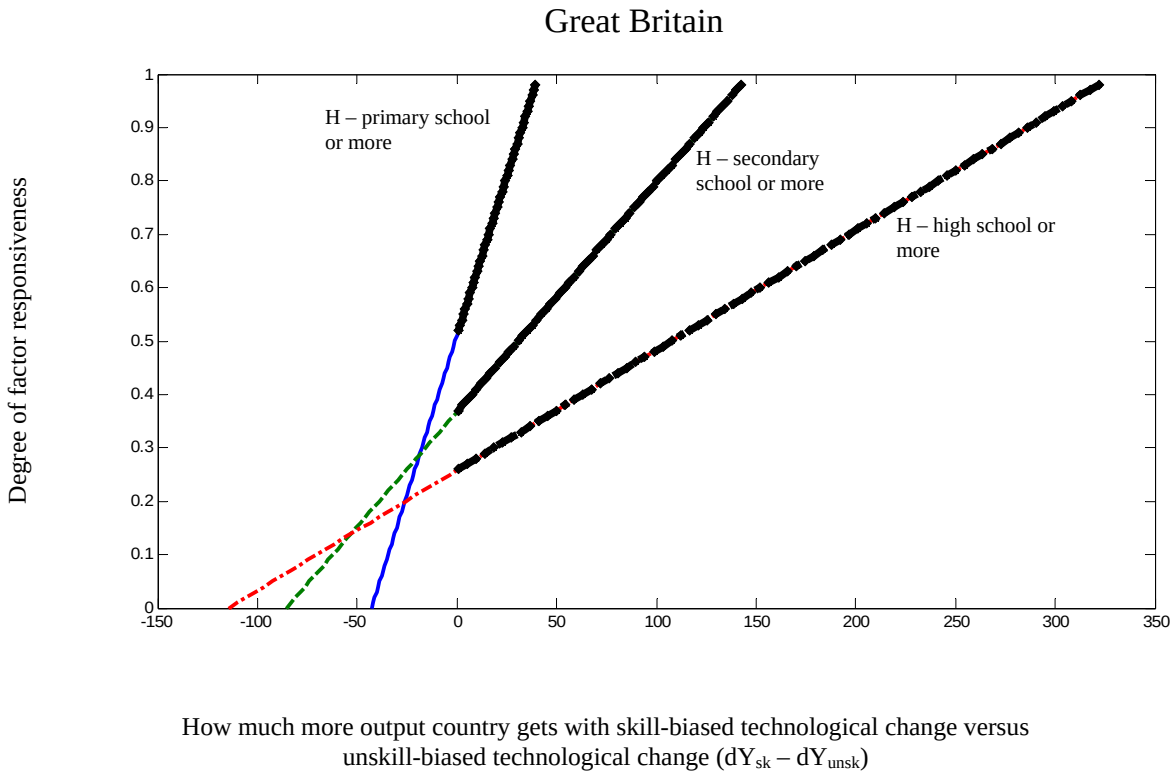
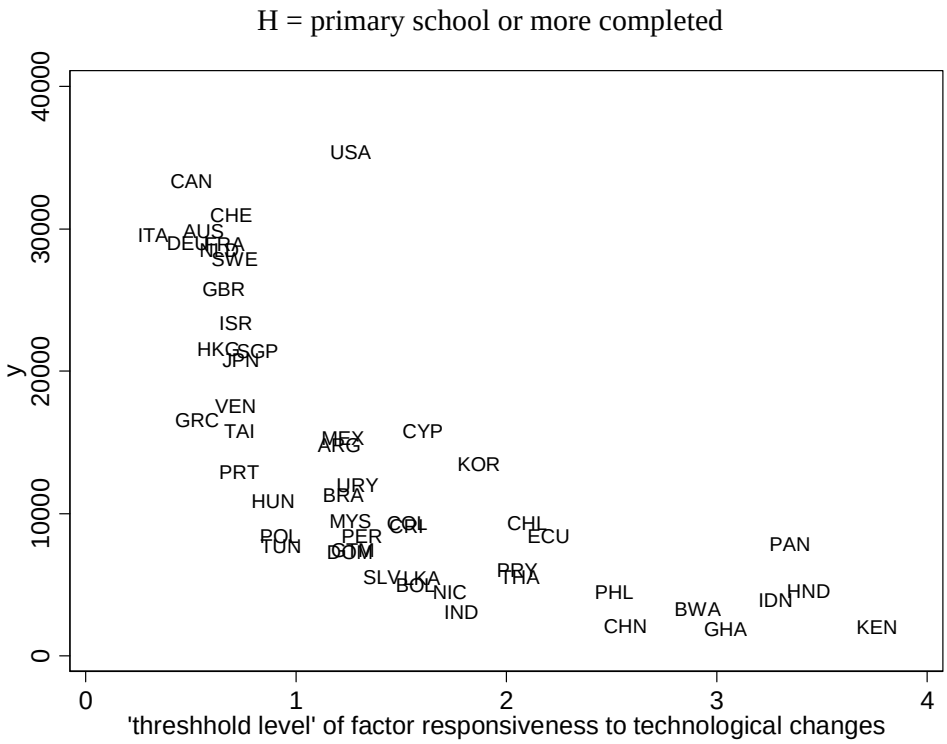
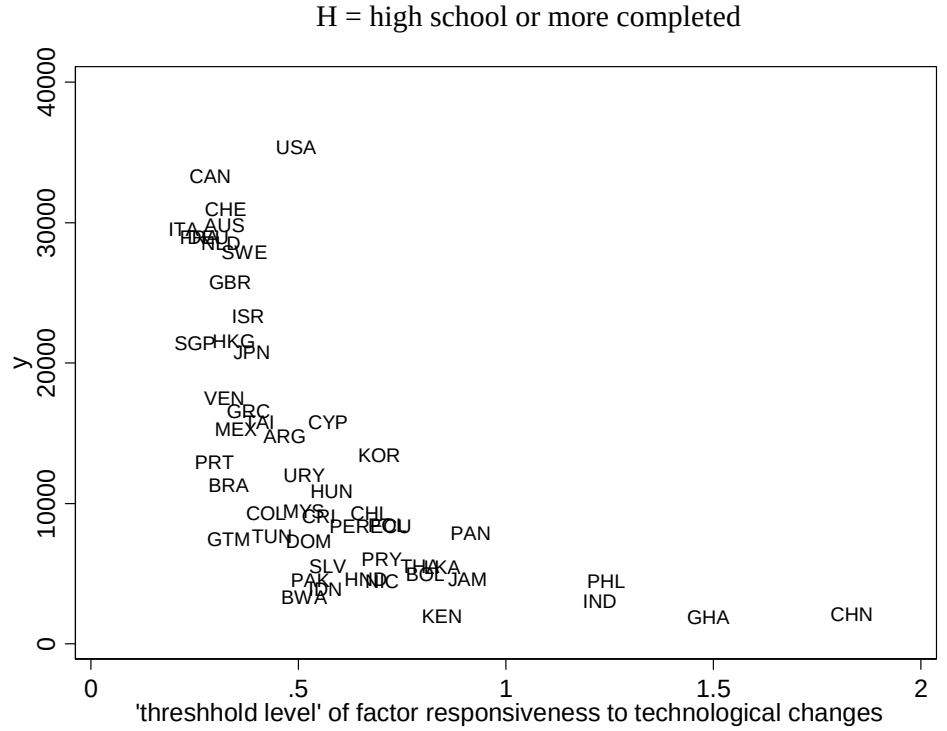


Figure 3: How Much Must Factors Respond To Make Skilled-Biased Technological Growth "Better?" - Cross Country Comparison





changes may themselves lead to subsequent changes in biased technologies. So we must move beyond this comparative static analysis to a model that endogenizes both factors and technologies in a general equilibrium framework. That is, by actually endogenizing the micro-economic incentives for researchers and families, we can simulate values for dA_l, dA_h, dL and dH for a hypothetical economy over time.

3 The Model

3.1 Production

Consider a discrete-time economy. We use the production function given by (1) but now we explicitly specify factor-specific technologies. Specifically production of aggregate output is specified as the following.

$$Y = [(A_l L)^\sigma + (A_h H)^\sigma]^{1/\sigma} \tag{6}$$

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L}\right)^\alpha dj \qquad A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H}\right)^\alpha dk \tag{7}$$

Here both types of labor (unskilled L, and skilled H) work with intermediate "machines" to produce a homogenous final output. A machine is designed for use either by skilled labor or unskilled labor, but not both. Machines (of type j) which complement unskilled labor are denoted by $x_l(j)$, while machines (of type k) which complement skilled labor are denoted by $x_h(k)$.

The parameter σ indicates the degree of substitutability between the skilled and unskilled-intensive "sectors" in aggregate production. As mentioned in section 2.1, estimates of this elasticity clearly place σ above zero; thus we will assume that these sectors are grossly substitutable.

Echoing the assumptions of Kiley (1999) and Acemoglu (2002), technological advance is assumed to come in two varieties. In the "unskilled labor sector," technical advance comes about from an expansion in the number of intermediate machines specialized for unskilled labor (that is, an increase in M_l). Similarly, in the "skilled labor sector," technical advance means an expansion in the number of intermediate machines specialized for skilled labor (an increase in M_h).

Final goods output produced by different firms is identical, and can be used for consumption, for the production of different intermediate machines, and for research and development to expand the varieties of skill-augmenting and unskilled-augmenting machines. For each time period (suppressing time subscripts) these firms endeavor to maximize:

$$\max_{\{L,H,x_l(j),x_h(k)\}} Y - w_l L - w_h H - \int_0^{M_l} p(j)x_l(j)dj - \int_0^{M_h} p(k)x_h(k)dk \tag{8}$$

where $p(j)$ is the price of machine $x_l(j)$ and $p(k)$ is the price of machine $x_h(k)$.

Endogenous growth theory suggests that research is generally profit motivated. However, modeling purposive research and development effort becomes difficult when prices and factors change over time, as they certainly do when growth is unbalanced. Endogenous growth theory typically assumes that the gains from innovation flow to the innovator throughout her lifetime, and this flow will depend on the price of the product being produced and the factors required for production at each moment in time.[13] If prices and factors are constantly changing (as they may in an economy where factors evolve endogenously), a calculation of the expected discounted profits from an invention may be impossibly complicated.

To avoid this complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last one time period only. More specifically, we assume that intermediate machines are produced either in monopolistic or competitive environments. An inventor of a new machine at time t enjoys monopoly profits for machine production only at t. After this patent rights expire, and subsequent production of this brand of machine is performed by many competitive manufacturers. Whether a machine is produced monopolistically or competitively will be conveyed in its rental price, denoted either as $p(j)$ for a unskilled-labor using machine j or $p(k)$ for a skilled-labor using machine k, and explained in the next sub-section. Also for simplicity, we assume that all machines depreciate completely after use, and that the marginal cost of production is simply unity in terms of the final good.

Given technology levels M_l and M_h and labor types L and H, an equilibrium can be characterized as machine demands for $x_l(j)$'s and $x_h(k)$'s that maximize final-good producers' profits (from equation 8), machine prices $p(j)$ and $p(k)$ that maximize machine producers' profits, and factor prices w_l and w_h that clear the labor market.

The first-order conditions for final-good producers yield intermediate-machine demands:

$$x_l(j) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(j)}{\alpha}\right)^{\frac{1}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_h(k) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(k)}{\alpha}\right)^{\frac{1}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{9}$$

Note that greater levels of employment of a factor raise the demand for intermediate goods augmenting that factor so long as $\sigma > \alpha$, an idea consistent with Acemoglu's so-called "market-size" effect. We will assume throughout the analysis that this condition is met.

The other first-order conditions for final-good producers illustrate that workers receive their marginal products:

[13] For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{10}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{11}$$

3.2 Research

In this section we describe the growth paths of M_l and M_h. Researchers expend resources (rather than time) to develop new types of machines, and these resource costs can change over time.[14] We make this modeling choice to stress that unbalanced growth can occur when research costs differ between different sectors. We will assume that these costs will depend both on the number of machine types already extant (indexed by M_l and M_h), and on some factor-specific technology variable (denoted by z_l and z_h, and discussed below). Specifically, the up-front cost of developing the blueprint of a new machine, c, is given simply by

$$c\left(\frac{M_l}{z_l}\right) = \frac{M_l}{z_l}$$

for an unskilled labor augmenting machine, and

$$c\left(\frac{M_h}{z_h}\right) = \frac{M_h}{z_h}$$

for a skilled labor augmenting machine. These functional forms illustrate that the costs of invention are negligible when there is little machine variety. As factor-specific technologies grow, however, costs can become increasingly prohibitive.[15]

Given these costs of technological advance, innovating firms must receive some profits from the development of a new technology in order to make research worth the expense. As mentioned above, we assume that developers of new machines receive monopoly rights to the production and sale of their machines for only one period. As a result, we must make a distinction between *old* machines (those invented before t) and *new* machines (those invented at t).

Assuming unitary marginal costs of machine production, the excess revenue generated from new machines of both types are given by the 'value' functions:

$$V_l = (p(j) - 1)\, x_l(j)$$

$$V_h = (p(k) - 1)\, x_h(k)$$

[14] This echoes Rivera-Batiz and Romer (1991)'s "lab-equipment" assumption for research.

[15] This approach of varying the cost of research echoes the leader-follower model illustrated in Barro and Xala-i-Martin 2003), where costs depend on the distance from the frontier of general knowledge.

Because demand is isoelastic, the price which maximizes monopolists' profits equals $1/\alpha$ for both skill- and unskilled-augmenting machines, so that demand for *new* intermediate machines (those invented at t) are:

$$x_{l,new}(j) = x_{l,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,new}(j) = x_{h,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{12}$$

On the other hand, because older machines are competitively produced, their prices equal unitary marginal costs, so that demand for *old* intermediate machines (those invented before t) are simply:

$$x_{l,old}(j) = x_{l,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,old}(j) = x_{h,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{13}$$

Thus factor-specific TFPs given by equation (6) can be re-written as an aggregation of two kinds of machines, illustrating the cumulation of all past and current innovation. If $M_{z,old}$, $M_{z,new}$, and M_z are, respectively, the number of existing old, new and total machine-types used by factor z, we can write factor productivity as:

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L}\right)^\alpha dj = \left[\int_0^{M_{l,old}} x_{l,old}(j)^\alpha \, dj + \int_{M_{l,old}}^{M_l} x_{l,new}(j)^\alpha \, dj\right] (1/L)^\alpha = \frac{M_{l,old}\, x_{l,old}^\alpha + M_{l,new}\, x_{l,new}^\alpha}{L^\alpha} \tag{14}$$

$$A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H}\right)^\alpha dk = \left[\int_0^{M_{h,old}} x_{h,old}(k)^\alpha \, dk + \int_{M_{h,old}}^{M_h} x_{h,new}(k)^\alpha \, dk\right] (1/H)^\alpha = \frac{M_{h,old}\, x_{h,old}^\alpha + M_{h,new}\, x_{h,new}^\alpha}{H^\alpha} \tag{15}$$

Substituting the monopoly price into our value functions yield:

$$V_l = \left[\frac{1-\alpha}{\alpha}\right] x_{l,new}$$

$$V_h = \left[\frac{1-\alpha}{\alpha}\right] x_{h,new}$$

where $x_{l,new}$ and $x_{h,new}$ are given by (12). Finally, an individual is free to research, guaranteeing that:

$$V_l\left(L, H, A_l, A_h\right) \leq c\left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{16}$$

$$V_h\left(L, H, A_l, A_h\right) \leq c\left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{17}$$

If resource costs of research were actually *less* than discounted profits, entry into research would occur, driving technology levels, and hence costs, up. We assume this happens quick enough so that valuations never exceed costs in any time period. Further, since applied research is irreversible (a society cannot forget how to make something once it is learned), the variety of machines remains unchanged when the inequalities in (16) or (17) do not bind with equality.

The levels of our technology variables z_l and z_h in the economy are key determinants of the costs of developing new "production processes;" higher levels of z_u lower the costs of developing intermediate machines which complement factor u. Conceivably the evolution of technological variables can be shaped by many things, such as factor endowments, government policies, trade patterns, institutional features, and technological diffusion from other countries. With this in mind, we will consider three cases:

$$Case\,1:\ Exogenous\,growth:\quad \frac{\Delta A_l}{A_l} = g \quad or \quad \frac{\Delta A_h}{A_h} = g$$

$$Case\,2:\ Semi-endogenous\,growth:\quad \frac{\Delta z_l}{z_l} = g \quad or \quad \frac{\Delta z_h}{z_h} = g$$

$$Case\,3:\ Endogenous\,growth:\quad \frac{\Delta z_l}{z_l} = \mu H^{\lambda} \quad or \quad \frac{\Delta z_h}{z_h} = \mu H^{\lambda}$$

where $0 < g < 1$, $\mu > 0$, and $\lambda > 0$. That is, we wish to compare the growth prospects of either path, looking at three alternative growth regimes.

A "steady-state" can be characterized as one where the share of labor devoted to each sector (skilled and unskilled) remains fixed, while output, the technology variables z_l and z_h, the varieties of skilled and unskilled machines, and wages all grow at the same rate, g (for Case 2). This will occur so long as equations (16) and (17) hold with strict equality. But as these inequalities imply there may be a considerable period of time when growth is *unbalanced*; this would occur if only one of the equations held with equality. What kind of unbalanced growth is likely to unfold will depend on a number of things, including the available supply of different factors (a relatively large L for example raises V_l and thus increases the chance that growth will be unskill-biased) and the relative "skewness" of the technology variables (a relatively large z_l for example lowers c_l and likewise increases the chance for unskill-biased growth).

No doubt unbalanced growth will be slower than balanced steady-state growth *ceteris paribus*, but it also seems logical that growth in the *bigger* sector (in this case, the unskilled sector) will produce faster growth than growth in the smaller sector (the skilled sector).[16] At the same time, there is wide recognition among development economists of the importance of skill accumulation in economic growth. The centrality of human capital in economic development is so established that most economists now treat education and modernity as going hand in hand.[17] From this perspective, a country's relative abundance in unskilled labor scarcely matters; the skill-intensive path is the *only* viable path to sustainable progress.

This paper suggests that forces that change the factors of production themselves are an important part of our answer to the question of which is the more appropriate growth path. Specifically, changes in the relative rewards to factors due to technological developments surely will alter the incentives to become educated or to remain an unskilled laborer. From the model we can write the "skill premium," the skilled wage relative to the unskilled wage, as

$$\frac{w_h}{w_l} = \left(\frac{\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}}{\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}} \right)^{\frac{\sigma-\sigma\alpha}{1-\sigma\alpha}} \cdot \left(\frac{H}{L} \right)^{\frac{\sigma-1}{1-\sigma\alpha}} \tag{18}$$

In the absence of any demographic response, skill-bias technological growth will raise the skill premium (by raising $M_{h,new}$), while unskill-bias technological growth will lower it (by raising $M_{l,new}$). But surely if unskill-intensive growth lowers the relative returns to skill, this will induce some people to remain unskilled. Conversely, increases in the returns to skills should induce individuals to increase human capital, and thus lower fertility rates through quality-quantity tradeoffs. Indeed, from the last section we suggest that the more responsive these factors are to changes in their relative returns, the more likely will skill-biased technological growth yield greater income per capita growth. But we have yet to analyze how such demographic responses can influence *subsequent* technological developments. These considerations compel us to merge this growth model with a simple theory of demography. The next sections do precisely that.

3.3 Endogenous Demography

To capture the symbiotic relationship between technologies and factors, we introduce households into the model in an over-lapping generations framework, where individuals have two stages of life: young and old. Only old people are allowed to make any decisions regarding demography. Specifically, the representative household is run by an adult who maximizes her utility by de-

[16] If $\frac{\Delta a}{a} = g$ and $\frac{\Delta b}{b} = 0$, $\frac{\Delta(a+b)}{a+b} = \frac{\Delta a}{a+b}$, which is smaller than, but converges to, g. The smaller is b relative to a, the closer will this growth be to g.

[17] By one recent study's account, "Anything that harms the accumulation of human capital harms our economic well-being" (Remler and Pema 2009). For a brief history of the study of human capital see Ehrlich and Murphy (2007).

ciding two things: how many children to have (denoted by n) and the fraction of these children that will receive an education (denoted by e).

An individual born at time t works either as an unskilled laborer (earning w_l) or as a skilled laborer (earning w_h). The individual becomes old at $t+1$. At this point she decides how many children to have herself, and the fraction of these children that will get an education and work as skilled workers.

Specifically, individuals wish to maximize both their own income and the income of their young.[18] Let utility for the household planner be described by the function

$$U = w_j\left(1 - c(n,e)\right) + ln\left[w_l(1-e)n + w_h en\right]$$

where $c(\cdot)$ is the function denoting child-rearing costs, and w_j is the wage of the parent (who could be either a skilled worker or an unskilled worker, depending on what *her* parent chose for her last time period, so $j = l, h$). Fraction $(1-e)$ of young work as unskilled workers, while fraction e of young work as skilled workers. This quasi-linear utility form simply conveys that adults face diminishing returns to enjoyment in their children's income, but not in their own.[19]

The first-order condition for the number of children is:

$$\frac{1}{n} = w_j c_n \tag{19}$$

where c_n is the derivative of the cost function with respect to fertility. The left-hand side illustrates the marginal benefit of an additional child (which falls with the total number of children), while the right-hand side denotes the marginal cost (the income foregone to raise an additional child).

The first order condition for education is:

$$\frac{w_h}{w_l(1-e) + w_h e} = \frac{w_l}{w_l(1-e) + w_h e} + w_j c_e \tag{20}$$

where c_e is the derivative of the cost function with respect to education. Again, the left-hand side is the marginal benefit and the right-hand side the marginal cost. At the optimum, the gains received from the added skilled income offsets the foregone unskilled- and adult-income requisite for giving more children an education.

Note that the results of this simple optimization problem are consistent with the negative correlations between income and fertility and between education and fertility that are observed in developing countries (Kremer and Chen 2002). For example, rising skilled wages induces

[18] This echoes Moav (2005), who models parents that decide both the number of children and the level of human capital of each child in order to simply maximize their potential income.

[19] Such quasi-linear utility functions to model demography have been used by, among other works, Kremer and Chen (2002) and Weisdorf (2007). Further, incorporating child-rearing costs strictly as an opportunity cost faced by the parent is standard in this literature (see Galor and Weil 2000).

households to increase education; the rise in child-rearing costs this produces however will also incentivise households to lower fertility.

Completing the model requires us to relate fertility and education rates to aggregate levels of unskilled labor and skilled labor. At time t, labor-types are given by:

$$L = N_t(1 - e_{t-1})\,(1 - c(n_t, e_t)) + N_t n_t (1 - e_t) \tag{21}$$

$$H = N_t e_{t-1}\,(1 - c(n_t, e_t)) + N_t n_t e_t \tag{22}$$

where N is simply the adult population. Note that each type of labor is comprised of both young and old workers, and that old workers spend only a fraction of their time in the workforce (spending the rest of their time raising children). Finally, population growth is given by

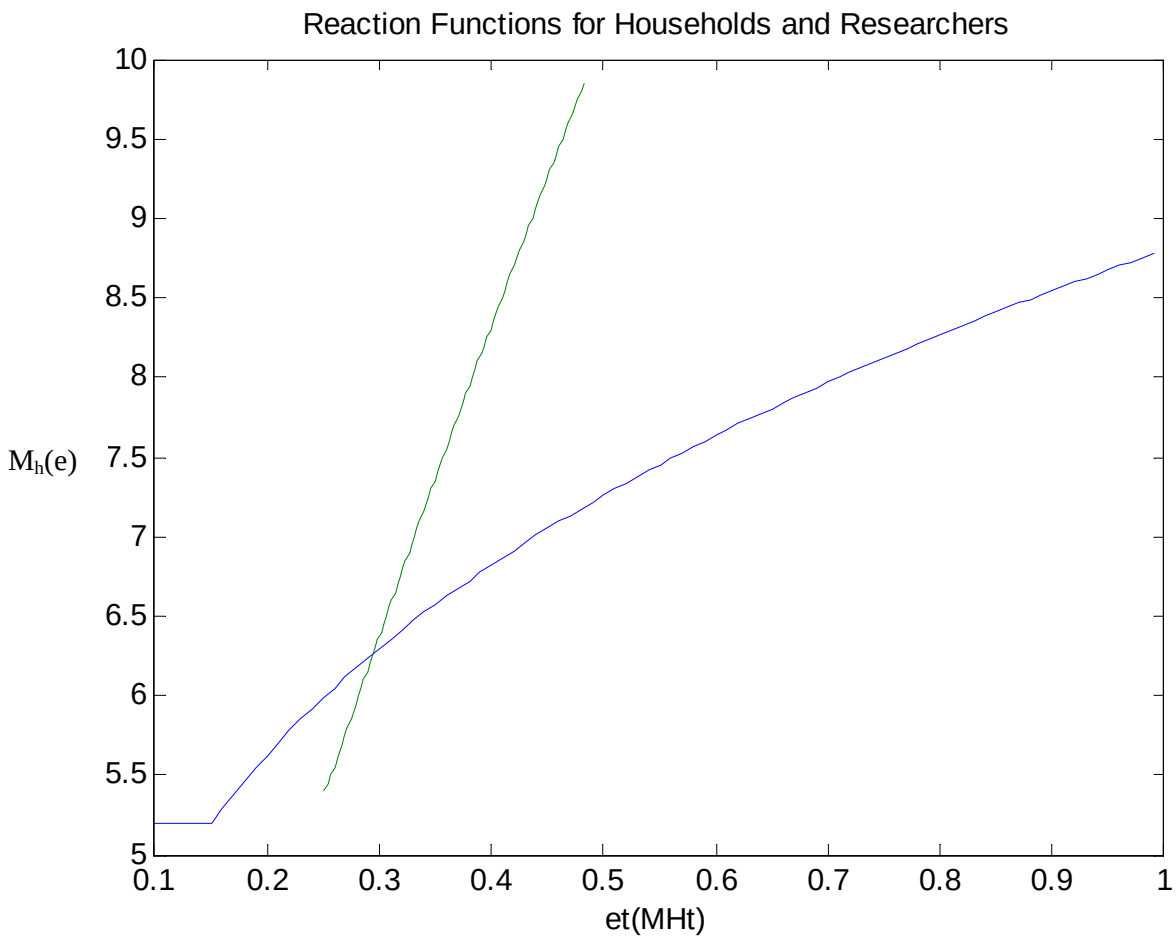
$$N_t = n_{t-1} N_{t-1} \tag{23}$$

Combining this model of demography with our model of biased technologies is straightforward. Through the simultaneous solving of (10), (11), (14), (15), (16), (17), (19), (20), (21), and (22), a unique set of variables w_l, w_h, A_l, A_h, M_l, M_h, n, e, L, and H can be determined for every time period.[20] We can perhaps synopsize our findings by initially focusing only on the economy's choice of e and M_h. If an adult expects researchers to develop new skill-biased technologies (and so to increase w_h), she will want to endow her children with more human capital. Similarly, if researchers anticipate a larger pool of human capital, they may wish to invent and build new skill-intensive machines, raising $M_{h,new}$ and thus M_h overall. Consequently we can plot the two "reaction functions" of each group as two upward-sloping curves; the development of new skill-using machines and the accumulation of skills are strategic complements. From the intersection of these reaction curves we find the unique simultaneous solution of the level of education and the new skill-biased technical coefficient. This is done in Figure 4. We can similarly plot two upward-sloping curves to determine an economy's choice of n and M_l.

To summarize, potential researchers look to the skill composition of the workforce (something influenced by households) to determine the direction and scope of technical change. Households look to wages (something influenced by researchers) to determine the levels of skilled and unskilled workers. Together they jointly determine the overall composition of the economy.

[20] This 10-by-10 system is reiterated with more detail in the Appendix.

Figure 4: "Reaction Curves"



- The steeper line represents the fraction of educated young a parent would choose for a given technological parameter M_h. The flatter curve represents the skill-biased technical coefficient that would result from a given fraction of educated young.

4 "Appropriate" Growth Paths for a Developing Country - Some Simulations[21]

With a model that endogenizes both technologies and factors, we may better assess the appropriateness of alternative development paths. Let us consider a hypothetical developing country endowed with a fairly sizeable amount of unskilled labor and a modest amount of skilled labor. We will set initial conditions such that $n > 1$ (to represent a growing population), and $e_{t-1} = e_t$ (to present a stable education rate). We can then test the effects of unbalanced growth on incomes per *capita* ($y_t = Y_t/N_t$) by allowing either only unskilled-labor technology or skilled-labor technology to rise, run the "horse-race," and compare the two paths. Each simulation is run for thirty time periods.

4.1 Case 1 - Simulation with Exogenous Unbalanced Growth[22]

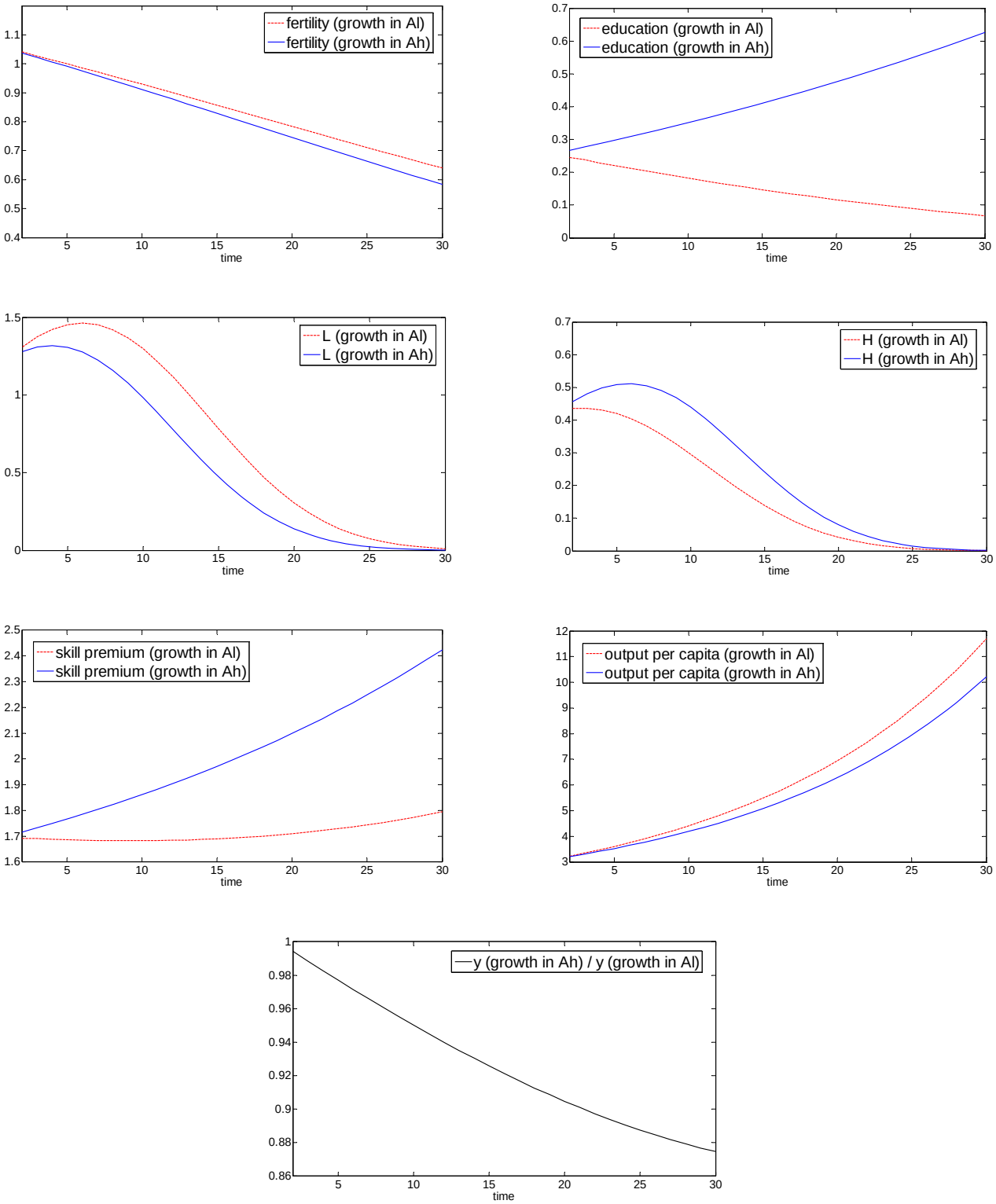
Our first horse-race is where we simply have either A_l or A_h grow exogenously, and compare the two paths. In other words, we ignore our discussion about endogenous technical growth in section 3.2 for the moment, and assume that unbalanced growth happens simply as some exogenous process, such as through technological diffusion from other countries (see the appendix for the system of equations being solved each time period). Specifically, each technological parameter grows 5% each period.

Figure 5 illustrates the results of these simulations. Red dotted lines are where only A_l grows; blue solid lines are where only A_h grows. Growth in both cases lowers fertility, since it raises the opportunity costs to raise children. However, it is clear that skilled-biased growth lowers fertility more dramatically, since it induces families to provide more of their offspring with education; this raises the costs of children even further. Unskilled-biased growth on the other hand puts downward pressure on skill premia, exerting upward pressure on fertility and downward pressure on education.

[21] Note that for the lessons of the simulations to hold, we require only that $0 < \alpha < \sigma < 1$. This simply means that factors of production must be substitutable "enough." Specifically we assume that $\alpha = 0.33$ and $\sigma = 0.5$. We also specify the simple cost function $c(n, e) = \gamma\left(n^2 + (ne)^2\right)$. γ is set to ensure that costs rise in both n and e, and that $c(n, e)$ remains bounded between 0 and 1. Results are not sensitive to the precise form of c, so long as costs are convex in n and e.

[22] We wish to establish plausible initial conditions for our hypothetical economy. Figure 1 strongly suggests that countries with large relative endowments of unskilled labor tend also to have low relative levels of skill-biased technologies. The high skill-premia we observe in developing countries is therefore mostly attributable to low relative endowments of skilled labor and not due to high levels of skill-biased technologies. With this in mind, we set initial values of A_l and A_h to 1 and normalize N to 1. For this and the next two cases, we set $\gamma = 0.5$; this gives an initial equilibrium where $n \approx 1.05$ and $e \approx 0.25$, producing initial factor endowments of $L = 1.3$ and $H = 0.45$, and initial wages of $w_l = 1.59$ and $w_h = 2.7$.

Figure 5: Simulation of Economy with Exogenous Unbalanced Growth



We can see how these demographic shifts affect the factors of production. Initial fertility rates above one induce increases in both labor types; once fertility falls below one, both labor types begin to fall. It is also clear that unbalanced growth creates changes in *relative* factors. We can see that H/L falls with unskilled-bias growth, and rises with skilled-bias growth. Recall from our discussion in section 2.2 that the latter means there is relative growth in the more productive factor (H is more productive even though we start with $A_l = A_h$ because it is more scarce than L), and this should be a boost to overall income. On top of this, the overall population grows faster with unskilled-bias growth than with skilled-bias growth.

And yet despite these apparent benefits of skill-bias growth, over time it actually generates *less* income per person than unskilled-biased growth. The final two graphs compare simulated per capita GDPs ($y = Y/N$) for the two paths. Why does skill-biased growth under-perform? Consistent with our earlier discussions, skill-biased technologies are not "appropriate" in the sense that they augment a relatively smaller workforce, so unskilled-bias technical growth generates relatively more overall growth. The roughly 3 to 1 ratio of unskilled to skilled workers assumed as the initial condition here is a conservative one; larger ratios would generate even greater divergence between the two growth paths.[23] Thus for reasonable skilled and unskilled labor endowments, unskilled-bias growth appears the more appropriate, despite the ostensibly negative effects of higher fertility and lower education. This result is also robust to different starting values of fertility and education (see the Appendix, where initial fertility is higher).

Note that with our quasi-linear utility specification, economic growth serves as a form of birth control - increases in income, whether from unskilled or skilled-biased technologies, induce parents to have fewer kids. This raises a question - what kinds of feedback effects will falling fertility have on *endogenous* technological growth? Do our conclusions change?

4.2 Case 2 - Simulation with Semi-endogenous Unbalanced Growth[24]

In this case we use the full system of equations that jointly solve for technological levels and for demographic variables. Technologies in this case are "semi-endogenous," in that we have research costs exogenously fall in order to observe the endogenous technological and demographic responses (see the appendix for the full system of equations). Specifically, for unskilled-intensive growth, we set z_l such that $V_l = c_l$ at the start of the simulation. Then we simply have z_l grow 5% each period, inducing research that produces new varieties of unskilled labor-using machines. For skill-intensive growth we do the same to z_h, V_h and c_h.

[23] For example, if we use the Caselli and Coleman (2006) country estimates for skilled workers (defined as those with secondary schooling or more) and for unskilled workers (those with anything less than secondary schooling), the world average over all countries is roughly 4 to 1, with obviously much higher ratios for developing countries.

[24] For this case initial levels of machines are set to $M_l = 0.87$ and $M_h = 0.75$. This gives us initial values of A_l and A_h of 1, thus giving us the same initial fertility, education, wages and factors as in Case 1.

Figure 6: Simulation of Economy with Semi-endogenous Unbalanced Growth

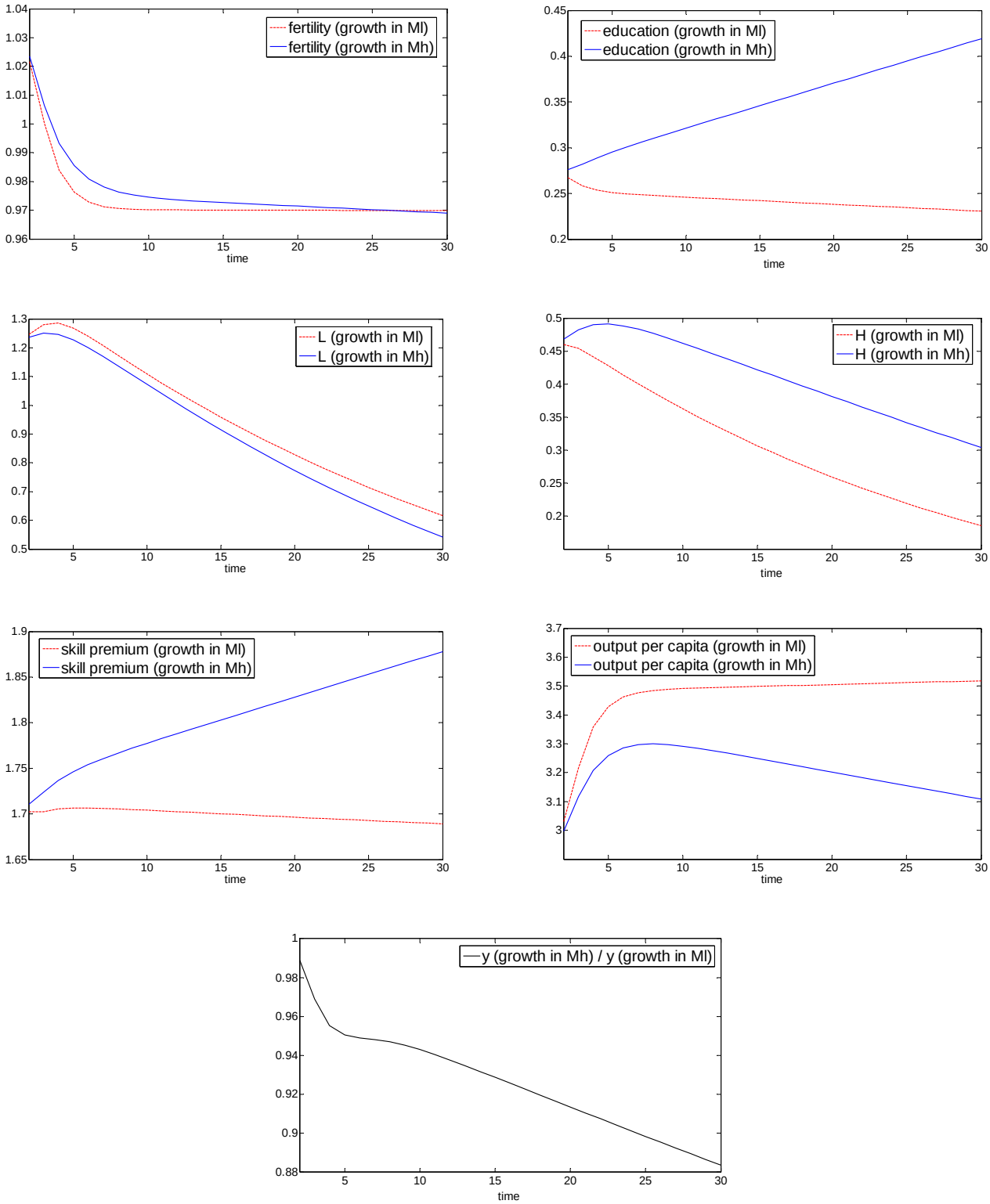


Figure 6 illustrates the results of these two simulations. In both cases unbalanced technological growth generates a robust demographic transition, lowering fertility and raising living standards. But soon thereafter growth in y slows dramatically; further, skill-bias growth actually lowers income per capita, while unskilled-bias growth continues to raise it, albeit slightly. Thus it seems that in this case the skill-biased path is *very* inappropriate. This may be surprising, because like Case 1 unskilled-intensive growth lowers H/L, and H is the more productive factor due to its relative scarcity. And these results once again buck conventional wisdom in development economics, which suggests that fertility declines should bolster per capita income, and that declines in education can be destructive for long-term prosperity.

In this case however, the fall in population due to the demographic transition contributes to subsequent technological stagnation in two ways. First, it induces final-goods producers to demand and use less *existing* machines, making workers less productive (eqn 9). Second, it shrinks the scale of the market for *new* innovation (eqns 16 and 17). These perverse effects on factor productivities offset the exogenous decreases in research costs, creating economic stagnation. Because unskilled-bias growth puts some upward pressure on fertility (after it lowers it significantly due to large productivity gains early on), the damage to technological progress is less severe.

A lesson here is that fertility declines, while inevitable in the process of economic development, can hurt subsequent growth when technologies are endogenous. Scale matters in this case, as indeed it does in nearly all semi-endogenous or endogenous growth models, since researchers require a large group of workers to purchase and use their new machines in order to recoup their fixed costs.[25] Because unskilled-labor biased growth limits the decline in fertility, it limits such negative effects to market size.

4.3 Case 3 - Simulation with Endogenous Unbalanced Growth[26]

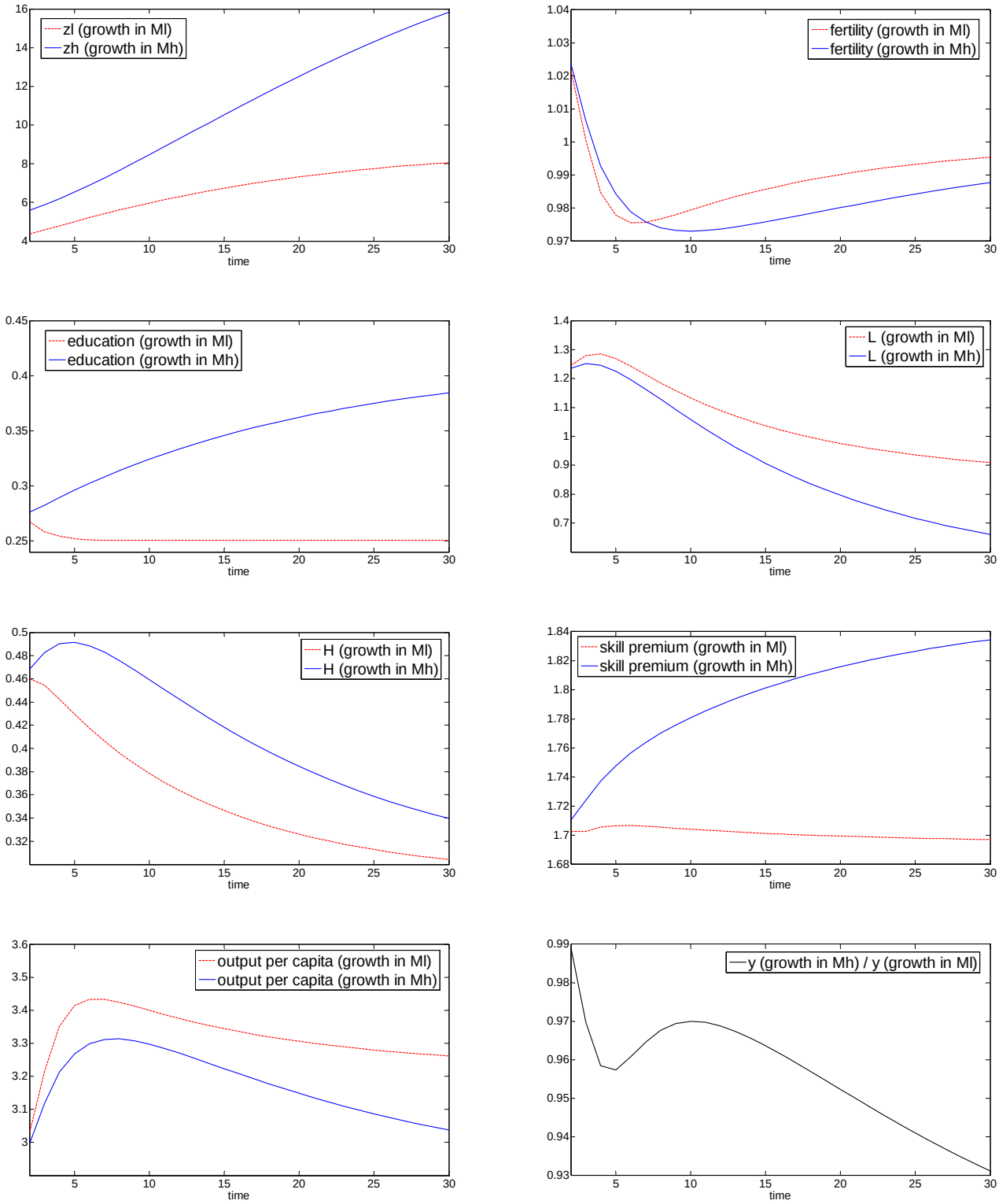
While the above case implies that unskilled-intensive growth produces more per capita income in the long-run than the alternative, we should acknowledge that this case is based on the assumption that growth in z_l or z_h is exogenous. If in fact such variables can rise only through a skilled workforce (for example, through a higher capacity to innovate [Nelson and Phelps 1966], or through human capital externalities [Lucas 1988]), the de-skilling effects of unskilled-intensive growth may hinder overall growth. That is, perhaps our previous cases under-emphasize the importance of the skill-intensive path?

Our final case explores this by assuming that growth in z_l or z_h is given by the endogenous

[25] More specifically, in such seminal endogenous growth models as Romer (1986, 1990), Segerstrom, Anant and Dinopolous (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991), a smaller labor force implies lower growth of technology. In "semi-endogenous" growth models such as Jones (1995), Young (1998), and Howitt (1999), a smaller labor force implies a lower level of technology.

[26] Initial values are the same as in Case 2. $\lambda = 0.1$, and μ is set so that growth in z_l or z_h starts at 5% per time period.

Figure 7: Simulation of Economy with Endogenous Unbalanced Growth



process suggested in section 3.2, where human capital generates beneficial spillovers to knowledge creation. Again, we allow for growth only in z_l or z_h, and compare trends created by each in Figure 7. Perhaps surprisingly, after the initial burst of economic growth from technological progress and demographic transition, per capita GDPs fall in both cases! Now the demographic transition adversely affects growth through two channels. The first is the same as before - factor productivities stagnate or outright shrink due to falling population. The second is a slowdown in growth in z_l and z_h due to declines in H. For a certain interval of time, skilled-biased growth generates faster overall growth than unskilled-biased growth; increases in z_h spur increases in education, and this creates a virtuous cycle of more human capital and technological progress. But through the quality-quantity tradeoff, it also generates a dramatic drop in fertility. Ultimately the declines in fertility outweigh the increases in education, such that overall human capital falls. Because this happens more dramatically with skilled-biased growth, income prospects deteriorate faster in this case.

At this point one could perhaps suggest that human capital externalities simply need to be sufficiently strong in order for the skilled-biased path to generate more growth (that is, have a larger μ and/or λ). But the dynamics remain the same (results not shown). A larger λ for example generates more income growth over the skill-intensive path early on, but also produces more dramatic decreases in fertility and thus *worse* growth prospects later on.

Cases 2 and 3 suggest an educational version of the "paradox of thrift." As skilled-biased technological growth incentivizes individual households to raise education levels, aggregate levels of human capital can outright shrink. The classic paradox suggests that individuals who raise their rates of savings can end up depressing aggregate demand and therefore lower savings and output (Keynes 1935). Similarly, individuals here can raise their rates of education only by lowering their rates of fertility; this ends up depressing technological growth and therefore lowers aggregate skilled labor and output.

5 Conclusion

This paper models the simultaneity of factors and technologies to evaluate different growth paths. Unlike approaches that credit either technological progress (Christensen and Cummings 1981) or factor accumulation (Young 1995) alone for economic success, the interaction of both can lend us new insights on which development path will breed the greatest rewards.

We see that the answer depends on the structure of the macro economy. Generally, a skill-intensive path will generate more benefits the more plentiful skilled labor already is. It also produces more benefits the more *responsive* are factors to technological changes, *provided* there is no or limited feedback from these changes on technologies. This is because the falling population growth caused by skill-intensive growth, normally a boom to income per capita, would hurt

economic growth if technologies are endogenous. Thus skill-biased technological diffusion, of the kind generated by the world-wide pervasiveness of skill-intensive technologies (Berman and Machin 2000; Berman, Bound and Machin 1998) can generate robust growth *because* of its exogenous nature, somewhat offsetting its apparent inappropriateness due to a low endowment of H.

Yet in general, unskilled intensive growth appears to be the best path for developing counties. Not only does it militate against factor income inequality (a focus of previous directed technical change papers), but it actually produces greater *aggregate* wealth for a variety of growth scenarios.

The bottom line is that the proper path to macro prosperity depends on lots of things - here we provide only the broadest brush-strokes delineating some major concerns. This subject however is relevant to all developing nations. Should India focus more on labor-heavy manufacturing or skill-heavy services? Should China's fiscal stimulus channel resources to build infrastructure using skill-intensive or labor-intensive techniques? Questions such as these dominate discussions over macro economic strategy in these countries; the answers will depend on some of the issues raised here.

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Appendix A: Simulations

First, for all simulations, we assume that the parent's wage, w_j, is simply an average of unskilled and skilled wages (specifically, we assume it is $w_l^{0.5} w_h^{0.5}$). This simplifying assumption allows us to treat all parents as the same. Alternatively, we could have skilled parents earn w_h and unskilled parents earn w_l. This would complicate the analysis without altering any of the qualitative results.

If technologies are exogenously determined, the simulation pre-determines A_l and A_h, and solves the following system of equations for w_l, w_h, n_t, e_t, L, and H for each time period t, given n_{t-1}, e_{t-1}, and N_t.

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{24}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{25}$$

$$\frac{1}{n_t} = 2\gamma w_j \left(n_t + n_t e_t^2\right) \tag{26}$$

$$\frac{w_h}{w_l(1-e_t) + w_h e_t} = \frac{w_l}{w_l(1-e_t) + w_h e_t} + 2\gamma w_j n_t^2 e_t \tag{27}$$

$$L = N_t(1-e_{t-1})\left(1 - \left(\gamma\left(n_t^2 + (n_t e_t)^2\right)\right)\right) + N_t n_t (1-e_t) \tag{28}$$

$$H = N_t e_{t-1}\left(1 - \left(\gamma\left(n_t^2 + (n_t e_t)^2\right)\right)\right) + N_t n_t e_t \tag{29}$$

If on the other hand technologies are semi-endogenously or endogenously determined by the process discussed in section 3.2, the following equations are solved for $M_{l,new}$, $M_{h,new}$, A_l, A_h, w_l, w_h, n_t, e_t, L, and H for each time period t, given n_{t-1}, e_{t-1}, N_t, z_l and z_h.

$$\left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{30}$$

$$\left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{31}$$

$$A_l = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}\right] \left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_l^{\frac{(\sigma-1)\alpha}{1-\alpha}} L^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{32}$$

$$A_h = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}\right] \left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_h^{\frac{(\sigma-1)\alpha}{1-\alpha}} H^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{33}$$

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{34}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{35}$$

$$\frac{1}{n_t} = 2\gamma w_j \left(n_t + n_t e_t^2\right) \tag{36}$$

$$\frac{w_h}{w_l(1-e_t) + w_h e_t} = \frac{w_l}{w_l(1-e_t) + w_h e_t} + 2\gamma w_j n_t^2 e_t \tag{37}$$

$$L = N_t(1-e_{t-1})\left(1 - \left(\gamma\left(n_t^2 + (n_t e_t)^2\right)\right)\right) + N_t n_t (1-e_t) \tag{38}$$

$$H = N_t e_{t-1}\left(1 - \left(\gamma\left(n_t^2 + (n_t e_t)^2\right)\right)\right) + N_t n_t e_t \tag{39}$$

(30) and (31) illustrate the benefits and costs of innovation; (32) and (33) are factor-specific TFP levels as functions of the demand for old and new machines and factors of production; (34) and (35) are wages; (36) and (37) are the benefits and costs of having children and educating them; (38) and (39) describe how fertility and education choices translate into aggregate factors of production. Note that if either of the first two equations holds with strict inequality, the algorithm sets the value of M_{new} to zero and simply solves the the rest of the system.

Appendix B: Simulations with higher initial fertility rates

Finally we demonstrate simulation results when fertility begins at a higher level. By setting $\gamma = 0.3$, we obtain $n_1 \approx 1.35$. Re-running all simulations with this condition does not alter our general story. Indeed, we can raise fertility by as much as possible - similar dynamics emerge. Thus, high-fertility and low-fertility countries face similar divergent growth path prospects.

Figure 8: Simulation of Economy with Exogenous Unbalanced Growth - with higher initial fertility

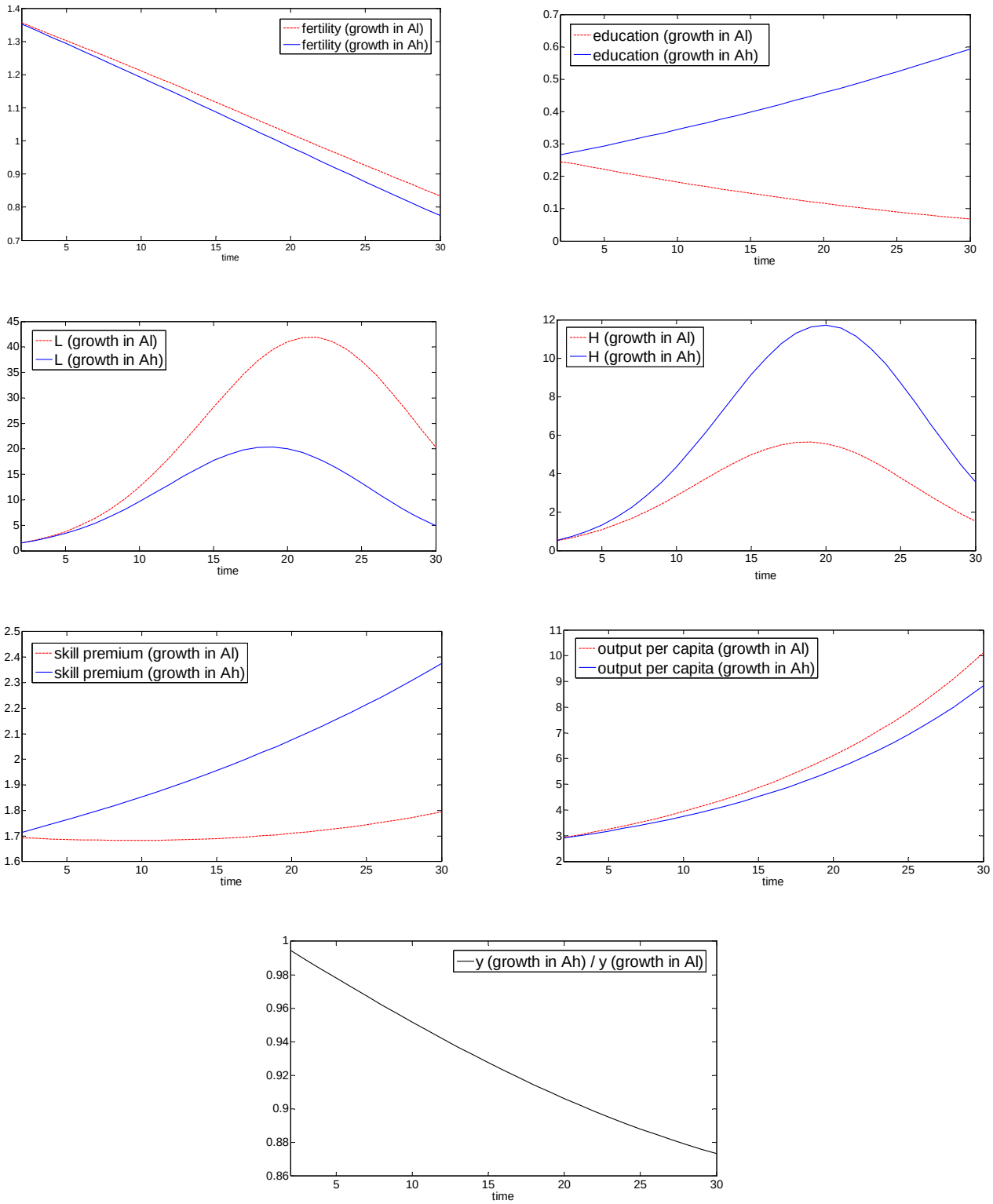

Figure 9: Simulation of Economy with Semi-endogenous Unbalanced Growth - with higher initial fertility

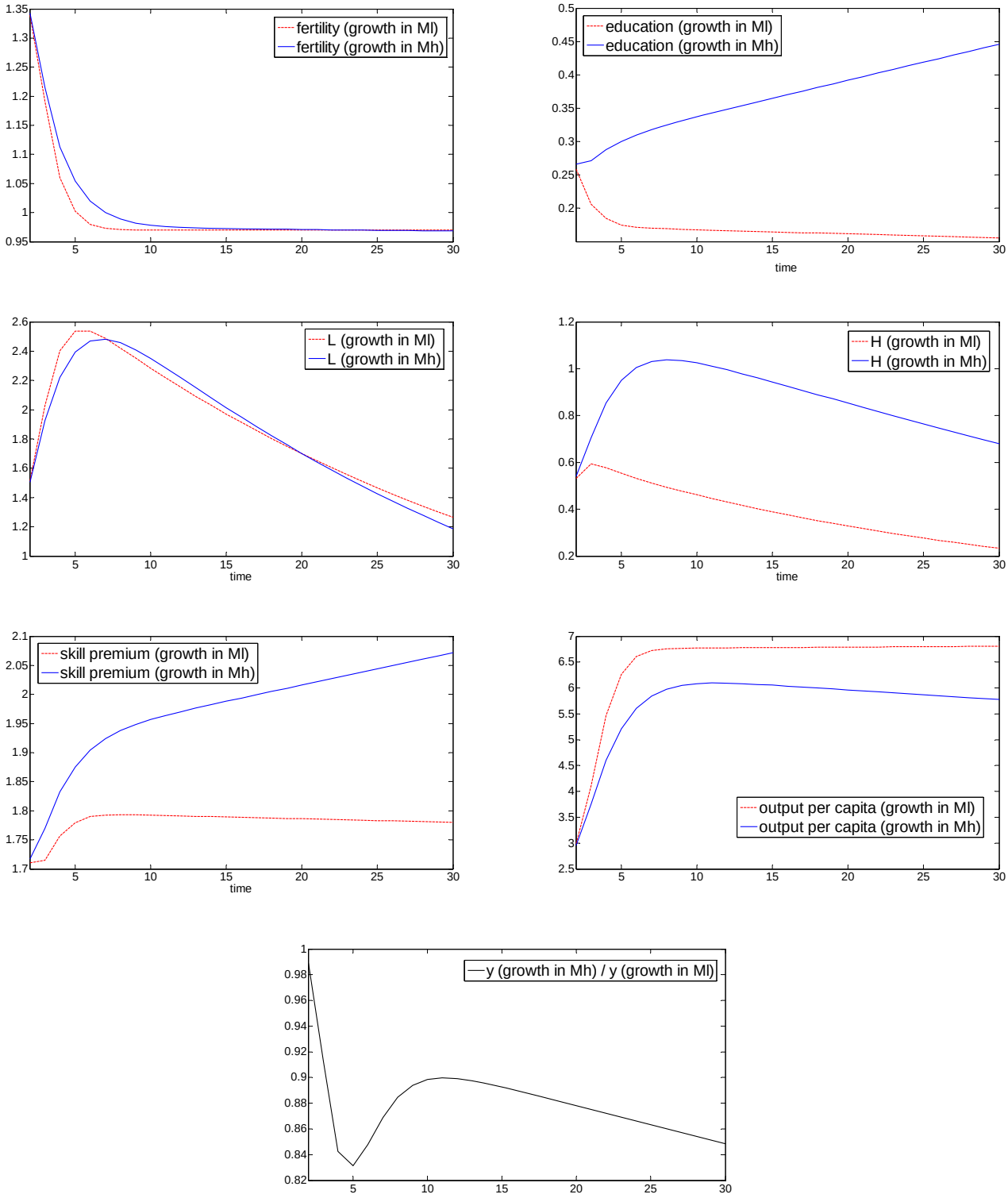
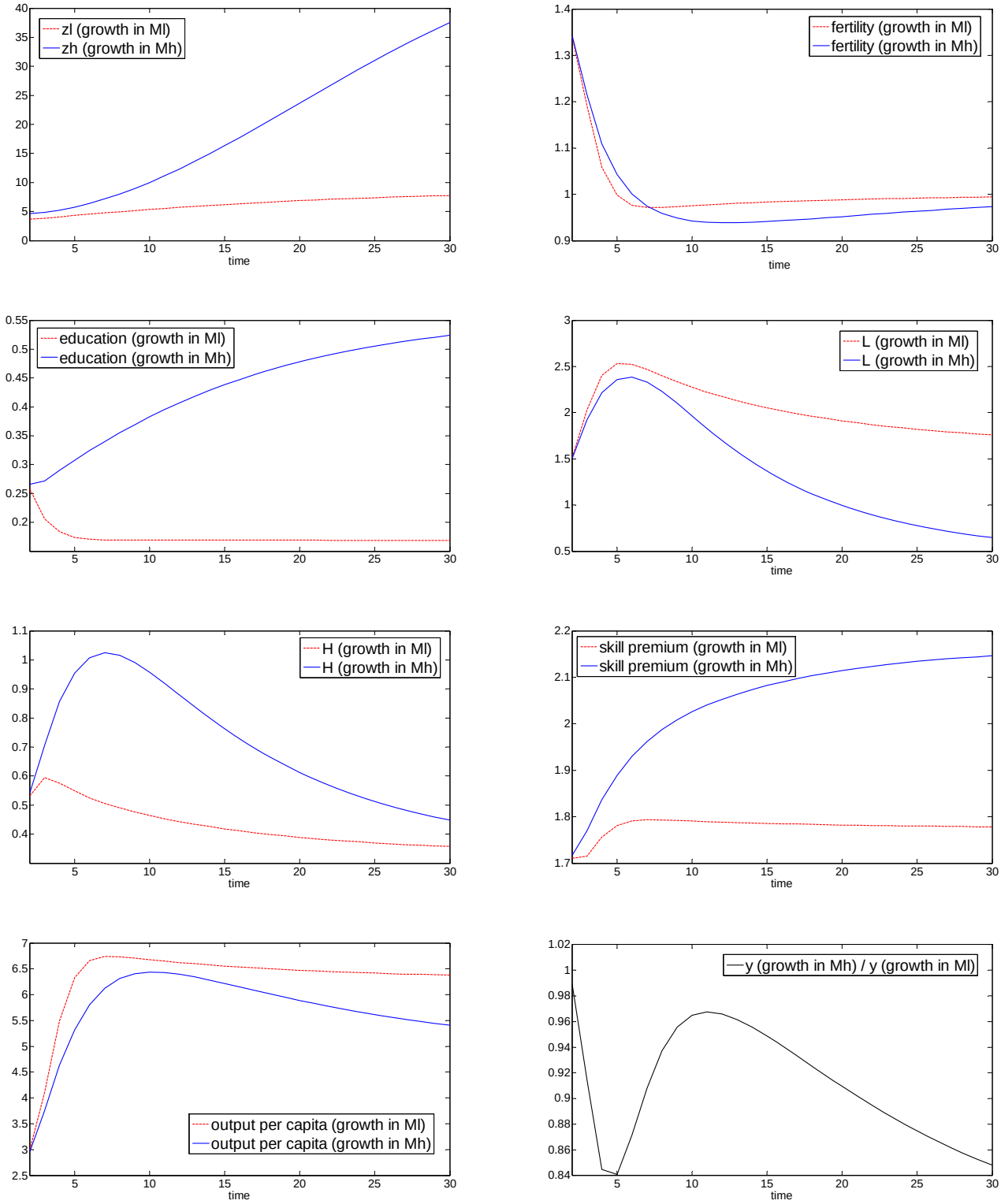
Figure 10: Simulation of Economy with Endogenous Unbalanced Growth - with higher initial fertility