Luddites and the Demographic Transition*

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Abstract

Technological change was unskilled-labor-biased during the early Industrial Revolution of the late eighteenth and early nineteenth centuries, but is skill-biased today. This fact is not embedded in extant unified growth models. We develop a model of the transition to sustained economic growth which can endogenously account for both these facts, by allowing the factor bias of technological innovations to reflect the profit-maximising decisions of innovators. Endowments dictated that the initial stages of the Industrial Revolution be unskilled-labor biased. Growth in "Baconian knowledge" allowed both for the takeoff of the Industrial Revolution and the transition to skill-biased technological change. Simulations show that the model does a good job of tracking British industrialization during the 18th and 19th centuries. In particular, we generate a demographic transition without relying on either rising skill premia or exogenous educational supply shocks.

Keywords: endogenous growth, demography, unified growth theory.
JEL Codes: O31, O33, J13, J24, N10.

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On March 11, 1811, several hundred framework knitters gathered in the Nottingham marketplace, not far from Sherwood Forest, to protest their working conditions. Having been dispersed by the constabulary and a troop of Dragoons, they reassembled that evening in nearby Arnold, and broke some 60 stocking frames. On November 10 of the same year, another Arnold mob gathered in Bulwell Forest, under the command of someone styling himself "Ned Lud," and the rapidly growing Luddite movement would suffer its first fatality that night when John Westley was shot dead during an attack on the premises of Edward Hollingsworth, a local hosier.

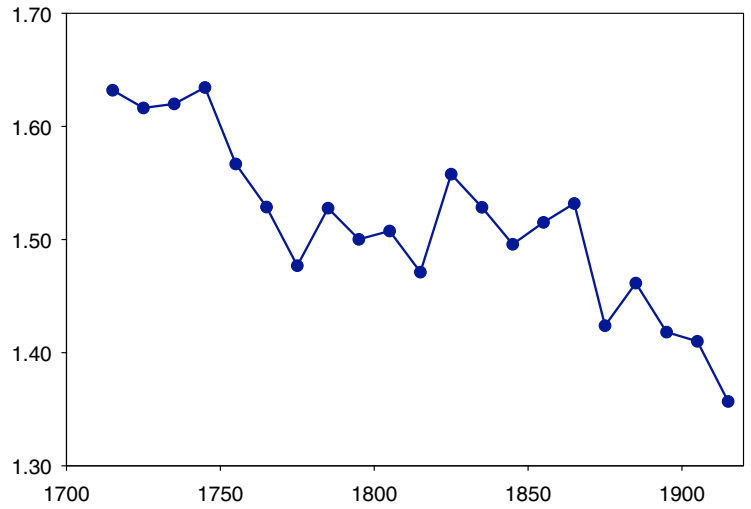
Today, the term Luddite often refers to opponents of technological progress for its own sake. At the time, however, Captain Ludd's followers were engaged in what Hobsbawm (1952, p. 59) has termed "collective bargaining by riot." "In none of these cases ... was there any question of hostility to machines as such. Wrecking was simply a technique of trade unionism" (ibid.) on the part of skilled textile workers whose living standards were being eroded by new machinery. This new machinery was making it possible for employers not just to produce cloth more efficiently, but to use cheaper unskilled workers, women, and even children, in the place of highly paid artisans. Technological change during the early Industrial Revolution hurt skilled workers, and as we can see from Figure 1, skill premia fell (Clark 2007; Katz and Autor 1999). Not surprisingly, skilled workers objected to this.

The emergence of Luddism occurred during what Galor and Weil (2000) have termed the "post-Malthusian regime." During this phase of British economic history, technological change was enabling the economy to slowly escape the Malthusian trap. However, living standards only rose slowly during this period, as population grew at an accelerating rate. But by the late 19th century, technological change was accelerating *and* living standards were growing more rapidly (Figure 2). Much of this acceleration was due to a dramatic and well-documented demographic transition, where fertility rates fell and educational standards rose (Figure 3). Many new technologies were now beginning to emerge which were skill-using rather than skill-saving, for example in modern chemical and metallurgical industries. But as Figure 1 shows, such a shift did not coincide with rising skill premia, since if anything they continued to fall.

All these pieces of historical evidence beg a unified explanation (Voth 2003). To theorists used to considering household fertility choices within a quantity-quality trade-off framework (Becker and Lewis 1973; Becker and Tomes 1976), the fact that the demographic transition, and the switch to what Galor and Weil call "modern economic growth," occurred during a period within which skill premia were falling poses a serious problem. If the skill premium is the crucial relative price which households take into account when deciding how many children to have, and how well to educate them, then ceteris paribus falling skill premia should have led to rising fertility rates and falling educational levels. A logical response to this dilemma is to argue that other things were not in fact equal. For example, one could argue, as does Galor (2005, pp. 255–56), that technological change was driving up the skill premium during the transition to modern economic growth, thus bringing about the demographic transition,

Figure 1: English Skill Premium, 1715–1915

The figure shows the ratio of the skilled to the unskilled wage in England in the 18th and 19th centuries. During the Industrial Revolution period, the skill premium fell.



Source: Clark (2007).

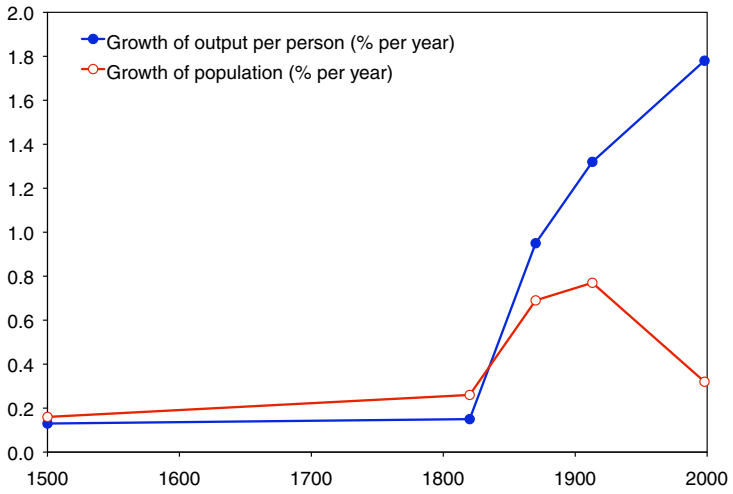
but that "the sizable increase in schooling that took place in the 19th century and in particular the introduction of public education that lowered the cost of education (e.g., The Education Act of 1870), generated significant increase in the supply of educated workers that may have prevented a rise in the return to education."

In this paper, we adopt a different approach, generating a demographic transition in the context of a model in which technological change is indeed initially unskilled-labor-biased, as was in fact the case; in which skill premia fall without subsequently rising, again as was the case; and in which household fertility choices do indeed reflect quantity-quality trade-offs. We do this without having to appeal to exogenous educational supply shocks, or indeed to exogenous shocks of any kind. However, in order to accomplish these objectives we need to go beyond the current so-called "unified growth theory" literature (e.g., Galor and Weil 2000; Jones 2001; Hansen and Prescott 2002; Lucas 2002; Weisdorf 2004), in several respects.

Most obviously, we need to incorporate two types of workers, skilled and

Figure 2: Annual Growth Rates of GDP per Capita and Population in Western Europe: 1500–2000

The figure shows the permanent acceleration in growth rates and the temporary boom in population which accompanied the Industrial Revolution.



Source: Galor (2005), based on Maddison (2001).

unskilled, so that we can track their relative earnings over time. Second, we need to allow for factor-biased technological change. Third, and most importantly, we need to allow the direction of factor bias to differ at different points in time, since we want to explain why the Industrial Revolution was initially so bad for skilled workers, rather than simply assume this was the case. Similarly, we want to explain why, by the end of the 19th century, new technologies that were skill-using were being invented, rather than just assume this was happening. We are thus going to have to explicitly model the choices facing would-be innovators. If the direction of technological change differed over time, this presumably reflected the different incentives facing these inventors.

In this paper, we thus delve into the microeconomics of technological change to a greater extent than previous unified growth theory papers, which have tended to model technological change in a reduced form manner as a function of scale affects (cf. Romer 1990, Kremer 1993) and/or human capital endowments. We propose a fully-specified research and development model driving

Figure 3: Fertility and Schooling in Four Advanced Countries

The figures show two important correlates of the Industrial Revolution: fertility decreased (the Demographic Transition) and education increased. However, the transtions did not occur until later in the 19th century.

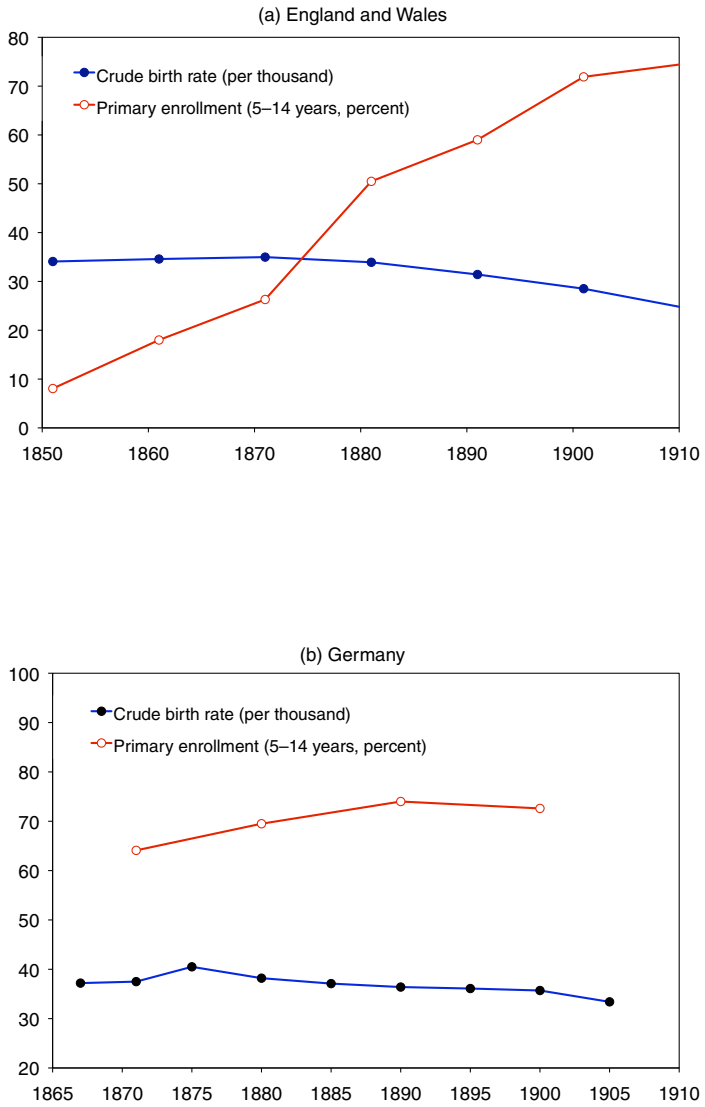
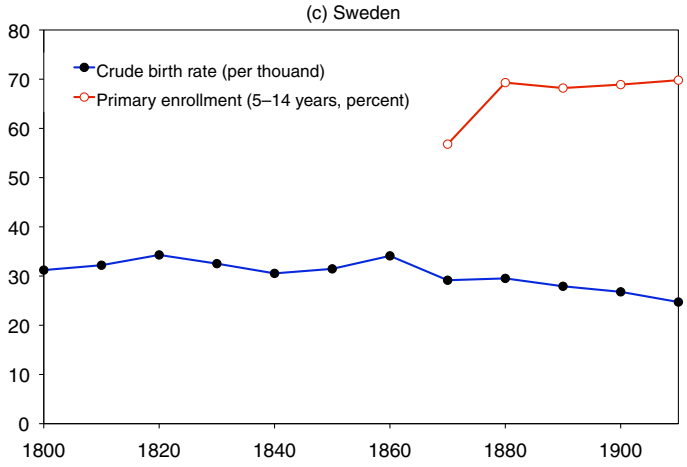
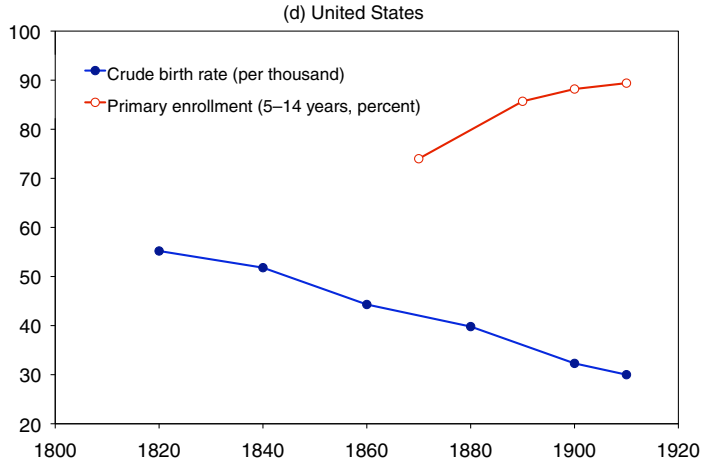


Figure 3 (continued): Fertility and Schooling in Four Advanced Countries





Source: Tan and Haines (1984). Trend lines are 2-period moving averages.

technological change, which is appropriate for this period since Allen (2006) has recently pointed out that British firms were investing significant resources in the search for technological breakthroughs during the Industrial Revolution. Building on the foundations of the benchmark Galor-Weil (2000) and Galor-Mountford (2004) models, we thus make several key changes to previous specifications.

The first key feature of our model, and the paper's main contribution, is that it endogenizes the direction of technological change. There are two ways to produce output, using either a low-skill technique (based on raw labor L) or a high-skill technique (based on educated labor or human capital H). For simplicity, these techniques are each linear in their sole input, and are characterized by their own, endogenous, productivity coefficients or technology levels.

Research by firms, which is patentable or otherwise excludable in the short run, can raise these technology levels and generate short-run monopoly profits. In the spirit of Acemoglu (1998), we allow potential innovators to look at the supply of skilled and unskilled labor in the workforce, and tailor their research efforts accordingly. The direction as well as the pace of technological change thus depends on demography. At the same time, demography is explicitly modeled as depending on technology, as is common in the literature (e.g., Galor and Weil 2000). Households decide the quality and quantity of their children (that is, the future supply of L and H) based directly on anticipated future wages, and thus (indirectly) on recent technological developments. As such the model allows for the co-evolution of both factors and technologies.

The second key feature of our approach is that we distinguish between two different types of technological progress: basic knowledge (B) and applied knowledge (A). In our model, the former grows according to the level of human capital in the economy and is a public good; the latter describes firms' techniques, which are subject (for a time) to private property rights, generate private profits, and hence create incentives for research. In our model, A is driven by research which generates benefits (increases in A) but also has costs (that are decreasing in B); thus basic knowledge drives the development of applied knowledge.

This distinction between basic and applied knowledge is inspired by Mokyr (2002, 2005a), who distinguishes between two knowledge types: the "propositional" episteme ("what") and the "prescriptive" techne ("how"). An addition to the former is for Mokyr a discovery, and an addition to the latter an invention. These categories can be thought of as close parallels to our B-knowledge (which we call "Baconian" knowledge) and A-knowledge (our sector-specific productivity levels, or TFP). We propose the term Baconian knowledge to honor Francis Bacon, since if Mokyr (2002, p. 41) is correct, then "the amazing fact remains that by and large the economic history of the Western world was dominated by materializing his ideals." Our model can provide a rationale for one of Mokyr's key claims, namely that "the true key to the timing of the Industrial Revolution has to be sought in the scientific revolution of the seventeenth century and the enlightenment movement of the eighteenth century" (p. 29). As will be seen, basic knowledge has to advance in our model for some time before

applied knowledge starts to improve. This helps model match reality: we find that Baconian knowledge B can increase continuously but applied knowledge or productivity A only starts to rise in a discontinuous manner once B passes some threshold.

The third key feature of our model is that it embodies a fairly standard demographic mechanism, in which parents have to trade off between maximizing current household consumption and the future skilled income generated by their children. We get the standard result that, ceteris paribus, a rising skill premium implies rising educational levels and falling fertility levels, while a falling skill premium implies the reverse. However, we further assume that rising wages makes education more *affordable* for households. Thus our model suggests that robust technological growth during the late 19th century fostered the dramatic rise in education and fall in fertility even with such low skill premia. A truly unified theory of industrialization should be able to capture all these trends; extent unified theories fall short in some important respects.

The next section of the paper presents the key aspects of the model, which endogenizes both technologies and demography. We then simulate this model to show how the theory can track the key features of the industrialization of Western Europe during the 18th and 19th centuries.

1 The Model

In this section we build a theoretical version of an industrializing economy in successive steps, keeping the points enumerated in the introduction firmly in mind. Section 1.1 goes over the production function and technologies. Here we develop a method for endogenizing the scope and direction of technical change, keeping endowments fixed. Section 1.2 then merges the model with an overlapping generations framework in order to endogenize demographic variables. These two parts form an integrated dynamic model which we use to analyze the industrialization of England during the 18th and 19th centuries.

1.1 Technology and Production

We begin by illustrating the static general equilibrium of a hypothetical economy. The economy produces a final good Y out of two "intermediate inputs" using a CES production function

$$Y = \left((A_l L)^{\frac{\sigma-1}{\sigma}} + (A_h H)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where A_l and A_h are technology terms, L is unskilled labor, H is skilled labor, and σ is the elasticity of substitution between the two intermediate inputs. Heuristically, one might think of the final good Y as being "GDP" which is simply aggregated up from the two intermediates.

By construction, A_l is L-augmenting and A_h is H-augmenting. We will assume throughout the paper that these intermediates are grossly substitutable,

and thus assume that $\sigma > 1$.[1] With this assumption of substitutability, a technology that augments a particular factor is also *biased* towards that factor. Thus we will call A_l *unskilled-labor biased* technology, and A_h *skill-biased* technology.

Economic outcomes heavily depend on which sectors enjoy superior productivity performance. Some authors use loaded terms such as "modern" and "traditional" to label the fast and slow growing sectors, at least in models where sectors are associated with types of goods (e.g., manufacturing and agriculture). We employ neutral language, since we contend that growth can emanate from different sectors at different times, where 'sectors' in our model are set up to reflect factor biases in technology. We argue that the unskilled-intensive sector was the leading sector during the early stages of the Industrial Revolution, while the skilled-intensive sector significantly modernized only from the mid-1800s onwards.

We assume that markets for both the final good and the factors of production are perfectly competitive. Thus, prices are equal to unit costs, and factors are paid their marginal products. Thus we can describe wages as

$$w_{l,t} = \left((A_{l,t} L_t)^{\frac{\sigma-1}{\sigma}} + (A_{h,t} H_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} L_t^{-\frac{1}{\sigma}} A_{l,t}^{\frac{\sigma-1}{\sigma}}, \tag{2}$$

$$w_{h,t} = \left((A_{l,t} L_t)^{\frac{\sigma-1}{\sigma}} + (A_{h,t} H_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} H_t^{-\frac{1}{\sigma}} A_{h,t}^{\frac{\sigma-1}{\sigma}}. \tag{3}$$

To endogenize the evolution of factor-specific technology levels A_l and A_h, we model technological development as improvements in the quality of machines, as in Acemoglu (1998). Specifically, we assume that researchers expend resources to improve the quality of a machine, and receive some positive profits (due to patents or first-mover advantage) from the sale of these new machines *for only one time period*.[2] We then define the productivity levels A_l and A_h to be amalgamations of quality-adjusted machines that augment either unskilled labor, or skilled labor, but not both.

In our model, costly innovation will be undertaken to improve some machine j (designed to be employed either by skilled or unskilled labor), get the blueprints for this newly improved machine, use these blueprints to produce the machine, and sell these machines to the producers of the intermediate good. After one period, however, new researchers can enter the market, and we find that newer, better machines will drive out the older designs. In this fashion we simplify the process of "creative destruction" (Schumpeter 1934, Aghion and Howitt 1992), where successful researchers along the quality dimension tend to eliminate the monopoly rentals of their predecessors.

[1] This has become a rather standard assumption in the labor literature, and is an important one for our analysis later.

[2] Conceptually one could also assume either that patent rights to innovation last only one time period, or equivalently that it takes one time period to reverse engineer the development of a new machine. In any case, these assumptions fit the historical evidence that profits from inventive activity have typically been short-lived.

Intermediate Goods Production

Let us now make technology levels explicit functions of these quality-adjusted machines. A_l and A_h at time t are defined as the following:

$$A_l \equiv \frac{1}{1-\beta} \int_0^1 q_l(j) \left(\frac{M_l(j)}{L} \right)^{1-\beta} dj, \tag{4}$$

$$A_h \equiv \frac{1}{1-\beta} \int_0^1 q_h(j) \left(\frac{M_h(j)}{H} \right)^{1-\beta} dj, \tag{5}$$

where $0 < \beta < 1$. M_l are machines that are strictly employed by unskilled workers, while M_h are machines that are strictly employed by skilled workers. $q_z(j)$ is the highest quality of machine j of type z. Note that these technological coefficients may thus be interpreted simply as functions of different types of capital per different types of workers; the capital however in this case is specialized and quality-adjusted. The specifications here imply constant returns to scale in the production of the skilled- and unskilled-intensive intermediate goods.

of a machine by a certain multiple. We assume that each 'quality ladder' with widely spaced rungs. These discrete and increments in quality are not assume that these increments are quality machines are always machines. Thus, when the the latest, older that

In the end, we care less about micro differences in machine qualities than about macro effects on total factor productivity. To draw conclusions about the latter we note that our problem is symmetric at the sector level, implying that aggregation is straightforward. In particular, machines along the (0,1) continuum will *on average* be of symmetrical quality, as inventors will be indifferent as to which particular machines along the continuum they will improve. As such we can alternatively write equations (4) and (5) as

$$A_l \equiv \left(\frac{1}{1-\beta} \right) Q_l \int_0^1 \left(\frac{M_l(j)}{L} \right)^{1-\beta} dj, \tag{6}$$

$$A_h \equiv \left(\frac{1}{1-\beta} \right) Q_h \int_0^1 \left(\frac{M_h(j)}{H} \right)^{1-\beta} dj, \tag{7}$$

where Q_k simply denotes the uniform and symmetric quality of all machines used in sector $k \in \{l, h\}$.

Increases in this index directly increase the total factor productivity of the sector. We also assume that machines last one period, and then depreciate completely.

Our modeling approach reflects the idea that different production techniques can be implemented only by particular factors. For example, by way of initial conditions, preindustrial textile production needed highly skilled labor such as spinners and weavers. Similarly, other preindustrial manufactures relied on their own skilled artisans of various sorts. But changes followed: implementing the technologies of the Industrial Revolution (in textile production, iron smelting and refining, mining and agriculture) required large labor forces with little to no

specialized training, and happy, highly-valued skilled craftsmen became angry, machine-breaking Luddites. These changes are proxied here as increases in A_l.

Much later, fortunes changed: the techniques developed in the latter half of the 19th century (for example in chemicals, electrical industries and services) raised the demand for a new labor force with highly specialized skills (Mokyr 1999). And Goldin and Katz (2008) document the introduction and adoption of electricity and capital-intensive technologies associated with continuous-process machinery during the late 19th century, technologies which dramatically raised the return of skills. These changes are proxied here as increases in A_h.

Returning to the model, let us consider a representative firm that competitively produces the unskilled intermediate. (Much of what follows will deal with only this sector. Parallel inferences can be made for the skilled sector.) Its maximization problem is stated as

$$\max_{\{L, M_l(j)\}} \quad p_l \cdot AL - \int_0^1 p(j) M_l(j) dj - w_l L, \tag{8}$$

where p_l is the price of the unskilled-intensive intermediate good, and $p(j)$ is the price of machine $M_l(j)$ faced by all producers of the intermediate. Hence the firm chooses an amount of unskilled labor to hire and amounts of complementary machines to employ, taking the price of its output, the price of machines, and the price of raw labor as given.

From the first order condition on L we have

$$p_l \beta A = w_l. \tag{9}$$

Solving for the price of the intermediate we have $p_l = \frac{w_l}{\beta A}$. From the first order condition on machine j we can get the total demand for machine $M_l(j)$

$$M_l(j) = \left(\frac{Q_l w_l}{\beta A_l p(j)} \right)^{\frac{1}{\beta}} L. \tag{10}$$

The Gains from Innovation

Innovation in a sector takes the form of an improvement in the quality of a machine by a certain multiple. Innovators expend resources up front to develop machine-blueprints, which they use to produce and sell a better-quality machine (all in the same time period). Assume that innovation is deterministic; that is, individuals who decide to research will improve the quality of a machine with a probability of one. We assume that there is a 'quality ladder' with discretely-spaced rungs. If innovation in sector j occurs, the quality of machine j will deterministically rise to εQ_l, where $\varepsilon > 1$ denotes the factor by which machine quality can rise. If on the other hand innovation does not occur, the quality of machine j remains at Q_l. If someone innovates, they have sole access to the blueprint for one period; after that the blueprint is public.

Once the researcher spends the resources necessary to improve the quality of machine j, she becomes the sole producer of this machine, and charges whatever

price (call it $p(j)$) she sees fit. Thus she receives total revenue of $p(j)M_l(j)$. Here we must make the distinction between the cost of *producing* a machine, and the cost of *inventing* a machine. We discuss the costs of innovation in the next sub-section. Here, we assume that the marginal cost of producing a machine is proportional to its quality, so that better machines are more expensive to make—a form of diminishing returns. Indeed, we can normalize this cost, so that total costs are simply $Q_l M_l(j)$.

Thus the producer of a new unskilled-using machine will wish to set the price $p(j)$ in order to maximize $V_l(j) = p(j)M_l(j) - Q_l M_l(j)$, where $V_l(j)$ is the value of owning the rights to the new blueprints of machine j at that moment in time. The question for us is what this price will be. Note that it will not be possible for the owner to charge the full monopoly markup over marginal cost unless the quality increase ε is very large. This is because all machines in sector j are perfect substitutes (they are simply weighted by their respective qualities); by charging a lower price, producers of older lower-quality machines could compete with producers of newer higher-quality machines.

We thus assume that producers of new machines can engage in Bertrand price competition, in the spirit of Grossman and Helpman (1991) and Barro and Sala-i-Martin (2003). In this case the innovator and quality leader uses a limit-pricing strategy, setting a price that is sufficiently below the monopoly price so as to make it just barely unprofitable for the next best quality to be produced.

This limit pricing strategy maximizes V_l and ensures that all older machine designs are eliminated from current production. The Appendix describes how we solve for this price; our solution yields

$$p(j) = p_{limit} = \varepsilon^{\frac{1}{1-\beta}} Q_l. \tag{11}$$

Note that this price is higher than the marginal cost of producing new machines, and so there is always a positive value of owning the blueprint to a new machine. Plugging this price into machine demands and these machine demands into our expression of V_l gives us

$$V_l = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_l}{\beta A_l}\right)^{\frac{1}{\beta}} L, \tag{12}$$

$$V_h = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_h}{\beta A_h}\right)^{\frac{1}{\beta}} H. \tag{13}$$

The Costs of Innovation

Before an innovator can build a new machine, she must spend resources on R&D to first get the blueprints. Let us denote these costs as c_l (for a new unskilled-labor using machine).

Conceivably the resource costs of research will evolve due to changing economic circumstances. Specifically, the "price" of a successful invention should

depend on things like how complicated the invention is and how "deep" general knowledge is. To capture some of these ideas, let us assume that the resource costs of research to improve machine j in the unskilled-intensive sector are given by

$$c_l = Q_l^{\alpha} B^{-\phi}, \tag{14}$$

and that the resource costs of research to improve machine j in the skilled-intensive sector are given by

$$c_h = Q_h^{\alpha} B^{-\phi}. \tag{15}$$

We include $\alpha > 1$ as a "fishing-out" parameter—the greater is the complexity of existing machines, the greater is the difficulty of improving upon them (see Jones 1998 on fishing out). The variable B is our measure of current general knowledge that we label *Baconian knowledge*. The general assumptions in each sector are that research is more costly the higher is the quality of machine one aspires to invent (another sort of diminishing returns), and the lower is the stock of general knowledge.

Growth of Baconian Knowledge

Baconian knowledge B can thus influence the level of technology A. But what are the plausible dynamics of B?

We allow general knowledge to grow throughout human history, irrespective of living standards and independent of the applied knowledge embedded in actual technology levels. According to Mokyr (2005b, pp. 291–2), Bacon regarded "knowledge as subject to constant growth, as an entity that continuously expands and adds to itself." Accordingly, we assume that the growth in basic knowledge depends on the existing stock. Furthermore, we assume that Baconian knowledge grows according to how much skilled labor exists in the economy;[3] specifically, we assume the simple form:

$$\triangle B_{t+1} = H_t \cdot B_t. \tag{16}$$

Thus we assume that increases in general knowledge (unlike increases in applied knowledge) do not arise from any profit motive, but are rather the fortuitous by-product of the existence of a stock of skilled workers, as well as of accumulated stocks of Baconian knowledge. But in our model, as we shall see, a skilled worker is just an educated worker, so it is here that the link between productivity growth and human capital is made explicit.

Thus we have a mechanism by which the growth in *general* knowledge (B) can influence the subsequent development of *applied* knowledge (A_l and A_h). Our functional forms (14) and (15) assume that low Baconian knowledge produces relatively large costs to machine improvement, while high Baconian knowledge generates low costs to machine improvement. In our model, B will never fall since (16) ensures that changes in B are nonnegative. Hence, the general

[3] Galor and Mountford (2004) make a rather similar assumption.

knowledge set always expands. This is not a historically trivial assumption, although it is accurate for the episode under scrutiny: Mokyr (2005b, 338–9) comments on the fact that knowledge had been lost after previous "efflorescences" (Goldstone 2002) in China and Classical Antiquity, and states that "The central fact of modern economic growth is the ultimate irreversibility of the accumulation of useful knowledge paired with ever-falling access costs."

Modeling the Beginnings of Industrialization

Turning to the decision to innovate in the first place, we assume that any individual can spend resources on research to develop and build machines with one quality-step improvement. Of course they will innovate only if it is profitable to do so. Specifically, if $\pi_i = V_i - c_i \geq 0$, research activity for i-type machines occurs, otherwise it does not. That is,

$$Q_{l,t} = \begin{cases} \varepsilon Q_{l,t-1} & \text{if } \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_l}{\beta A_l}\right)^{\frac{1}{\beta}} L \geq Q_l^{\alpha} B^{-\phi}, \\ Q_{l,t-1} & \text{otherwise,} \end{cases} \tag{17}$$

$$Q_{h,t} = \begin{cases} \varepsilon Q_{h,t-1} & \text{if } \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_h}{\beta A_h}\right)^{\frac{1}{\beta}} H \geq Q_h^{\alpha} B^{-\phi}, \\ Q_{h,t-1} & \text{otherwise.} \end{cases} \tag{18}$$

As these expressions make clear, applied innovations will not be profitable until Baconian knowledge reaches a certain critical threshold where benefits exceed costs. The natural world needs to be sufficiently intelligible before society can begin to master it (Mokyr 2002). Thus our model embodies the idea that growth in general Baconian knowledge is a necessary but not sufficient condition for output growth.

Finally, which type of applied innovations happen first? It turns out that due to a "market size" effect, the sector that innovates first will be the one using the abundant factor, since it is there that there is the greatest potential demand for new machines. We can state the following proposition:

Proposition 1 *If $Q_l = Q_h$ and $L > H$, initial technological growth will be unskilled-labor biased if and only if $\sigma > 1$.*

Proof: Given $Q_l = Q_h$, the costs of innovation are the same for skilled and unskilled using machines, and are falling at the same rate. Thus in order to illustrate that initial growth will be unskill-intensive, we must demonstrate that initial conditions are such that $V_l > V_h$.

First, note that we can plug the limit price from (11) into our machine demand equation (10), and plug this expression into our technology coefficient expressions (4) and (5) to get

$$A_l = \left(\frac{1}{1-\beta}\right)^{\beta} \left(\frac{1}{\beta}\right)^{1-\beta} Q_l^{\beta} w_l^{1-\beta}, \tag{19}$$

$$A_h = \left(\frac{1}{1-\beta}\right)^{\beta} \left(\frac{1}{\beta}\right)^{1-\beta} Q_h^{\beta} w_h^{1-\beta}. \tag{20}$$

We then can plug these expressions into our value expressions (12) and (13). After simplifying a bit, we get

$$V_l = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right) \left(\frac{1}{Q_l}\right) \left(\frac{1-\beta}{\beta}\right) w_l L, \tag{21}$$

$$V_h = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right) \left(\frac{1}{Q_h}\right) \left(\frac{1-\beta}{\beta}\right) w_h H. \tag{22}$$

Thus we see that our condition is equivalent to

$$V_h < V_l \Rightarrow \frac{V_h}{V_l} < 1 \Rightarrow \frac{V_h}{V_l} < \frac{w_h H}{w_l L}.$$

From this we can see that the relative gains for the innovator is larger when the factor capable of using the innovation is large (the so-called "market-size" effect) and when the price of the factor is large (the so-called "price" effect). To get everything in terms of relative factors, we can use (2) and (3), and our productivity expressions (19) and (20), to get an expression for *relative* wages:

$$\frac{w_h}{w_l} = \left(\frac{L}{H}\right)^{\frac{1}{\sigma}} \left(\frac{A_h}{A_l}\right)^{\frac{\sigma-1}{\sigma}} = \left(\frac{L}{H}\right)^{\frac{1}{(1-\beta)(1-\sigma)+\sigma}} \left(\frac{Q_h}{Q_l}\right)^{\frac{\beta(\sigma-1)}{(1-\beta)(1-\sigma)+\sigma}}. \tag{23}$$

Finally, plugging this relative wage into the above inequality, using the fact that $Q_l = Q_h$ and simplifying, we have

$$\frac{V_h}{V_l} = \left(\frac{L}{H}\right)^{\frac{(1-\beta)(\sigma-1)-\sigma+1}{(1-\beta)(1-\sigma)+\sigma}} < 1.$$

Given that $L/H > 1$ and $0 < \beta < 1$, this can only hold if the exponent is negative, which is true only when $\sigma > 1$. Q.E.D.

In other words, provided that labor-types are grossly substitutable, the initial stages of industrialization had to be unskill-intensive simply because there were so many more unskilled laborers in the workforce than skilled laborers. So, to grossly simplify the history of innovation, we maintain that for most of human existence the inequalities $\pi_l < 0$ and $\pi_h < 0$ held strictly. Once $\pi_l \geq 0$, industrialization could occur. Once *both* $\pi_l \geq 0$ and $\pi_h \geq 0$, robust modern economic growth could occur. As we will suggest below, this sequence was precisely how we believe history played out.

Relative Wages during Industrialization

Let us take stock. At the start of the Industrial Revolution, the Luddites became history and a high-skilled artisanal class gave way to lower-skilled factory workers. But during the later nineteenth century, the patterns of economic growth did not subsequently restore the skill premium, even as skill acquisition gathered pace. England found itself at the turn of the twentieth century with a large endowment of skilled labor and an historically low skill premium (Clark 2007, van Zanden 2004). But this presents us with a modeling challenge: if skill premia were low and falling, then what induced families to limit fertility and invest in education? In order to understand this, we need to better understand the feedback from relative factor endowments (demography) to relative factor rewards (the skill premium) and vice versa. The demographic micro-foundations of how the factors of production endogenously react to changes in wages is the subject of the next subsection.

1.2 Endogenous Demography

We adopt a variant of a fairly standard overlapping generations model of demography suited to unified growth theory (cf. Galor and Weil 2000).

In a very simplified specification, we assume that 'adult' agents maximize their utility, which depends both on their current household consumption and on their children's expected future income. In an abstraction of family life, we assume that individuals begin life naturally as unskilled workers, accumulate human capital, and then become skilled workers as adults. Consequently the skilled and unskilled are divided into two distinct age groups. That is, an agent evolves naturally from a 'young' unskilled worker into an 'adult' skilled worker; thus his welfare will be affected by both types of wages.

Only 'adults' are allowed to make any decisions regarding demography. Specifically, the representative household is run by an adult who decides two things: how many children to have (denoted n_t) and the level of education each child is to receive (denoted e_t). The number of children must be nonnegative and to keep things simple all households are single-parent, with $n = 1$ being the replacement level of fertility. The education level is constrained to the unit interval and is the fraction of time the adult devotes to educating the young.

Our modeling of demography is as follows. An individual born at time t spends fraction e_t of her time in school (something chosen by her parent), while spending the rest of her time as an unskilled laborer. At $t + 1$, the individual (who is by this time a mature adult) works strictly as a skilled laborer, using whatever human capital she had accumulated as a child.

The Adult Household Planner

Allowing for the time cost of child-rearing, we assume that the household consumes all the income that the family members have generated. At time t there are L_{t-1} adults, each who is an individual household planner. These planners

wish to maximize the sum of current household consumption and the future skilled income generated by their children. That is, the individual born at time $t-1$, and now an adult at time t, faces the problem

$$\max \quad \log(c_t) + \log\left(w_h \frac{H_{t+1}}{L_{t-1}}\right), \tag{24}$$

where c_t denotes current consumption per household and H_{t+1} is the total future level of human capital "bequested" by parents to their children.

We now must specify how household consumption, population, and human capital are functions of rates of fertility (n_t) and education (e_t). We assume the following functional forms:

$$c_t = w_{h,t}\left(\frac{H_t}{L_{t-1}}\right) + w_{l,t}(1-e_t)n_t - w_{h,t}\Gamma n_t^{\mu} - x n_t e_t, \tag{25}$$

$$L_t = n_t L_{t-1}, \tag{26}$$

$$H_{t+1} = (e_t L_t)^k = (e_t n_t L_{t-1})^k, \tag{27}$$

where $\Gamma > 0$, $x > 0$, $\mu > 1$ and $0 < k < 1$. The first term in (25) is the income generated by the parent (this is total skilled income generated at time t, $w_{h,t}H_t$, divided by the total number of adults L_{t-1}). The second term in (25) is the unskilled income generated by the children (each child spends $1-e$ of their childhood as an unskilled worker). Thus we see that when children are not being educated for a fraction of time $1-e$, they increase the family's unskilled income, but this will reduce their own future skilled income because they will receive a lower endowment of H.

The final two terms in (25) are the overall costs of child-rearing. Note that while having more children involves an *opportunity* cost (since these costs depend on the skilled wages of the parent), each unit of education per child involves a *resource* cost (of some amount x). On the one hand, having more children typically requires one to spend more time on child rearing. This explains the standard opportunity cost term $w_h\Gamma n_t^{\mu}$. On the other hand, formal education also typically embodies resource costs (books and other educational overhead) that rise in proportion to the amount of educational services supplied. This explains the resource cost term $x n_t e_t$.

Notice that we assume a particular functional form for human capital, where the term $e_t L_t$ represents the total educational input of the economy. Also notice that increases in fertility rates will *immediately* translate into increased levels of unskilled labor, while increases in education will *eventually* translate into increased levels of skilled labor next period. Thus given our discussion above, wage changes will immediately change the overall population level, and will eventually change the level of human capital in the economy. Given (25) - (27), we can rewrite the household objective function as

$$\max_{n_t, e_t} \quad \log\left[w_{h,t}\left(\frac{(e_{t-1}n_{t-1}L_{t-2})^k}{L_{t-1}}\right) + w_{l,t}(1-e_t)n_t - w_{h,t}\Gamma n_t^{\mu} - x n_t e_t\right]$$

$$+\log\left[w_{h,t+1}\frac{(e_t n_t L_{t-1})^k}{L_{t-1}}\right] \tag{28}$$

subject to: $n_t \geq 0$, and $0 < e_t \leq 1$.

The individual born at $t-1$ will choose a pair of $\{n_t, e_t\}$ that maximizes (24), taking perceived wages as given.

decisions as an adult at time t, but $w_{h,t+1}$, something that is simulations below, we (28) individuals forecast of the future forecasts

From (28), the first-order condition for the number of children is:

$$w_l(1-e_t) + c_t\frac{k}{n_t} = \mu w_h \Gamma n^{\mu-1} + xe_t. \tag{29}$$

The left-hand side illustrates the marginal benefit of an additional child, while the right-hand side denotes the marginal cost. At the optimum, the gains from an extra unskilled worker in the family and more skilled income for children in the future precisely offsets the foregone income that results from child-rearing.

The first order condition for education is:

$$c_t\frac{k}{e_t} = (w_l + x)\, n_t. \tag{30}$$

Again the left-hand side is the marginal benefit and the right-hand side is the marginal cost, this time of an extra unit of education per child. At the optimum, the gains received from more skilled income by children offset the foregone unskilled-labor income *and* the extra resource cost associated with an extra educational unit for all children at t.

2 A Tale of Two Revolutions

Sections 1.1 and 1.2 summarize the long-run co-evolution of factors and technologies in the model. Once innovation occurs and levels of Q_l and Q_h are determined, the complete general equilibrium can be characterized by simultaneously solving (2), (3), (19), (20), (26), (27), (29) and (30) for wages, productivity-levels, fertility, education, and factors.

We are now ready to see how well our model can account for what happened in England (and other northwestern European economies) during the eighteenth and nineteenth centuries.

2.1 The Industrial Revolution

A critical part of the argument offered here is that the Industrial Revolution was really a sequence of unskilled-labor intensive technological developments. These developments first appeared in England and Wales in the latter half of the eighteen century, and then spread to other parts of continental Europe and European "offshoots" in the early part of the nineteenth century.

Our theory here suggests a number of things concerning this revolution. First, implicit in our model is that the institutional framework protecting intellectual property rights had been in place far before the onset of the Industrial Revolution . Hence we do not rely on an exogenous institutional shock to launch the Industrial Revolution. Rather, we rely on basic scientific (Baconian) knowledge to rise above a certain threshold level in order for applied innovation to become feasible. Once this happened in certain northwestern European economies, the growth of technologies and output became possible. Second, technological developments tended to heavily employ unskilled labor, for this factor of production was in relatively great supply in these areas. Finally, by increasing the relative earnings of unskilled labor, these technological developments spurred population growth and at the same time limited the growth of human capital.

We can see these propositions within the context of the model. An economy before its launch into the Industrial Revolution may be described by the one in section 1.1, with technological coefficients A_l and A_h constant. Here wages are fixed, and thus the levels of raw labor and human capital remain fixed as well. Both output and output per capita remain stagnant.

If we assume that the evolution of technological coefficients A_l and A_h are described by the relationships in section 1.1, then the economy must wait until Baconian knowledge grows to a sufficient level before applied innovation becomes possible. Further, technological growth will initially be unskilled-labor biased (that is, there is growth in A_l) so long as it becomes profitable to improve machines used in the unskilled sector *before* it becomes profitable to improve machines used in the skilled sector

Thus if the economy begins such that $A_l = A_h$ (as we maintain in the simulations), initial technological growth will be unskilled labor biased so long as there is relatively more unskilled labor than skilled labor in the economy, which was surely the case in the eighteenth century (by Proposition 1).

Furthermore, these technological developments change the wage structure, and by implication the evolution of factor endowments. The growth of A_l lowers the skill premium, increasing the future ratio of unskilled wages to skilled wages and thereby inducing families to have more children to take advantage of higher unskilled wages. This is a major emphasis of our model and this paper. Unlike all extant "unified" models of the Industrial Revolution and Modern Economic Growth, we try to take the Luddites seriously: population boomed, and skilled labor was initially hurt by the Industrial Revolution, a historical fact that many current theories fail to explain.

This story of unbalanced growth in early industrialization seems consistent with history. Well-known studies such as Atack (1987) and Sokoloff (1984) describe the transition of the American economy from reliance on highly-skilled artisans to the widespread mechanization of factories. And Goldin and Katz (1998) assert that technological advances which led to standardization and assembly-line production processes inevitably replaced skilled workers with raw labor. This paper further argues that the boom in fertility that the industrializing areas experienced was both the cause and consequence of these technological

revolutions arising, in the British case, in the late 1700s and early 1800s. According to Folbre (1994), the development of industry in the late eighteenth and early nineteenth centuries led to changes in family and household strategies. The early pattern of rural and urban industrialization in this period meant that children could be employed in factories at quite a young age. The implication is that children became an asset, whose labor could be used by parents to contribute income to the household. In English textile factories in 1835, for example, 63% of the work force consisted of children aged 8-12 and women (Nardinelli 1990). This is not to say that attitudes toward children were vastly different in England then compared with now; rather economic incentives were vastly different then compared with now (Horrell and Humphries 1995). As a result of these conditions, fertility rates increased during the period of early industrialization.

While our approach may help explain the population growth that coincided with the initial stages of the Industrial Revolution,[4] we are still faced with the challenge of explaining the demographic transition that followed it.

2.2 The Demographic Transition

Human capital presents a challenge for unified growth theories: it appears to hardly play any role at all in the Industrial Revolution, yet clearly is central to the story of growth both in the late nineteenth and throughout the twentieth centuries. We argue that industrialization took on a new form around the mid-1800s, and that this development shifted the world economy in ways that continue to manifest themselves today.

The educational stagnation in England of the late eighteenth and early nineteenth centuries starkly contrasts with the large school enrollment rates of the late nineteenth century. Schofield (1968) reveals very modest rises in signature rates at marriage (a mark of society-wide illiteracy) from 1780 to 1830, and Mitch (1982) highlights the 1818 and 1833 parochial surveys of elementary schooling which indicate that the proportion of all students enrolled in schools remained constant at 42%. In contrast by the turn of the twentieth century in England there was virtually universal literacy and primary school enrollment.

At the same time there was a marked decrease in fertility rates in England and other regions of western Europe. Crude birth rates in England declined by 44% from 1875-1920, while those for Germany, Sweden and Finland between 1875 and 1920, and France between 1865 and 1910, declined by 37%, 32%, 32% and 26% respectively (Andorka 1978; Kuczynski 1969). These trends suggest a reversal in the relationship between income and fertility, corresponding to an increase in the level of resources invested in each child.

Part of our argument is that biases inherent in technological innovation fostered this reversal. As Baconian knowledge rose further and skill-biased production grew in importance, the labor of children became less important as a source

[4] Of course there are a host of other explanations, including falling death rates related to health improvements, and the passage of various Poor Laws. Naturally we are abstracting from these possibilities without dismissing them as inconsequential.

of family income, and this was reflected in economic behavior. It is true that legislation limited the employment possibilities for children (Folbre 1994), and introduced compulsory education. However,the underlying economic incentives were leading in the same direction.

Recent studies make a variety of related points which can also explain the demographic transition. Hazan and Berdugo (2002) suggest that technological change at this stage of development increased the wage differential between parental labor and child labor, inducing parents to reduce the number of their children and to further invest in their quality, stimulating human capital formation, a demographic transition, and a shift to a state of sustained economic growth. In contrast, Doepke (2004) stresses the regulation of child labor. Alternatively, the rise in the importance of human capital in the production process may have induced industrialists to support laws that abolished child labor, inducing a reduction in child labor and stimulating human capital formation and a demographic transition (Doepke and Zilibotti 2003; Galor and Moav 2006).

2.3 Simulations

How well does the model track these general historical trends? To answer this we numerically simulate the model.

For each time period, we solve the model as follows:

1) Baconian knowledge grows according to (16).

2) Based on this new level of Baconian knowledge, if $\pi_l > 0$, Q_l rises by a factor of ε; if not Q_l remains the same value. Similarly, if $\pi_h > 0$, Q_h rises by a factor of ε; if not Q_h remains the same value.

3) Given levels of Q_l and Q_h, solve the equilibrium. This entails solving the system of equations (2), (3), (19), (20), (26), (27), (29) and (30) for w_l, w_h, A_l, A_h, n, e, L, and H.

Parameters and Initial Conditions

Parameters are set to the following: $\sigma = 2$, $\beta = 0.7$, $\alpha = 2$, $\varepsilon = 1.1$, $\phi = 2$, $k = 0.5$, $\mu = 5$, $\Gamma = 0.1$, $x = 2$. Many other parameterizations will produce qualitatively similar results; the critical assumptions here are that $\sigma > 1$ (skill and unskill-intensive produced goods have enough substitutability to satisfy Proposition 1),[5] $0 < k < 1$ (diminishing marginal returns to education), and $\mu > 1$ (child-rearing costs convexly rise with the number of children).[6]

[5] The degree of substitutability between skilled and unskilled labor has been much explored by labor economists. Studies of contemporary labor markets in the U.S. (Katz and Murphy 1992, Peri 2004), Canada (Murphy et al. 1998), Britain (Schmitt 1995), Sweden (Edin and Holmlund 1995), and the Netherlands (Teulings 1992) suggest aggregate elasticities of substitution in the range of one to three. See Katz and Autor (1999) for a detailed review of this literature.

[6] Other parameters simply scale variables (such as Γ and x) or affect the speed of growth (such as α, ε and ϕ). Changing the values of these parameters however would not shift the direction of technology, nor would it affect the responsiveness of households. Thus these specific values are not crucial for the qualitative results and our underlying story.

We start with $Q_l = Q_h = 0.1$. For other initial conditions, we set $n = 1$ (replacement fertility) and solve the 4 by 4 system of (2), (3), and the two first-order conditions from (24) for initial values of w_l, w_h, L and H. Given $Q_l = Q_h$ and ensuring that $w_h/w_l > 1$, it must be that $L > H$. By Proposition 1, this implies that the first stage of the Industrial Revolution *must be unskilled in nature*. To see this we turn to modeling the economy as it evolves in time.

Dynamic Simulation

We simulate our hypothetical northwestern European economy through 30 time periods, roughly accounting for the time period 1750-1910. The results are given in Figures 4–9, using the parameterizations summarized above.[7]

Figure 4 illustrates the market for innovation. Initial Baconian knowledge B is set low enough so that the costs of innovation are larger than the benefits in the beginning. As a result technology levels remain stagnant at first. But through Baconian knowledge growth costs fall, catching up with the benefits for research first for technologies designed for unskilled labor; hence, at $t = 5$ Q_l begins to grow (that is, π_l becomes positive). By contrast, $\pi_h < 0$ early on, so Q_h remains fixed. Note that this results solely because L is larger than H through Proposition 1. In other words, endowments dictate that the Industrial Revolution will initially be unskilled-biased. As Q_l rises the costs of subsequent unskilled innovation rise because it gets increasingly more expensive to develop new technologies the further up the quality ladder we go (due to the "fishing-out" effect from the α term); however, the benefits of research rise even faster due to both rising unskilled wages and unskilled labor (see equation 21). And further growth in Baconian knowledge keeps research costs from rising too fast (see equation 14). As a result unskilled-intensive growth persists.

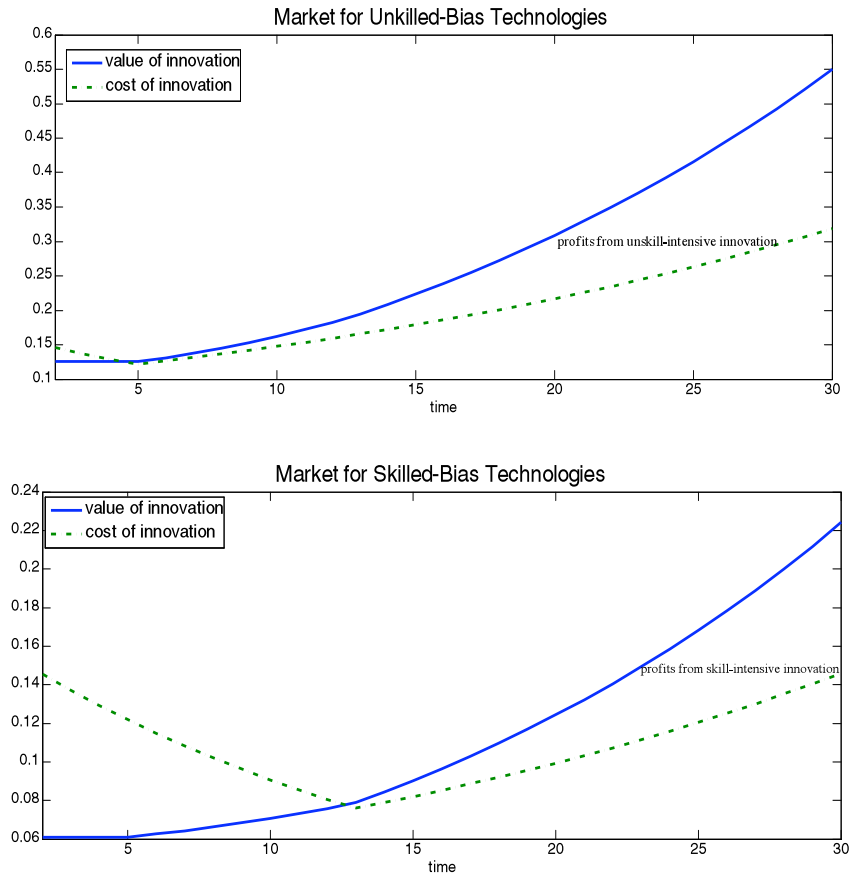
Skill-biased technologies, on the other hand, remain stagnant during this time. Their eventual growth however is inevitable; Baconian knowledge growth drives down the costs to develop them, and rising skilled wages (through growth of unskilled technologies) drives up the value to develop to them. By $t = 12$, π_h becomes positive as well, allowing Q_h to climb. At this point growth occurs in both sectors, with advance in all periods given by the constant step size on the innovation ladders. Note that this induces something of an endogenous demographic transition; fertility rates stabilize and eventually begin to fall, while education rates rise, as we can see in Figures 5 and 6.

Figures 5 through 9 depict historical and simulated time series for fertility rates, education rates, wages, skill-premia and income per capita. As can be seen, our model reproduces the early rise in fertility, followed by falling fertility and rising education. Education rates very modestly during the early stages Industrial Revolution, and rise only after growth occurs in both sectors. To understand the fertility and education patterns depicted in Figures 5 and 6, one must observe the absolute and relative wage patters depicted in Figures 7 and 8. The early stages of industrialization are associated with very limited wage

[7] The initial level of B is chosen such that growth starts after a few periods.

Figure 4: Incentives for Innovation in Each Sector

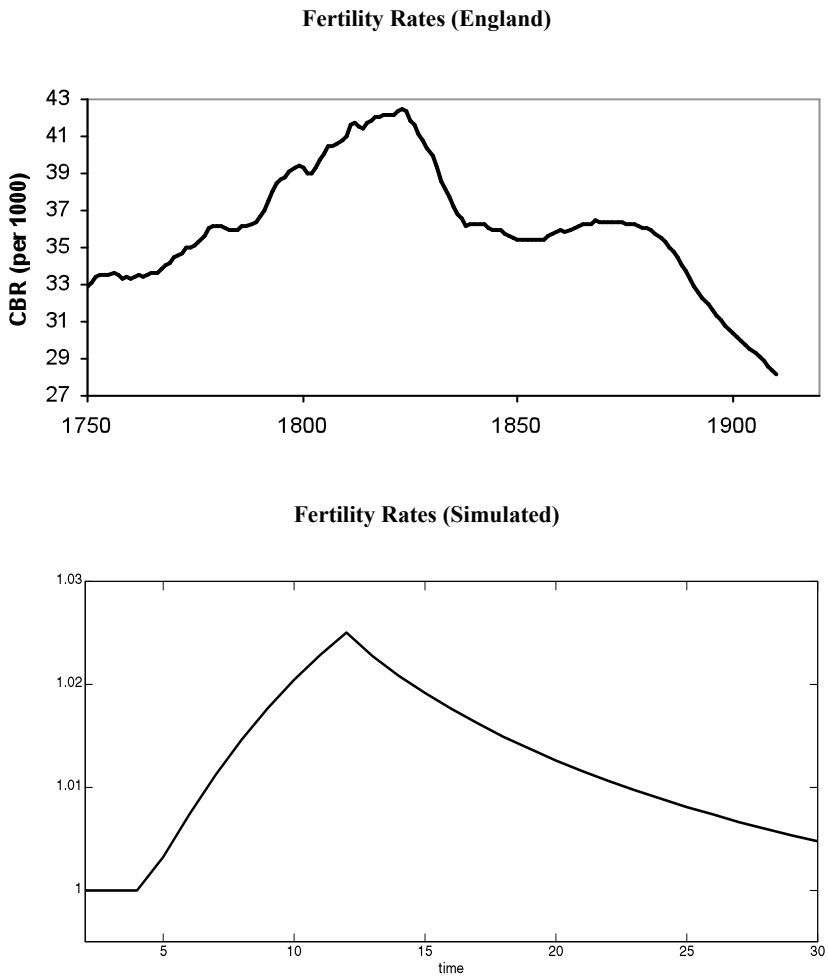
The figure shows the value of innovating and the cost of innovating in each period for each sector in our simulated model. The economy is initially endowed with much more unskilled labor L than skilled labor H. Hence, unskilled-intensive technologies are the first to experience directed technological change.



Notes: Parameterizations are as follows: $\sigma = 2, \beta = 0.7, \alpha = 2, \varepsilon = 1.1, \phi = 2, k = 0.5, \mu = 5, \gamma = 0.1, x = 2$. These values ensure that Propositions 1 and 2 hold for all t. Initial technological conditions are as follows: $Q_L = Q_H = 0.1, B = 1$.

Figure 5: Fertility Rates: Data versus Model

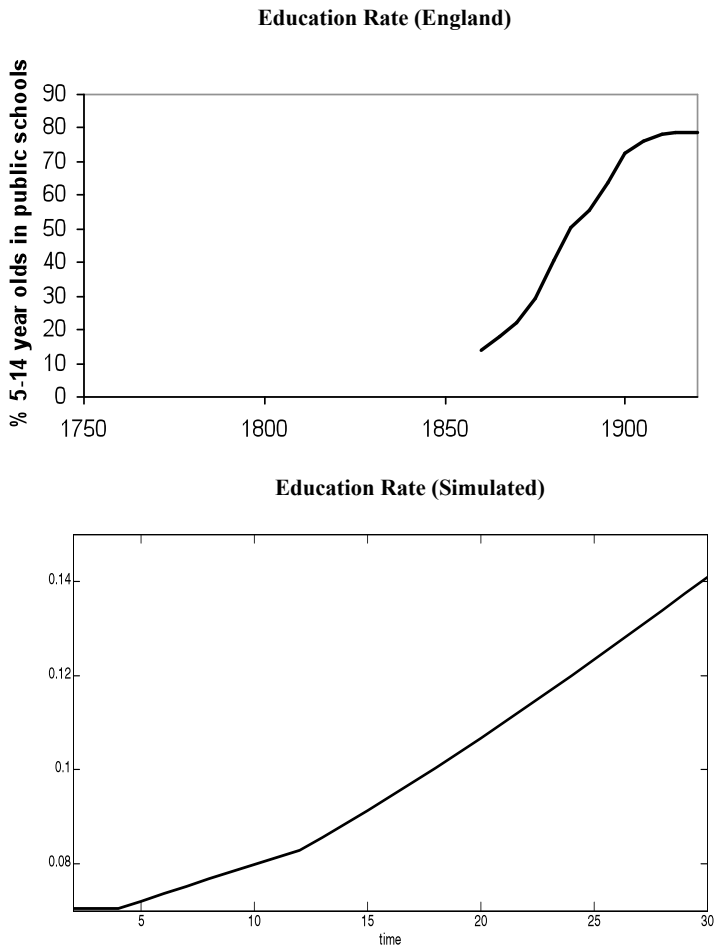
The figure shows the fertility levels in the English data and in the simulated model.



Source: The data are 10-year moving averages from Wrigley and Schofield (1981) and Andorka (1978).

Figure 6: Education: Data versus Model

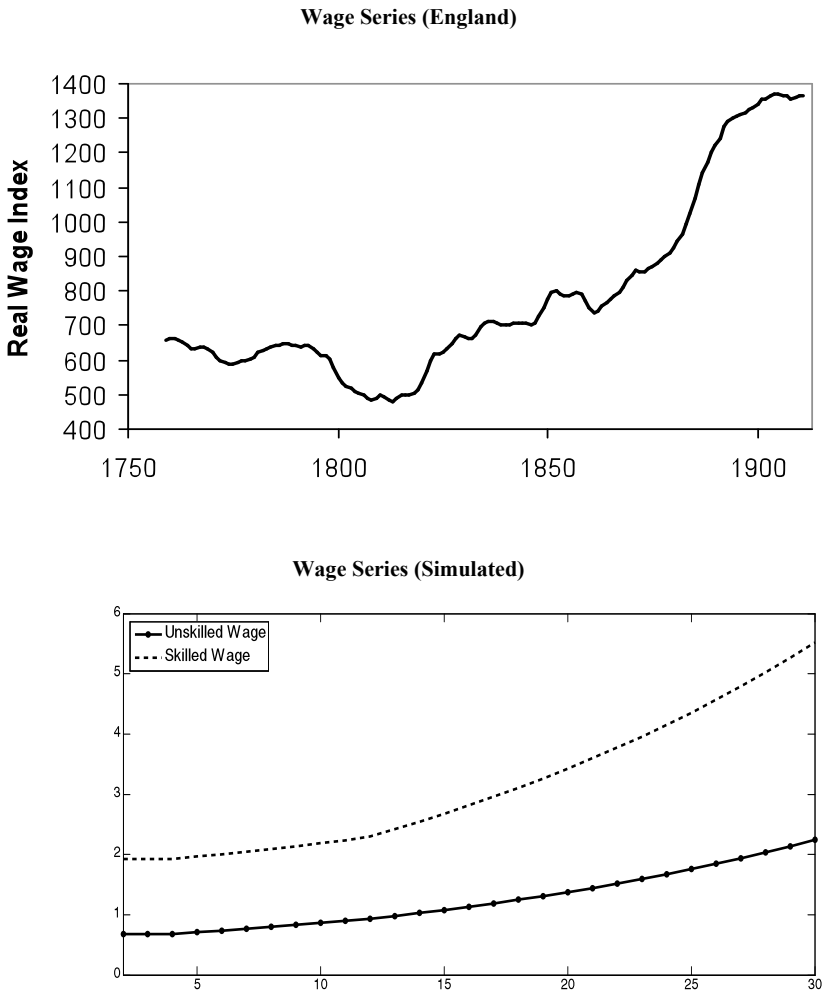
The figure shows the education levels in the English data and in the simulated model.



Source: The data are 10-year moving averages from Flora et al. (1983).

Figure 7: Wages: Data versus Model

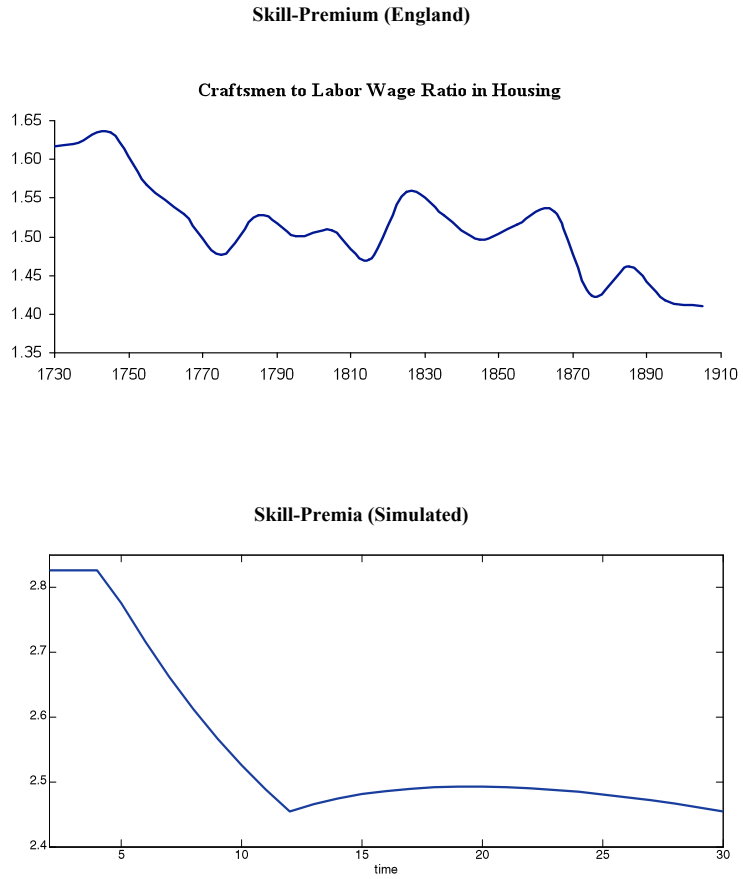
The figure shows wage levels in the English data and in the simulated model.



Source: The data are 10-year moving averages from Wrigley and Schofield (1981).

Figure 8: Skill Premium: Data versus Model

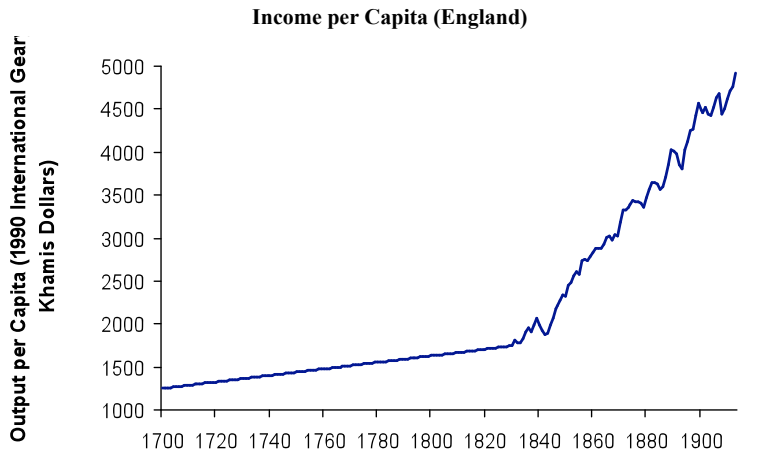
The figure shows the skill premium in the English data and in the simulated model.

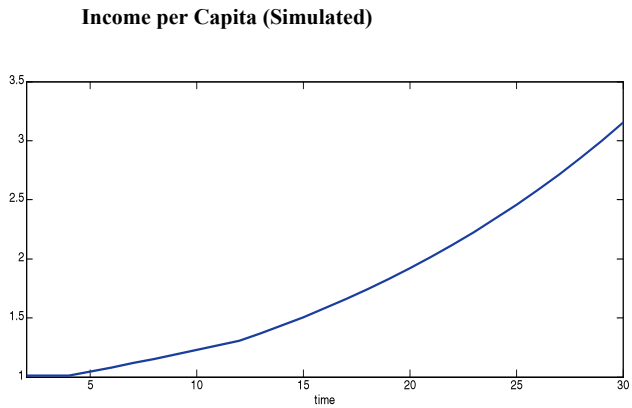


Source: The data are from Clark (2007).

Figure 9: GDP per Person: Data versus Model

The figure shows GDP per person in the U.K. data and in the simulated model.





Source: The data are from Maddison (2003).

growth and a declining skill premium. Of course this makes sense: what growth in applied knowledge does occur is confined to the unskilled sector.

The evolution of wages that this produces then have implications for education and fertility—while the falling skill premium puts *downward* pressure on education (education limits the children's ability to earn unskilled income), rising wages puts *upward* pressure on education (to take advantage of the higher skilled income that can be generated). These forces roughly cancel each other out, keeping any growth in education during this period modest. Fertility however is free to rise: since slow-rising education keeps the resource costs of each child low, parents can take advantage of rising unskilled wages by having more kids. Of course this produces more L and keeps H fairly low. Thus the model can replicate the rapid fertility growth, low education growth, and falling skill premia consistent with the early stages of the Industrial Revolution.[8]

Once skill-biased technologies grow as well, the patterns of development change entirely. Growth in both A_l and A_h allows wages to rise more rapidly and keeps *relative* wages fairly constant (from equation 23 we know that relative wages are a function of relative quality indices and relative factors—with balanced growth relative quality indices remain stable, leaving only relative factors to influence changes in the skill premium). In this case, the relatively stable skill premium and rising overall wages changes behavior, as households lower their rates of fertility in order to invest in education. The reason is that with greater and greater skilled wages, the opportunity cost for having children grow larger, since more fertility implies more foregone parental income. There then is an endogenous switch from child quantity to child quality. The demographic transition of the mid to late 19th century is launched.

Figures 4 through 8 thus depict an integrated story of western European development fairly consistent with the historical record. The theory of endogenous technical change can indeed motivate a unified growth theory. Increases in income induces increases in education, which then through the quality-quantity tradeoff eventually induces a fall in fertility. And increases in H/L put downward pressure on the premium through supply-side forces. Thus, as implied by (23), continued supply increases can outstrip demand-side forces: a modern economy can have both high relative skilled labor and a low skill premium even in the context of directed endogenous technological change.

Finally, Figure 9 shows the evolution of output per person in the data and the model. In both theory and reality, the population growth spurred by initial technological growth kept per capita income growth very modest (making the Industrial Revolution appear to be a fairly *un*-revolutionary event). With technological advance occurring in more sectors, incomes rose faster; this in turn provoked fertility decreases and made per capita growth faster still.

[8] Acemoglu (2006) makes the distinction between "weak technological bias" (referring to how factor prices change at *given* factor proportions) and "strong technological bias" (referring to how factor prices change with *changing* factor proportions). Our modeling rests on the idea that early industrialization was *strongly* unskilled labor biased: even with continuous injections of unskilled labor, quality-step increases in unskilled-intensive technologies allowed relative returns to skilled labor to fall over time.

3 Conclusion

We believe that by explicitly modeling research and development, thus endogenizing the direction of technical change, we have been able to shed some valuable light on the transition to modern economic growth. Like most unified growth models, our model is subject to the criticism that it makes a take-off "inevitable," a proposition to which many historians, more comfortable with notions of chance and contingency, might object. Our model makes another claim, however, which seems much more robust: if a take-off took place, it should, inevitably, have first involved unskilled-labor-using technologies, for the simple reason that unskilled labor was the abundant factor of production at this time.

From this simple prediction, as we have seen, flow a whole series of consequences. The Industrial Revolution should have seen skill premia fall, which they did. It should therefore have seen an initial increase in fertility rates, which again it did. Our model predicts that these two phenomena would have continued to reinforce each other indefinitely, barring some countervailing force. One such force was the continuing growth in Baconian knowledge, which would eventually lead to the growth of the science-based and skill-intensive sectors of the Second Industrial Revolution. While this should have caused skill premia to stop falling and start rising, massive endogenous increases in education kept the relative wages of skilled labor low.

Our simulations indicate that our model does a pretty good job of explaining the concomitants of industrialization during the eighteen and nineteenth centuries. Other theories often rely on exogenous forces to reconcile falling skill premia with their theories of the demographic transition. For example Galor and Moav (2006) highlight the fact that at precisely this time European and North American governments embarked on a massive programme of public education, thus exogenously raising skill endowments and lowering skill premia. Furthermore, public primary education programmes were later followed by two world wars, rising union strength, and public secondary and tertiary education programmes, all of which served to further reduce skill premia, in the U.S. case at least through the "Great Compression" of the 1940s (Goldin and Margo 1992; Goldin and Katz 1999, 2008).

In the context of this paper, it seems possible that these so-called exogenous factors leading to greater equality were actually endogenously influenced by growth, even though powerful endogenous technological factors were pushing economies towards greater inequality. This is striking, since as Acemoglu (1998) points out, in the context of a model like this one, long run exogenous increases in skill endowments lead to more rapid skill-biased technical change, thus increasing the upward pressure on skill premia in the long run. What is remarkable, therefore, is that western labor markets remained on an egalitarian path until well into the twentieth century. In this context, current inegalitarian trends in the U.S. and elsewhere can be seen as late nineteenth century chickens finally coming home to roost.

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Appendix:
Limit Pricing and the Gains from Innovation

Here we solve for the price new innovators would charge for newly invented machines in the face of competition from producers of older machines. First, let us describe the producers of the unskilled-intensive good, $A_l L$; analogous results will hold for the skill-intensive good. The production function for these goods are

$$\left(\frac{1}{1-\beta}\right) Q_l L^{\beta} \int_0^1 M_l(j)^{1-\beta} dj.$$

Producers wish to maximize profits or, equivalently, to minimize unit costs. The unit costs for producers who buy new machines, denoted as uc, can be written as

$$uc = \beta^{-\beta} (1-\beta)^{\beta} \left(\frac{1}{\varepsilon Q_l}\right) w_l^{\beta} \int_0^1 p(j)^{1-\beta} dj,$$

where $p(j)$ is the price of machine j and w_l is the wage of L.[9]

The question for us is, what maximum price could an innovator charge and drive out the competition at the same time? The "competition" in this case are those who hold the blueprints of the next highest-quality machines of quality Q_l. The *lowest* price they can charge is their marginal cost, Q_l; if all old machine-producers charge this, unit costs can be written as

$$uc_{old} = \beta^{-\beta} (1-\beta)^{\beta} \left(\frac{1}{Q_l}\right) w_l^{\beta} Q_l^{1-\beta}.$$

Traditional endogenous growth theories that use quality ladders typically have producers of new machines charge a monopolistic mark-up over marginal cost. In our case producers of new machines would charge a price $p_{monop} = \frac{\varepsilon Q_l}{(1-\beta)}$ for a machine of quality εQ_l. However, in order for this to be a profitable strategy, unit costs for producers of $A_l L$ must be at least as low when they buy new machines compared with when they buy the older, cheaper machines. In other words, $uc_{old} \geq uc_{monop}$, which requires that

$$\beta^{-\beta} (1-\beta)^{\beta} \left(\frac{1}{Q_l}\right) w_l^{\beta} Q_l^{1-\beta} \geq \beta^{-\beta} (1-\beta)^{\beta} \left(\frac{1}{\varepsilon Q_l}\right) w_l^{\beta} \left(\frac{\varepsilon Q_l}{(1-\beta)}\right)^{1-\beta}.$$

This simplifies to the condition $\varepsilon \geq (1-\beta)^{\frac{\beta-1}{\beta}}$. Thus, in order for monopoly pricing to prevail, quality improvements must be large enough for goods-producers to be willing to pay the higher price. If this condition does not hold, the monopoly-priced machine will be too expensive, and producers will opt for the older machines.

However, producers of newer machines can charge a price lower than this and still turn a profit. How low would they have to go to secure the market? They certainly could go no lower than εQ_l, which is their own marginal cost of machine production.

[9] If our production function is Cobb-Douglas of the form $q = \Phi X^a Y^b$, the general formula for the unit cost of producing q is $\left(\frac{a+b}{(\Phi a^a b^b)^{\frac{1}{a+b}}}\right) \left(w_x^a w_y^b\right)^{\frac{1}{a+b}}$, where w_x and w_y and unit costs for X and Y, respectfully. If we have constant returns to scale where $a = 1-b$, unit costs simplify further to $\left(\frac{1}{\Phi a^a (1-a)^{1-a}}\right) w_x^a w_y^b$.

Fortunately they would not have to go that low; they could charge a price p_{limit} low enough such that $uc_{old} \geq uc_{limit}$ (see Barro and Sala-i-Martin 2003 and Grossman and Helpman 1991 for similar limit-pricing treatments). That is, producers of new machines could undercut their competition so that goods-producers would prefer the higher-quality machines to the older lower-quality machines. And to maximize prices, new machines producers would charge a price such that this held with equality:

$$\beta^{-\beta}(1-\beta)^{\beta}\left(\frac{1}{Q_l}\right)w_l^{\beta}Q_l^{1-\beta} = \beta^{-\beta}(1-\beta)^{\beta}\left(\frac{1}{\varepsilon Q_l}\right)w_l^{\beta}p_{limit}^{1-\beta}.$$

Solving for this limit price gives us

$$p_{limit} = \varepsilon^{\frac{1}{1-\beta}}Q_l > \varepsilon Q_l.$$

Thus, producers will always opt for newer machines, no matter the size of quality steps. So our approach would be valid for any values of $\varepsilon > 1$ and $0 < \beta < 1$. Given our paramerization described in section 2.3, we assume this limit pricing strategy is used.

Finally, plugging in this limit price for the machine price in (10), and plugging in (10) for machine demands in our expression of $V_l(j) = p(j)M_l(j) - q(j)M_l(j)$, the current values for new blueprints (for unskilled and, analogously, skilled machines) can be written as

$$V_l(j) = V_l = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_l}{\beta A_l}\right)^{\frac{1}{\beta}} L;$$

$$V_h(j) = V_h = \left(1 - \frac{1}{\varepsilon^{\frac{1}{1-\beta}}}\right)\left(\frac{w_h}{\beta A_h}\right)^{\frac{1}{\beta}} H.$$