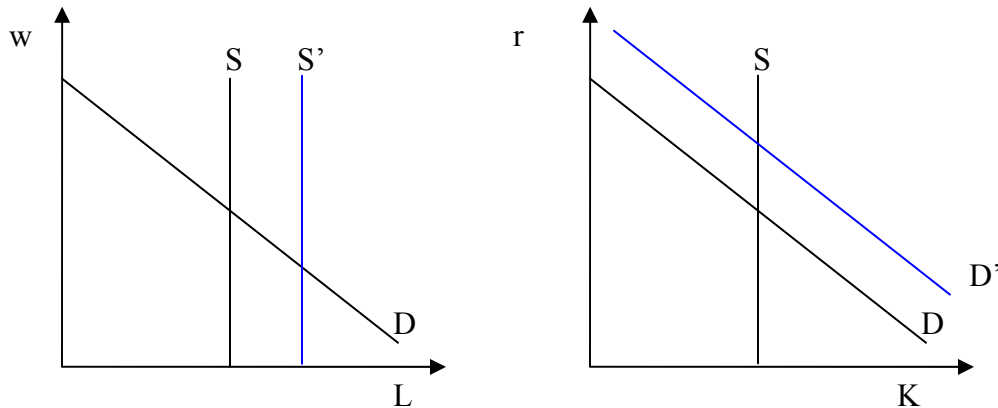


Some Answers

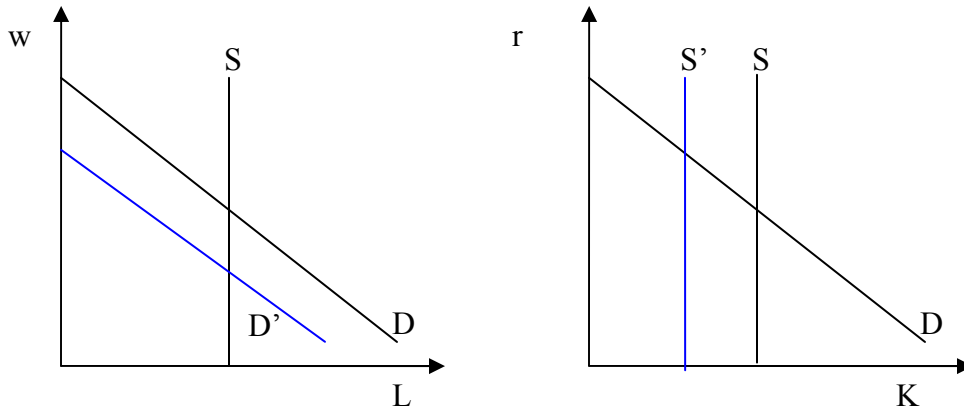
1) Use the neoclassical theory of distribution to predict the impact on the real wage and the real rental price of capital of each of the following events (illustrate graphically, using two graphs for each case, one for the labor market and one for the capital market, and explain in one or two sentences):

a. A wave of immigration increases the labor force.



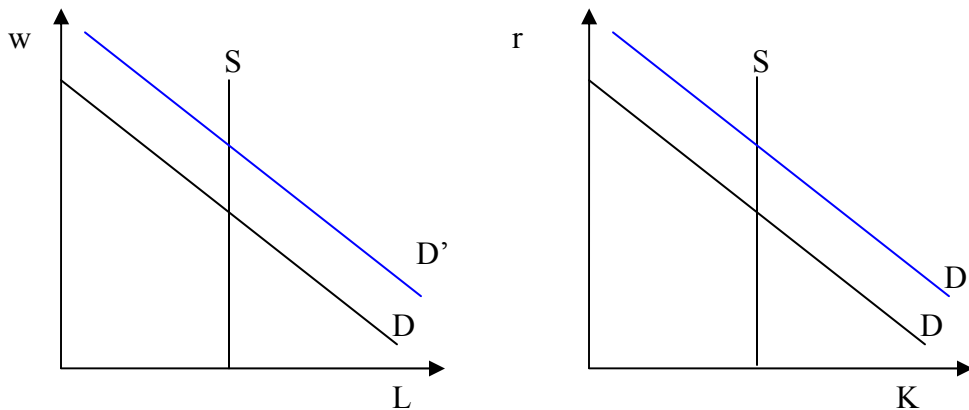
According to the neoclassical theory of distribution, the real wage equals the marginal product of labor. Because of diminishing returns to labor, an increase in the labor force causes the marginal product of labor to fall. Hence, the real wage falls. Because capital is now RELATIVELY more scarce, it becomes more productive – hence demand for capital by firms rises.

- b. An earthquake destroys some of the capital stock.



*The real rental price equals the marginal product of capital. If an earthquake destroys some of the capital stock (yet miraculously does not kill anyone and lower the labor force), the marginal product of capital rises and, hence, the real rental price rises. . Because labor is now **RELATIVELY** more abundant, it becomes less productive – hence demand for labor by firms rises.*

- c. A technological advance improves the production function.



If a technological advance improves the production function, this is likely to increase the marginal products of both capital and labor. Hence, the real wage and the real rental price both increase through increases in demand for both factors.

2) Suppose that the production function is Cobb-Douglas. That is, the production function is $Y = F(K, L) = AK^\alpha L^{1-\alpha}$.

a) Solve for the fraction of income paid to capital (show your work).

$MPK = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1} L^{1-\alpha}$. *Of course, $K \cdot MPK$ is the total income paid to capital. Thus the FRACTION of income paid to capital is simply $\frac{K \cdot MPK}{Y}$, which is simply α .*

b) Say $\alpha = 0.3$, and suppose that immigration raises the labor force by 10 percent. What happens to total output (in percentage change)? The real rental price of capital (in percentage change)? The real wage (in percentage change)?

$$Y_1 = AK^{0.3} L^{0.7}, \quad Y_2 = AK^{0.3} (1.1L)^{0.7}$$

$$Y_2 / Y_1 = (1.1)^{0.7} = 1.069$$

$$(R/P)_1 = MPK = 0.3AK^{-0.7} L^{0.7}, \quad (R/P)_2 = 0.3AK^{-0.7} (1.1L)^{0.7}$$

$$(R/P)_2 / (R/P)_1 = 1.069$$

$$(W/P)_1 = MPL = 0.7AK^{0.3} L^{-0.3}, \quad (W/P)_2 = 0.7AK^{0.3} (1.1L)^{-0.3}$$

$$(W/P)_2 / (W/P)_1 = 0.972$$

Thus we see that while output and the rental rate of capital each rise by 6.9%, the wage falls by 2.8%.

c) Say $\alpha = 0.3$, and suppose that a gift of capital from abroad raises the capital stock by 10 percent. What happens to total output (in percentage change)? The real rental price of capital (in percentage change)? The real wage (in percentage change)?

$$Y_1 = AK^{0.3}L^{0.7}, \quad Y_2 = A(1.1K^{0.3})L^{0.7}$$

$$Y_2 / Y_1 = (1.1)^{0.3} = 1.029$$

$$(R/P)_1 = MPK = 0.3AK^{-0.7}L^{0.7}, \quad (R/P)_2 = 0.3A(1.1K^{-0.7})L^{0.7}$$

$$(R/P)_2 / (R/P)_1 = 0.935$$

$$(W/P)_1 = MPL = 0.7AK^{0.3}L^{-0.3}, \quad (W/P)_2 = 0.7A(1.1K^{0.3})L^{-0.3}$$

$$(W/P)_2 / (W/P)_1 = 1.029$$

Thus output rises by 2.9%, rental rates fall by 6.5%, and wages rise by 2.9%

- d) Say $\alpha = 0.3$, and suppose that a technological advance raises the value of the parameter A by 10%. What happens to total output (in percentage change)? The real rental price of capital (in percentage change)? The real wage (in percentage change)?

Everything goes up by 10%.

- 3 a) Suppose an automobile manufacturer is choosing between two production options. It can produce 100 cars with 200 workers and 50 machines, or it can produce 166 cars with 300 workers and 75 machines. Would you describe the manufacturer's production function as exhibiting decreasing, constant, or increasing returns to scale? Explain.

The capital stock and the labor force increase by 50% and the output increases by 66%. This is increasing returns to scale.

- b) Regardless of what your answer was in part a, we can say that this manufacturer faces diminishing marginal productivity for both machines and workers. Explain how this is possible.

Increasing returns to scale refers to the change in output when both factors are increased in the same percentage. Diminishing marginal productivity describes the change in output when only one factor is altered.

4) Consider two competitive economies that have the same quantities of labor ($L = 400$) and capital ($K = 400$), and the same technology ($A = 100$). The economies of the countries are described by the following Cobb-Douglas production functions:

North Economy: $Y = AL^{0.3} K^{0.7}$

South Economy: $Y = AL^{0.7} K^{0.3}$

a) Which economy has the larger production? Explain.

Output is the same in both economies, given the symmetry of the parameters of the production functions and the equal quantities of labor and capital.

b) In which economy is the marginal product of labor larger? Explain.

The MPL is large in the South Economy. The MPL depends on the value of $(1-\alpha)$ and the average productivity of labor (Y/L). Since the average productivity of labor is the same in both countries, differences in the MPL depend on the value of $(1-\alpha)$, which is larger in the South.

c) In which economy is the real wage larger? Explain.

Since factors are paid according to the values of their marginal products, the real wage is larger in the South Economy, because the MPL is larger in the South.

d) In which economy is labor's share of income larger? Explain.

Labor's share of incomes equal $(1-\alpha)$, which is again larger in the South.