Some Answers

1) Assume that production is a function of capital and labor, and that the rate of savings and depreciation are constant, as described in Chapter 7's version of the Solow Model. Further, assume that the production function can be described by the function:

$$Y = K^{\left(\frac{1}{2}\right)} L^{\left(\frac{1}{2}\right)}$$

where K is capital and L is labor.

a. What is the per-worker production function $y=f(k)$? Show your work.

$$\frac{Y}{L} = \frac{K^{\left(\frac{1}{2}\right)} L^{\left(\frac{1}{2}\right)}}{L} = \frac{K^{\left(\frac{1}{2}\right)}}{L^{\left(\frac{1}{2}\right)}} \frac{L^{\left(\frac{1}{2}\right)}}{L^{\left(\frac{1}{2}\right)}} = \frac{K^{\left(\frac{1}{2}\right)}}{L^{\left(\frac{1}{2}\right)}} = k^{\left(\frac{1}{2}\right)}$$

b. Solve for steady-state capital per worker, production per worker, and consumption per worker with $s = 0.4$? (Note: you need to set $\Delta k = 0$, to get an equation in s, δ, and k, and then solve for k).

When k is in steady-state, $sk^{*0.5} = \delta k^*$ *This gives us* $k^* = \left(\frac{s}{\delta}\right)^2$, *which means that*

$y^* = \left(\frac{s}{\delta}\right) = \frac{0.4}{\delta}$. *Consumption per worker in steady-state then is just*

$c^* = (1-s)\left(\frac{s}{\delta}\right) = \frac{0.24}{\delta}$.

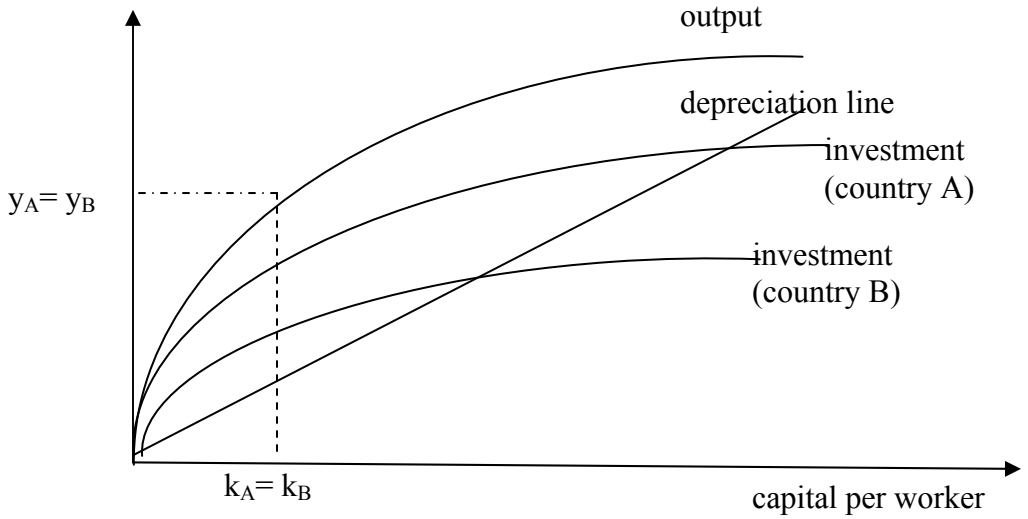
2) Assume once again that production is given by: $Y = K^{\left(\frac{1}{2}\right)} L^{\left(\frac{1}{2}\right)}$. First, write the production function in per person terms ($y=f(k)$). Next, assume that the per person level of capital in the steady state is 4, the depreciation rate is 5% per year, and population growth is 5% per year. Does this economy have "too much" or "too little" capital? How do you know? [Show your work].

Recall that when the economy is at the golden rule steady-state, $MPK^* = \delta . + n$ Given that $f(k) = k^{1/2}$, this means that $(1/2) * k_{G.R.}{}^{*-1/2} = \delta + n$. Using the numbers given to us above, our solution gives us $k_{G.R.}{}^* = 25$. Given that our steady state capital stock is at 4, it means that we have WAY too little capital (at least compared with the golden rule level).

3) Suppose that two countries are exactly alike in every respect (meaning they have the same levels of capital, output, depreciation, etc.) except that the citizens of country A have a higher saving rate than the citizens of country B.

a. Which country will have the higher level of output per worker in the steady state? Illustrate graphically.

b. Which country will have the faster rate of growth of output per worker?

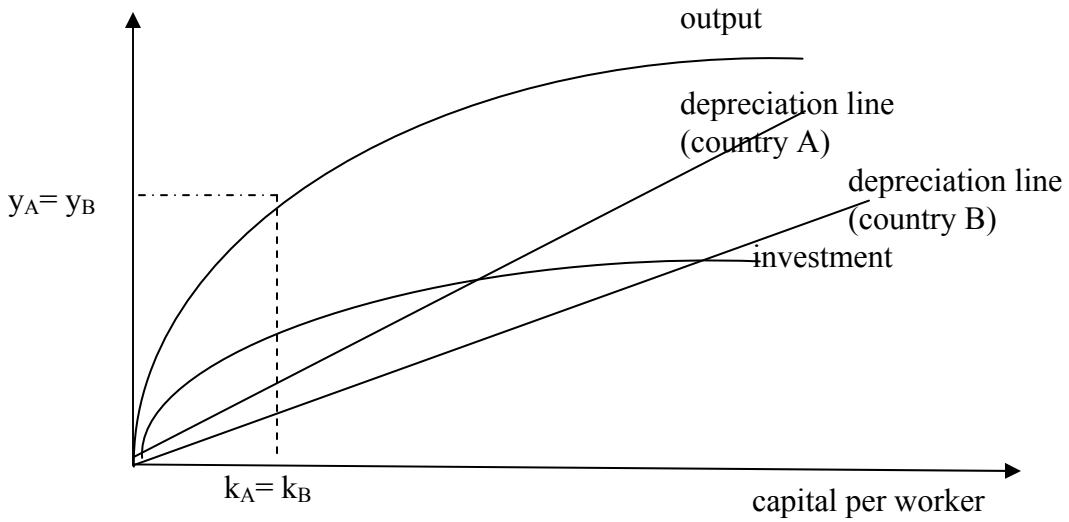


While each country starts with the same amount of capital per worker (and therefore starts with the same output per worker), we can see that country A has the higher steady state. Notice also that at this initial position, $\Delta k_A > \Delta k_B$. Therefore country A will grow faster as well.

4) Suppose that two countries are exactly alike in every respect (meaning they have the same levels of capital, output, depreciation, etc.) except that population grows at a faster rate in country A than in country B.

a. Which country will have the higher level of output per worker in the steady state? Illustrate graphically.

b. Which country will have the faster rate of growth of output per worker?



While each country starts with the same amount of capital per worker (and therefore starts with the same output per worker), we can see that country B has the higher steady state. Notice also that at this initial position, $\Delta k_B > \Delta k_A$. *Therefore country B will grow faster as well.*

5) The initial steady-state level of capital per worker in Macroland is 5. The Golden Rule level of capital per worker in Macroland is 8.

a. What must change in Macroland to achieve the Golden Rule steady state?

Macroland clearly needs more capital per person. It can get this by increasing its rate of savings.

b. Why might the Golden Rule steady state be preferred to the initial steady state? (two or three sentences)

Golden rule means that long run consumption is maximized. Since conceivably we care not only about our consumption, but also the consumption of our children, their children, and so on, that seems like a good thing.

c. Why might some current workers in Macroland prefer the initial steady state to the Golden Rule steady state? (two or three sentences)

In order to get to the golden rule, we will have to cut our consumption levels NOW. Some workers might view the short-run costs as bigger than the long-run gains, and therefore opt not to increase savings.

By the way, another approach to get closer to the golden rule would be to lower population growth (provided n > 0). To the extent that society can effectively influence rates of growth in population, what are the costs and benefits of doing so? Think about it.

6) The economy of Alpha can be described by the Solow growth model. The following are some characteristics of the Alpha economy:

savings rate (s)	0.20
depreciation rate (δ)	0.12
steady-state capital per worker (k^*)	4
population growth rate (n)	0.02
steady-state output per worker	20,000

a. What is the steady-state growth rate of output per worker in Alpha?

In the steady state, capital per worker is constant, so output per worker is constant. Thus, the growth rate of steady-state output per worker is 0.

b. What is the steady-state growth rate of total output in Alpha?

In the steady state, population grows at 2 percent (0.02). Capital must grow at a rate of 2 percent in order to maintain a constant capital per worker ratio in the steady state; therefore, given the constant returns to scale production function, total output must increase at a 2 percent rate.

c. What is the level of steady-state consumption per worker in Alpha?

If the saving rate is 20 percent, then the consumption rate is 80 percent. Steady-state consumption per worker is 16,000.

d. What is the steady-state level of investment per worker in Alpha?

4,000.