

Practice Problems for FE 486B – Thursday, February 2, 2012

1) *Suppose you win the lottery. You have a choice between receiving \$100,000 a year for twenty years or an immediate payment of \$1,200,000.*

a) *Which choice should you make if the interest rate is 3 percent? If it is 6 percent?*

For both interest rates you need to figure out the present value of the 20 annual payments of \$100,000. Assuming that each annual payment is received at the end of the year (so that the first payment is received after one year), the equation for the discounted present value of the 20 annual payments reads: Present value = $\$100,000/(1 + i) + \$100,000/(1 + i)^2 + \$100,000/(1 + i)^3 + \dots + \$100,000/(1 + i)^{20}$.

For the 3 percent interest rate ($i = 0.03$), the present value is: $PV = 1,487,747.49$.

For the 6 percent interest rate ($i = 0.06$), the present value is: $PV = 1,146,992.12$.

Note that the present value of fixed future cash flows is lower the higher the interest rate. With an interest rate of 3 percent, you should take the 20 annual payments of \$100,000. With an interest rate of 6 percent, you are better off taking the immediate payment of \$1,200,000.

b) *For which range of interest rates should you take the immediate payment?*

From your answer in part (a), it is clear that you should take immediate payment for any interest rate above 6 percent. Even with an interest rate just below 6 percent (e.g., 5.8 percent), you will still be better off in present value terms with the immediate payment. In fact, you can solve for the interest rate at which the present value of the 20 annual payments exactly equals \$1,200,000. This is difficult without a financial calculator, but you can check that an interest rate of 5.45 percent would make you indifferent between the two options. You should take immediate payment for any interest rate above 5.45 percent.

2) *Describe how (in some detail) each of the following events affects stock prices and bond prices (explain the different effects between the two) :*

a) *The economy enters a recession.*

Often during recessions, interest rates tend to decrease. This would increase both stock and bond prices positively. However, company's earnings will fall as well. Lower interest rates and lower earnings impact stock prices in opposite directions. The earnings impact usually outweighs the interest rate impact on stock prices during a recession.

b) *A genius invents a new technology that makes factories more productive.*

Future expected earnings of the company will increase, likely increasing the dividend paid to the stockholders. This increases the value of the company's stock. Bond prices are not affected because the coupon payments are not affected.

c) *The Federal Reserve raises its target for interest rates.*

Higher interest rates reduce both stock and bond prices.

d) *People learn that major news about the economy will be announced in a few days, but they don't know whether it is good news or bad news.*

This event increases uncertainty and can be thought of as increasing the risk premium that gets added to the safe interest rate to determine the present value of future income. With a higher interest rate, future asset income gets discounted at a higher rate and both stock and bond prices will fall.

3) *Consider two bonds. Each has a face value of \$100 and matures in ten years. One has no coupon payments, and the other pays \$10 per year.*

a) *Calculate the price of each bond if the interest rate is 3 percent and if the interest rate is 6 percent.*

No coupon bond:

$$\text{Price of bond} = \$100/(1 + 0.03)^{10} = \$74.63$$

$$\text{Price of bond} = \$100/(1 + 0.06)^{10} = \$55.87$$

Coupon bond:

$$\text{Price of bond} = \$10/(1 + 0.03) + \$10/(1 + 0.03)^2 + \dots + \$110/(1 + 0.03)^{10} = \$159.70$$

$$\text{Price of bond} = \$10/(1 + 0.06) + \$10/(1 + 0.06)^2 + \dots + \$110/(1 + 0.06)^{10} = \$129.43$$

b) *When the interest rate rises from 3 percent to 6 percent, which bond price falls by a larger percentage? Explain why.*

$$\text{Percentage change (no coupon)} = (55.87 - 74.63)/74.63 = -0.25 = -25\%$$

Percentage change (coupon) = $(129.43 - 159.70)/159.70 = -0.19 = -19\%$

The drop in the no-coupon bond is larger in percentage terms. On average, the coupon bond does not get discounted as heavily as the no-coupon bond when interest rates increase.

4) Consider two stocks. For each, the expected dividend next year is \$100 and the expected growth rate of dividends is 3 percent. The risk premium is 3 percent for one stock and 8 percent for the other. The economy's safe interest rate is 5 percent.

a) What does the difference in risk premiums tell us about the dividends from each stock?

The stock with the higher risk premium has dividends that are more variable. Just because the expected dividend for each stock is the same, this does not mean that the variance on the dividend return is the same.

b) Use the Gordon growth model to compute the price of each stock. Why is one price higher than the other?

The Gordon growth model says that the price of the stock (P) can be calculated as follows:

$$P = D / (i - g)$$

where D is the expected dividend, i is the risk-adjusted interest rate, and g is the expected growth rate of the dividend. The two stock prices are:

Price of stock with 3% risk premium = $\$100 / (0.05 + 0.03 - 0.03) = \2000 .

Price of stock with 8% risk premium = $\$100 / (0.05 + 0.08 - 0.03) = \1000 .

The stock that is less risky has a higher price due to the fact that it yields a more certain return. In addition, note that the 3% growth rate of the dividend exactly offsets the 3% risk premium, resulting in a fairly certain 5% return.

5) Suppose it is 2020 and the one-year interest rate is 4 percent. The expected one-year rates in the following four years (2021 to 2024) are 4 percent, 5 percent, 6 percent and 6 percent.

a) Assume the expectations theory of the term structure, with no term premium. Compute the interest rates in 2020 on bonds with maturities for one, two, three, four and five years. Draw a yield curve.

The interest rates are calculated as follows:

1-year rate in 2020: 4%

2-year rate in 2020: $(4\% + 4\%)/2 = 4\%$

3-year rate in 2020: $(4\% + 4\% + 5\%)/3 = 4.33\%$

4-year rate in 2020: $(4\% + 4\% + 5\% + 6\%)/4 = 4.75\%$

5-year rate in 2020: $(4\% + 4\% + 5\% + 6\% + 6\%)/5 = 5\%$

Drawing the yield curve will show a flat yield curve for the first two years. After year 2, the yield curve is upward sloping.

b) *Redo part (a) with term premiums. Assume the term premium for an n -year bond, τ_n , is $(n/2)$ percent. For example, the premium for a four-year bond is $(4/2)\% = 2\%$.*

The interested rates are calculated as follows:

1-year rate in 2020: $4\% + (1/2)\% = 4.5\%$

2-year rate in 2020: $(4\% + 4\%)/2 + (2/2)\% = 5\%$

3-year rate in 2020: $(4\% + 4\% + 5\%)/3 + (3/2)\% = 5.83\%$

4-year rate in 2020: $(4\% + 4\% + 5\% + 6\%)/4 + (4/2)\% = 6.75\%$

5-year rate in 2020: $(4\% + 4\% + 5\% + 6\% + 6\%)/5 + (5/2)\% = 7.5\%$

Drawing the yield curve will show an upward-sloping yield curve throughout. Adding term premiums results in a steeper yield curve.

6) *Explain what happens in an asset-price bubble. Explain what happens in an asset-price crash.*

In an asset price bubble, something else other than a change in the interest rate or a change in the expected income of the asset causes the asset price to rise. In most cases, the asset price rises because people expect it to rise. If, for example, we all expect the price of stock A to rise, then there is an increase in the demand for stock A and this causes the price of stock A to actually rise. It does not matter if the expectation was justified or not as long as we all believe it.

A bubble forms when the increase in the price of the asset causes further increases in demand for the asset and further increases in the price of the asset.

Inevitably, the price rises so high that people fear it cannot continue and demand for the asset falls. As demand falls, the price will fall; this leads to an asset-price crash. Eventually the price of the asset returns to what the classical theory would predict.

7) *Which asset prices are most volatile? Why?*

Long-term bond prices are more volatile than short-term bond prices because changes in the interest rate will have a larger impact on payments that occur further into the future. Since the expected income flow of the bond is known at the time of purchase (barring default), only changes in the interest rate will change the present value of the expected income of the bond.

Stock prices are more volatile than bond prices, long or short term, because the stock price depends not only on the interest rate but also on expected income.