

A Simple Theory on the Effects of Industrialization*

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Abstract

Historically, industrialization has been associated with falling relative returns to skills. This fact is at odds with most unified theories of industrialization, which tend to imply rising skill premia as natural concomitants to economic growth. This paper develops a very simple model of historical growth to help solve this puzzle. Assuming that human capital is both a consumption good and an investment good, the model demonstrates how gradually rising investments in human capital, non-monotonic fertility rates, and falling skill premia can all be explained within one theory.

- *Keywords:* endogenous demography, skill premium, unified growth theory
- *JEL Codes:* J11, J31, N33, O33, O41

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1 Introduction

World economic history can be told simply - for the last few hundred years technological progress has allowed some societies to lift themselves out of their timeworn Malthusian-based poverty (Clark 2007). Yet while this story is simple, the challenge to theoretically explain it has been anything but. One major difficulty has been to capture the *effects* of these technological changes. In particular, many features of growth today are very different from those at the dawn of robust technological progress. The world's first foray into modernity two and a half centuries ago saw exploding populations, stagnant rates of education and only modest increases in income. A century later growth was associated with falling fertility rates and rapid rises in both education and average incomes. Furthermore the relative returns to human capital appear to have fallen through all these developments. Traditional growth theories, able to capture present-time growth, are nevertheless consistent only with a small fragment of human history.

This paper attempts to link different periods of development by employing a very simple model of growth and demography. Producing "unified" theories of growth has been the *cri de coeur* of Oded Galor, who entreats growth economists to use micro-founded theories to capture various stages of development, not merely one episode of it (Galor 2005, 2009, 2011a). This creates a formidable but important intellectual challenge - constraining oneself to a single theory to account for various stages of growth will arguably enhance the viability of growth theory overall.

An even greater challenge is to link different episodes of industrialization in different economies. First the U.K. and then the U.S. led the world in labor productivity from the mid-eighteenth to mid-twentieth centuries (Broadberry 1994), yet few theories can reasonably capture the growth stories of both. In particular, the industrialization histories of these and many other economies show falling relative returns to skilled labor (at least until the latter half of the 20th century). Yet most unified theories of industrialization imply that technological transition tends to raise the returns to human capital (see for example Galor and Weil 2000, 2002; Jones 2001; Hansen and Prescott 2002; Lucas 2002). And theories which do model human capital increases that would be consistent with falling skill premia fail to account for other important features of historic industrialization.¹

Specifically, we develop an overlapping-generations model where households choose the number of their children and their levels of education. To this we add two simple and plausible assumptions. One is that skilled and unskilled labor are grossly *substitutable* in production (so

¹For example, seminal papers such as Cervellati and Sunde (2005) and Galor and Moav (2006) focus on human capital formation but do not model fertility. Other important works dealing with human capital in historical development include Boucekkine et al. (2002), which does not model fertility as an optimized decision by individual households, and Cervellati and Sunde (2009), which does not model the non-monotonicity of fertility, only its decline during the Demographic Transition.

that increases in either can produce large aggregate income gains). The other is that income and human capital are grossly *complementary* in household utility (so that human capital can be considered a consumption good as well as an investment good). With these assumptions, we demonstrate how technological progress can produce the characteristics of historical growth in Western societies (roughly from the early eighteenth century to the early twentieth century) in one parsimonious theory - gradual increases in education, non-monotonic changes in fertility, and declines in skill premia.

We hasten to mention that we are only studying the consequences of technological progress, not the sources.² Nor are we attempting to account for the timing of industrialization, or motivate the reasons why it happened in England, and not China, India or Japan. For these reasons we treat technological advances as exogenous.³ Thus this is not precisely a unified theory of growth, but rather a theory that unifies certain aspects of growth in history. It does however investigate the *deus ex machina* of both the Industrial Revolution and the Demographic Transition simultaneously by suggesting that the balance between education and income in preferences can explain key features of industrialization. This should help us reinterpret certain aspects of existing unified growth theories as well as help motivate new ones.

The rest of the paper proceeds as follows. Section 2 enumerates some of the “stylized facts” of industrialization in history. Section 3 goes through the model of technology and demography. Section 4 simulates the model to see if it can adequately capture the major qualitative aspects highlighted in section 2. Section 5 concludes.

2 The Facts to Fit

Galor (2005,2011a) asks some of the fundamental questions that unified growth theories aim to answer, one of the most important being “What are the underlying behavioral and technological structures that can simultaneously account for these distinct phases of development?” In this paper we impose a certain behavioral structure, motivated by available historical evidence, that uniquely accounts for these phenomena. The phenomena we wish to address constitute a new set of “stylized facts” that beg explanation by unified growth theories on industrialization in England and other regions.

²See however O’Rourke et al. (2008) for a recent study that endogenizes technological growth and demography in history. See also Heijdra and Ligthart (2006) and Rahman (forthcoming) for potential long-run growth effects of fertility declines.

³Mokyr and Voth (2010) draws from Isaiah Berlin’s “The Hedgehog and the Fox” to distinguish between two kinds of researchers - “hedgehogs” looking for a single Theory of Everything, and “foxes” looking for solutions to smaller problems. In keeping certain things exogenous in order to explain some other large phenomena, this paper strives to merge the grand vision of the hedgehog with the talents of the fox.

2.1 The Rise and Fall of Fertility

The first challenge for a model of the Industrial Revolution is to account for its apparent *un*-revolutionary beginnings, for personal income increases were very minor in the early stages of industrialization. One major reason for this was the link between productivity and population growth during the years 1750-1850 (often called the “first Industrial Revolution”). The English population for example rose from six million in the 1740s (roughly the maximum attained throughout the previous millennium) to twenty million by the 1860s. Most of this population increase came from increases in fertility, as mortality declined very modestly during the first Industrial Revolution (falling only to where it had been during the mid-seventeenth century). Crude birth and death rates for England are depicted in Figure 1.

The relationship between income per capita and population growth however evolved non-monotonically, not just in England but in Germany, Sweden, Finland, the Netherlands, and Belgium (Mokyr and Voth 2010). While the first Industrial Revolution witnessed a dramatic increase in birth rates along with increases in per capita incomes, this pattern reversed during the “second Industrial Revolution” (roughly 1850-1910), where further per capita income gains accompanied rapidly falling birth rates (see Figure 1).

For the United States, Jones and Tertilt (2006) document both the sharp fall in fertility throughout most of the 20th century as well as the strong negative relationship between income and fertility for all cohorts. Thus it appears that 20th century American growth continued the late 19th century European trend of general demographic transition (see Figure 3).

2.2 The Role of Human Capital Investment

One of the most difficult challenges for unified growth theory is in attempting to explain the minimal role played by human capital investments during the early stages of industrialization. While most theories of industrialization imply that rising investments in labor quality and growth in living standards go hand in hand,⁴ the first Industrial Revolution appears to be compatible with fairly flat rates of human capital investment. There is much evidence in fact that suggests parents neither spent a lot more time nor invested many more resources on their children during early growth (Clark 2007, Mokyr and Voth 2010). For example, David Mitch suggests that in key expanding sectors of the British economy, such as cotton textiles, rates of education were outright declining. For the general economy, elementary school enrollment figures based on parochial surveys between 1818 and 1833 show enrollment perfectly steady at 42 percent (Mitch 1982). Flora et al. (1983) documents that merely 11 percent of English children aged 5-14 were enrolled in primary school in 1855; obviously education could not have risen much if they were so low a full century after the start of industrialization.

⁴See for example Becker et al. (1990), Hansen and Prescott (2002), and Lucas (2002).

Yet at the same time, many scholars have suggested that human capital itself (the *outcome* of these investments) did notably rise during this period. In fact some forms of human capital were rising even before the Industrial Revolution, due in part to slowly rising incomes (Mokyr and Voth 2010). The two best measures we have for human capital formation in history are literacy and numeracy. West (1978) shows clear improvements in literacy after 1760.⁵ Schofield (1973) argues that early improvements in literacy in England occurred despite no noticeable increases in formal education. Numeracy also seems to have improved from the late 18th century to the mid 19th century throughout most of Europe and the U.S. (A’Hearn et al. 2009). This leaves us with another related puzzle - how could early industrialization have caused increases both in the quantity and quality of people, but very little increases in formal investments in children?⁶

This makes the contrast between the first and second Industrial Revolutions all the more striking, for investments in human capital grew rapidly only later on. For example, Flora et al. (1983) documents the explosion of education from 11 percent in 1855 to 74 percent by the turn of the century, with other European countries experiencing similar rises in education rates (see Figure 2). And America’s growth story of the 20th century only reaffirms the importance of human capital in the post mid-19th century world. Goldin (1999) documents the prominent role education played in the U.S. economy during the 20th century (see Figure 3). At the start of the century very few people could afford to attend school; by the end of the century very few could afford *not* to attend school. This dramatic transformation in America’s focus on education leads Goldin and Katz (2007, 2008) to dub the 20th century the “human capital century.” Any theory attempting to unify the stages of growth has to account for this transformation of education’s immaterial role in the eighteenth century to its apparent indispensability in the twentieth.

2.3 Inequality and the Skill Premium

Based on the evidence on education, one would perhaps suspect that earnings for educated people were quite low during England’s first Industrial Revolution and at the dawn of the American 20th century, thus inducing families to keep their children uneducated. Higher relative earnings for educated children would induce parents to provide their children a formal schooling (Clark 2007). But the evidence suggests just the opposite - from 1700 up to the Great War, the premium on education in England was at its peak *before* industrialization and modernization ever happened. Similarly in the United States, the relative return of a high school diploma was at its peak at the turn of the *20th* century (the earliest period consistent measures are available),

⁵“It is generally agreed by all participants that people were more literate at the end of the Industrial Revolution period, 1760-1840, than they were at the beginning” (West 1978).

⁶A broader measure of human capital would include health. The evidence suggests that during early industrialization England had a much more productive labor force compared to France due to better health, with low levels of caloric intake in *both* regions compared to today (see Fogel 1991, Kelly et al. 2011).

not the 21st.

As can be seen in Figure 4, the skill premia generally fell during both of England's Industrial Revolutions. Williamson (1982) produces a variety of skill premia for different classifications of workers - here we see that even when farmers are included as unskilled laborers (a group whose wages were relatively stagnant in the first half of the 19th century), skill premia do not reach their 1755 highs even a century later. These figures also indicate falling premia between 1781 and 1815, right during the heart of the Industrial Revolution. And when farmers are excluded from the sample, skill premia remain low right through the mid-19th century. Mitch (1999) reaffirms this, showing that premia were either stagnant or declining during this time. And Clark (2005) illustrates that premia continue to fall even during the second Industrial Revolution, a time that many suggest was when human capital played a critical role in production.⁷

As for the United States, an extensive literature exists on the pre-Second World War wage premium for skilled to manual workers. Almost all papers in this literature find a compression of the wage structure before 1950.⁸ Goldin and Katz (2008) suggest a general pattern of falling premia prior to 1950, relatively stagnant premia from 1950 to 1980, and rising premia only after 1980 (see Figure 5).

Thus historically industrialization appears to be conducive to falling returns to skilled labor relative to the returns to unskilled labor. This is also true when looking across countries at different *stages* of industrialization (see Figure 5). There are still other sources of evidence. For example, there is no sign that the rewards to numeracy and literacy were any higher in England in 1800 than they were in 1200. The premium for these and other skills in the labor market seems to have outright declined through the Industrial Revolution. There is simply no evidence of any market signal to parents that they needed to invest more in the education or training of their children during any part of the 19th century (Clark 2005, 2007).

A rather formidable puzzle emerges when attempting to reconcile all these pieces of industrialization. Although human capital did increase during the early stages of industrialization, and it is often center stage in any story of modernization, we see a very poor match between the elements that enter into a true human capital story of early growth: the role of education, the average size of families, and the premium paid in the labor market for skills. Explaining the evolution of the relative returns to skilled labor is particularly challenging. Why does the skill premium fall with an economy's launch into sustainable economic growth?

⁷Feinstein (1978) does suggest that premia in England likely remained flat through the entire course of industrialization.

⁸A partial list of this literature includes Bell (1951), Keat (1960), Lebergott (1947), Ober (1948), Woytinsky (1953), and Lindert and Williamson (1980).

3 A Simple Model of Industrialization

Here we offer a theoretical solution to the above-mentioned puzzles. Two general assumptions are necessary to achieve this. First, final output is produced both by skilled and unskilled labor. The second is that households derive benefits from both income (generated from both skilled and unskilled labor) and human capital (generated from the education obtained by children). The next sections make specific some of these ideas, and simulate an economy to replicate the key features of Western industrialization.

3.1 Production

Total production in the economy combines the efforts of both unskilled and skilled labor. These labor-types are imperfectly substitutable; thus we assume that aggregate production can be described by the CES production function:

$$Y = A_t \left(L_t^{\frac{\sigma-1}{\sigma}} + H_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

where L_t , H_t , and A_t are respectively unskilled labor, skilled labor and the Hicks-neutral technology level at time t . Following many labor studies on the elasticity of substitution between skilled and unskilled labor, we will assume that $\sigma > 1$, so that these factors are grossly substitutable in aggregate production.⁹

Factors are paid their marginal products in competitive markets. Assuming this, we derive unskilled and skilled wages, w_l and w_h , as

$$w_{l,t} = A_t \left(L_t^{\frac{\sigma-1}{\sigma}} + H_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} L_t^{-\frac{1}{\sigma}} \quad (2)$$

$$w_{h,t} = A_t \left(L_t^{\frac{\sigma-1}{\sigma}} + H_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} H_t^{-\frac{1}{\sigma}} \quad (3)$$

Note that here *relative* factor payments are simply inversely related to relative factors (that is, technology is not *biased* in any way towards any particular factor, so technology does not directly influence relative factor payments).

3.2 Endogenizing Demography

Here we attempt to model stable family preferences that are consistent with the evolving patterns of demographic behavior observed in history. We will assume that agents care both about their current consumption of the final good, and the level of human capital of their

⁹See for example Katz and Murphy (1992), Autor et al. (1998), Heckman et al. (1998), Ciconne and Peri (2005), and Autor et al. (2009).

children. We assume an over-lapping generations framework with two stages of life - young and old. Young individuals work strictly as unskilled workers, but also accumulate human capital. Old individuals work as both skilled and unskilled laborers, and have children of their own. Only old individuals make decisions regarding demography. Specifically, the representative household is run by an old person who decides two things: how many children she wishes to have (denoted n_t) and the human capital investment each child will receive (denoted e_t).

Our modeling of demography is as follows. An individual born at time t spends fraction e_t of her time in school (something chosen by her parent), while devoting the rest of her time as an unskilled laborer in the unskilled sector. At $t + 1$, the individual (who by this time is mature) provides one unit of labor inelastically in the unskilled sector, and uses whatever human capital she accumulated as a child in the skilled sector. After incurring the resource costs of child-rearing, the adult consumes all the income she and her family have generated. After this the individual dies.

Given this, we specify a household objective function. We assume that agents care about both their income and their children's future level of human capital, and that these two things are imperfectly substitutable.¹⁰

The utility of the parent U is given by

$$U_t = (\lambda (I_{p,t} + I_{c,t} - C_t)^\varepsilon + (1 - \lambda) (h_{t+1})^\varepsilon)^{1/\varepsilon} \quad (4)$$

where I_p is the income generated by the parent, I_c is the income generated by her children, C is the opportunity cost associated with child-rearing, and h is the average human capital endowed to each child. These variables are functions of fertility and education choices made by the parent, as well as functions of wages paid to skilled and unskilled labor. Specifically, we assume that $\frac{\partial I_{c,t}}{\partial n_t} > 0$, $\frac{\partial I_{c,t}}{\partial e_t} < 0$, $\frac{\partial I_{p,t}}{\partial e_{t-1}} > 0$, $\frac{\partial C_t}{\partial n_t} > 0$, $\frac{\partial C_t}{\partial e_t} > 0$, and $\frac{\partial h_{t+1}}{\partial e_t} > 0$. That is, increasing fertility will raise the income produced by children and raise the costs of child-rearing; increasing education on the other hand will raise bequests of human capital to children and raise the costs of child-rearing yet at the same will lower income from children (essentially pulling children out of work and into school).¹¹

Thus we are treating human capital as both an *investment* good (as adults rely on their education to generate greater returns to their labor) and a *consumption* good (as families derive

¹⁰Utility based on the education of children need not solely be motivated by altruism. For example, educated children may produce a pleasant and stimulating living environment, or may produce a source of retirement income. Acemoglu, in his recent book *Introduction to Modern Economic Growth*, would call this an example of “impure altruism,” or “warm glow preferences.” “Warm glow preferences assume that parents derive utility not from the future utility of their offspring, but from some characteristic of their offspring.” (Acemoglu 2008, pg 840). A sample of other works which consider human capital directly entering utility include Michael (1973), Schaafsma (1976), Kodde and Ritzen (1984), Jacob and Lefgren (2007), and Alstadsaeter and Sievertsen (2009).

¹¹In order to produce an interior solution, we also require that $\frac{\partial^2 C_t}{\partial n_t^2} > 0$, $\frac{\partial^2 C_t}{\partial e_t^2} > 0$, and $\frac{\partial^2 h_{t+1}}{\partial e_t^2} < 0$.

benefits from educated children unrelated to family income). Goldin (1999) summarizes the complex nature of education this way: “Education directly enhances productivity, and thus the incomes of those who receive schooling, by providing individuals with useful skills....Schooling is also a pure consumption good, enabling people to better understand and enjoy their surroundings.”¹² The approach taken here is simple enough to be incorporated tractably into an inter-generational setup, yet still able to highlight this multi-dimensional nature of education.

The parent will maximize this expression with respect to fertility and education. The first order condition with respect to fertility is simply

$$\frac{\partial C_t}{\partial n_t} = \frac{\partial I_{c,t}}{\partial n_t} \quad (5)$$

This states that the marginal cost of an additional child (in the form of higher child-rearing costs) must equal the marginal benefit of an additional child (in the form of greater unskilled-labor income). Note that because fertility is only in the first term of equation (4), the first order condition for fertility is simple and takes no account of education levels.

The first order condition for education on the other hand is slightly more involved:

$$\frac{\partial C_t}{\partial e_t} - \frac{\partial I_{c,t}}{\partial e_t} = \left(\frac{1 - \lambda}{\lambda} \right) \left(\frac{I_{p,t} + I_{c,t} - C_t}{h_{t+1}} \right)^{1-\varepsilon} \frac{\partial h_{t+1}}{\partial e_t} \quad (6)$$

The left hand side is the marginal cost of education. This cost arises from two sources - increasing the level of education per child raises the opportunity cost of child-rearing ($\frac{\partial C}{\partial e}$) and lowers the income generated from unskilled-child labor ($\frac{\partial I_c}{\partial e}$, which is negative). The right hand side is the marginal benefit of education. Education raises the level of human capital per child, which is a positive input in the parent’s welfare function. Notice however that these gains from education are augmented by the term $\left(\frac{I_{p,t} + I_{c,t} - C_t}{h_{t+1}} \right)^{1-\varepsilon}$ - which captures the importance of *balance* between total income and average human capital per child. The greater is net household income relative to human capital per child, the greater are the marginal benefits from additional education. The exponent $1 - \varepsilon$ magnifies this effect - the smaller is ε (that is, the more complementary are net income and human capital per child), the greater are the net benefits from education when net income is large relative to human capital. Indeed, these income-human capital complementarities will be a key feature that drives human capital accumulation throughout the Industrial Revolution.

Finally, income levels change with wage changes. These wage shocks arise from technological developments and are exogenous to the family planner.

¹²Other scholars have suggested that historically status also played a role - “Nothing could show more clearly that the economic status of the parents was the determinant of schooling in early modern England than the hierarchical distribution of signature literacy - for to send a child to school long enough to be able to write correlated with a degree of prosperity” (Cressy 1980).

In order to simulate the economy we will need to explicitly state how the variables in the welfare function ($I_{p,t}$, $I_{c,t}$, C_t , and h_{t+1}) are functions of both control variables (n_t and e_t) and current wages ($w_{l,t}$ and $w_{h,t}$). We specify the following simple forms:

$$I_{p,t} = w_{l,t} + w_{h,t}h_t \quad (7)$$

$$I_{c,t} = w_{l,t}(1 - e_t)n_t \quad (8)$$

$$C_t = w_{h,t}\phi n_t^\gamma (1 + e_t)^\gamma \quad (9)$$

$$L_t = pop_t + n_t(1 - e_t)pop_t \quad (10)$$

$$H_t = h_t pop_t = e_{t-1}^\alpha pop_t \quad (11)$$

$$pop_{t+1} = n_t pop_t \quad (12)$$

where $0 < \alpha < 1$, $\gamma > 1$ and pop is the adult population. These functional forms and parameter restrictions satisfy our first and second derivative conditions. (7) - (9) respectively depict the unskilled and skilled income generated by the parent, the unskilled income generated by children, and forgone skilled income due to child rearing.¹³ The functional form of (9) simply reflects the fairly standard idea that child-rearing costs are convex in both the number of children and the level of education per child.¹⁴ (10) and (11) illustrate how fertility and education translate into unskilled and skilled factors of production (note that L includes both adults and children, while H includes only adults). Here we have a simple production function for human capital that increases in education but experiences diminishing returns, and costs of child rearing (in the form of foregone skilled income for the parent) that rise in both fertility and education. Further, notice that (8) captures our fertility-education tradeoff mentioned above - more education, while increasing h_{t+1} , will necessarily decrease $I_{c,t}$. So long as these relationships are true our results will hold, so the qualitative conclusions we get will not be sensitive to the precise forms or parameter values in (7) - (11). Finally, (12) shows the law of motion of the adult population, dictated by the fertility rate.

¹³This implicitly assumes that adults pay a fraction of their *skilled* earnings for child-rearing. This is done only for convenience - we could alternatively have $C_t = \tilde{w}_t \phi n_t^\gamma (1 + e_t)^\gamma$, where \tilde{w}_t is some weighted average of $w_{l,t}$ and $w_{h,t}$. None of the qualitative results are affected by such a change.

¹⁴The idea goes back at least to Becker (1960, 1981), who suggests that the interaction between quality and quantity of children implies that costs are convex in each.

4 Simulating the Past

We now want to see if this simple model can replicate the key features of historic industrialization, including declining skill premia. Note that since technological growth is Hicks-neutral, the skill premium can fall only if $\frac{H}{L}$ rises. Given (5) and (6), we can write this condition as:

$$\partial H > \partial L \quad \Rightarrow \quad \alpha e^{\alpha-1} \partial e > (1-e) \partial n - n \partial e \quad (13)$$

where ∂H and ∂L are the total differentiations of skilled and unskilled labor. We know that in early stages of growth $\partial n > 0$ and ∂e is small but also greater than zero. Thus in order for (13) to hold, the rate of return to education in terms of human capital (the left hand expression) must be fairly large. On the other hand, later stages of growth typically display a classic quality-quantity tradeoff with child-rearing, such that $\partial n < 0$ and $\partial e > 0$. (13) should hold for this case as well.

With exogenous growth in A , steady-state is achieved when $e_{t-1} = e_t = 1$; in this case $L_t = H_t = pop_t$ (child labor ceases to exist), and income per capita simply grows at the rate of growth in A . However, more interesting to us are the transitory dynamics towards this “modern” state which govern the evolutions of ∂n and ∂e . Despite the simplicity of the model, (5) and (6) do not lend themselves to closed-form solution. We thus turn to simulation.

4.1 Static Equilibrium - Before the Industrial Revolution

In order to simulate the economy we must first establish the appropriate initial conditions. Here we will treat the onset of industrialization as the moment when technological growth becomes positive. Thus we treat our pre-industrial economy as a purely static one, where technology coefficient A is fixed at some pre-determined level. This is of course not an entirely accurate depiction of pre-industrial society, as technologies glacially improved for millennia prior to industrialization. The fact that technological growth was much slower before the Industrial Revolution is what is important, however, and hence we lose nothing in assuming the extreme case of zero growth as our starting point.

Beyond this, the static equilibrium requires a stable demographic structure. The conditions necessary for this are: $n_t = 1$, and $e_t = e_{t-1}$ at some very small level. That is, the typical dynasty simply reproduces itself, and parents and children have the same low education levels. The four equations we must satisfy are the first order conditions on production (2) and (3), and the first order conditions on utility (5) and (6). To obtain the static equilibrium, we set $n_t = 1$ and $e_t = e_{t-1} = 0.0001$, and solve for $w_{l,t}$, $w_{h,t}$, A_t , and α for $t = 1$, setting all other parameters equal to plausible values.

Specifically, parameters are set as follows: $\sigma = 2$, $\lambda = 0.5$, $\varepsilon = -10$, $\phi = 0.5$, $\gamma = 2$. Note that the key parameters here are those that affect the substitutability of skilled and unskilled labor in production (σ) and the substitutability of income and human capital in utility (ε). While

values of ε are purely speculative, much work has been done to estimate σ . The literature tends to value this somewhere between 1 and 2.5 for contemporary labor markets.¹⁵

Our specific functional forms allows for a solution, and this constitutes our static equilibrium. We produce initial values of $\alpha \approx 0.13$, $A_1 \approx 0.05$, $w_{l,1} \approx 0.07$, $w_{h,1} \approx 0.17$, $L_1 \approx 2$, and $H_1 \approx 0.3$. Note that the parameter α is then held fixed at its original value, while all other variables evolve with technological progress. We would suggest that this value of α , which governs the rate of return to education, is a realistic measure. For example, an increase of e from 0.1 to 0.2 increases human capital by roughly 9.5%, while an increase of e from 0.2 to 0.3 increases human capital by roughly 5.5%. Given that Mincerian-based studies for contemporary labor markets find an *annual* rate of return to education from anywhere between 6% and 10% (Krueger and Lindahl 2001), we suggest that this fairly low value of α is defensible.¹⁶

With these values as our initial condition, all remains static - as households have no incentive to change demographic behavior given current wages, L and H , and thus wages themselves, remain fixed. That is, until technologies begin to improve.

4.2 Dynamic Equilibria - the Industrial Revolution and the Demographic Transition with Exogenous Technological Growth

Here we run the simulation for 50 time periods to roughly capture economic and demographic trends for our hypothetical industrializing economy. Each time period we grow A exogenously by 2.5 percent and pop by the rate n_{t-1} , and solve the system of equations (2), (3), (5) and (6) for $w_{l,t}$, $w_{h,t}$, n_t and e_t .

As we are dealing with roughly 200 years of history, each time period can be considered roughly four years. Estimates of productivity growth in England during early industrialization over 4-year periods range from 1.5 to 3.1 percent (based on numbers reproduced in Voigtlander and Voth 2006). 2.5% growth in A is defensible based on historic estimates. Also note that we could increase this growth rate (say to 10%) to better reflect the average lifespan of people (so that each period could be considered 20 years or so); the simulation here implicitly assumes that individuals only live for a couple of years. This is strictly for presentation sake; increasing the rate of productivity growth and producing a simulation for 10 periods demonstrates the same patterns.

Figures 6 and 7 illustrate the results of the simulation. At the moment of technological growth, the population is stable, education is extremely low and the skill premium is quite large. With

¹⁵see Katz and Autor 1999 for a review of this literature.

¹⁶However, we can increase α by increasing the initial value of e . For example, starting with an e ten times the value used in our baseline specification ($e = 0.001$), we produce initial values of $\alpha \approx 0.16$, $A_1 \approx 0.06$, $w_{l,1} \approx 0.08$, $w_{h,1} \approx 0.21$, $L_1 \approx 2$, and $H_1 \approx 0.32$. Simulating the evolution of this economy produces very similar results compared with those displayed in section 4.2.

early productivity increases, fertility rises and education rises very slightly as well. How is this possible in the context of a quality-quantity model of demography? Given (11) and the fact that $0 < \alpha < 1$, we know that the production of human capital follows the *Inada conditions* ($\lim_{e \rightarrow 0} \frac{\partial h}{\partial e} = \infty, \lim_{e \rightarrow \infty} \frac{\partial h}{\partial e} = 0$). Thus if early rates of education are “small enough,” human capital will rise even with very small increases in education. Households can then raise the human capital of their offspring as their income rises. But because it does not cost them a lot to do this, they can also afford to have more children (to take advantage of increases in the unskilled wage). This captures the simple conceit that when human capital is low, small investments can yield large returns. Hence industrializing societies saw improvements in the quality of their workforce, despite any marked increase in human capital investments.¹⁷ Finally, the low level of education allows (13) to hold, so that technological progress makes relative levels of human capital rise and the skill premium fall.

Note that Galor (2005,2011a) labels this period the “post-Malthusian” era; this is because productivity increases still translate into population increases. This is precisely what we show here. Even though education is explicitly modeled as a normal good, increases in productivity creates a lot of fertility with only minuscule increases in education. Income *per capita* growth is very modest as a result of population growth. Thus early growth dynamics appear to be dictated by a classic “Malthusian trap.”¹⁸

As exogenous technological progress continues, however, there is an endogenous transition where further increases in productivity result in less fertility and more education. Why does the switch occur? Once education reaches some critical level, diminishing returns to further education are such that these increases become quite expensive. Households are willing to pay this expense as their incomes rise further. But to lower the overall expense for education they have fewer and fewer children. So households become less reliant on the unskilled income generated by their offspring, and more reliant on their own income. This is in essence the transition to modernity; a classic quantity/quality tradeoff of child rearing emerges. And as technology continues to rise, income per capita begins to grow faster than overall GDP. Note that the model does not rely on stronger assumptions concerning technology-skill or capital-skill complementarities to generate the transition (O’Rourke et al. 2008, Maoz and Moav 2004).

The two critical parameters that generate this transition are those that govern the substitution between factors in production (σ) and between income and human capital in utility (ε).

¹⁷Including health as a broader measure of human capital, we know that small increases in household incomes in England in the 18th century translated into large nutritional gains, creating a more productive workforce (Komlos 1990). This would *a fortiori* suggest that human capital should be treated in part as a consumption good.

¹⁸This simple structure also mimics so-called Malthusian dynamics prior to industrialization. So long as education rates are low, any discrete increase in A will increase the population but leave incomes per capita virtually unchanged. Such discrete increases (what Goldstone 2002 calls “efflorescences”) would also induce glacial increases in human capital, as it did for centuries prior to industrialization (see figure 2 for literacy, and A’Hearn et al. (2009) for a discussion on numeracy).

To see how we rerun each simulation but increase the degree of substitutability in each case. Figure 8 shows the case where σ is increased to (an arguably implausible level) three.¹⁹ Exogenous technological growth does produce fertility increases and some educational increase, but no demographic transition occurs during the 50-period interval due to the fact that education never rises to a sufficiently high level (education eventually reaches this level when productivity continues growing beyond fifty periods). There are two reasons for this. One is that wages in this case start out lower (due to the fact that parents start out with less human capital), and thus technological growth produces proportionally smaller increases in wages, producing no impetus to raise education. The other is that our solved value for α is higher in this case. This implies that diminishing returns to education are not as strong as before, so increasing human capital is not as costly. This of course delays the transition, as households can afford to increase both the quality and the quantity of their children for a longer duration. The model thus implies that *ceteris paribus* countries with faster wage growth and/or higher diminishing returns to education experienced their demographic transitions earlier (Galor 2011b).

Figure 9 shows the case where ε is increased to -1, so that income and human capital are only weakly complementary in preferences.²⁰ Again, growth produces no demographic transition due to the simple fact that now there is less incentive to invest in children with income growth. Whereas in figure 8 the demographic transition is only somewhat delayed, the transition in this case happens *much* later (hundreds of periods later when productivity is allowed to continue to grow). In both cases, lack of human capital growth limits overall growth in income, further limiting incentives to invest and producing a vicious cycle of underdevelopment. This approach highlights the potential of explaining the different timings of demographic transitions through differences in such things as factor substitution or income-human capital complementarities.

The simulation also stresses that in order to truly unify the stages of economic growth, models must account for constant injections of skilled labor over time. An implicit result of important unified theories such as Galor and Weil (2000), and Galor and Mountford (2006, 2008) is that transition to modern growth is associated with rising relative returns to skilled labor (Voth 2003). But we know this did not happen - skill premia at best remained stagnant. Galor (2005) acknowledges the role of supply of human capital in explaining low skill premia, but must rely on exogenous injections of skilled labor (such as those from compulsory schooling laws enacted in the late 19th century). But one might suggest that these so-called exogenous shocks did not come randomly, but rather were the results of political pressures both from industry (who increasingly saw the importance of a skilled work force to exploit productivity improvements, discussed in Galor and Moav 2006) and from households (who saw the “value” of educated children in increasing the well-being of the family, discussed in Horrell and Humphries 1995).

¹⁹Initial values for this case are $\alpha \approx 0.22$, $A_1 \approx 0.03$, $w_{l,1} \approx 0.03$, $w_{h,1} \approx 0.08$, $L_1 \approx 2$, and $H_1 \approx 0.13$.

²⁰Initial values for this case are $\alpha \approx 0.12$, $A_1 \approx 0.00007$, $w_{l,1} \approx 0.0001$, $w_{h,1} \approx 0.00025$, $L_1 \approx 2$, and $H_1 \approx 0.32$.

These were changes in the economic incentives for education that arguably should be *endogenous* in any model of unified growth.

5 Conclusion

Explaining the non-monotonic evolution of fertility, initial stagnation and subsequent growth of education, and fall in relative returns to skilled labor in the Industrial Revolution has constituted one of the major puzzles of economic history. Here we have offered an hypothesis to explain the evolution of these variables, suggesting that the substitutability between skilled and unskilled labor and the familial preferences for educated children must play pivotal roles in the explanation.

Extant unified growth theories rely on interactions between the rate of technological progress and the size and composition of the population to produce both an escape from the Malthusian world and a demographic transition (Galor and Weil 2000, Galor and Moav 2002, Galor and Mountford 2008, Galor 2011a). These works also suggest a modest role for human capital during the early stages of industrialization; they are however not the only tales that can be told that suggest this (Mokyr and Voth 2010). Here we propose an alternative mechanism for producing a transition to a modern economy that does not rely on scale effects or faster technological progress. Results here provide testable hypotheses which should further our understanding of the true dynamics of growth in history.

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Figure 1: Birth and Death Rates in England (top); Birth Rates in Europe (bottom) [sources: Galor (2005), based on Wrigley and Schofield (1981), Andorka (1978) and Kuzynski (1969)]

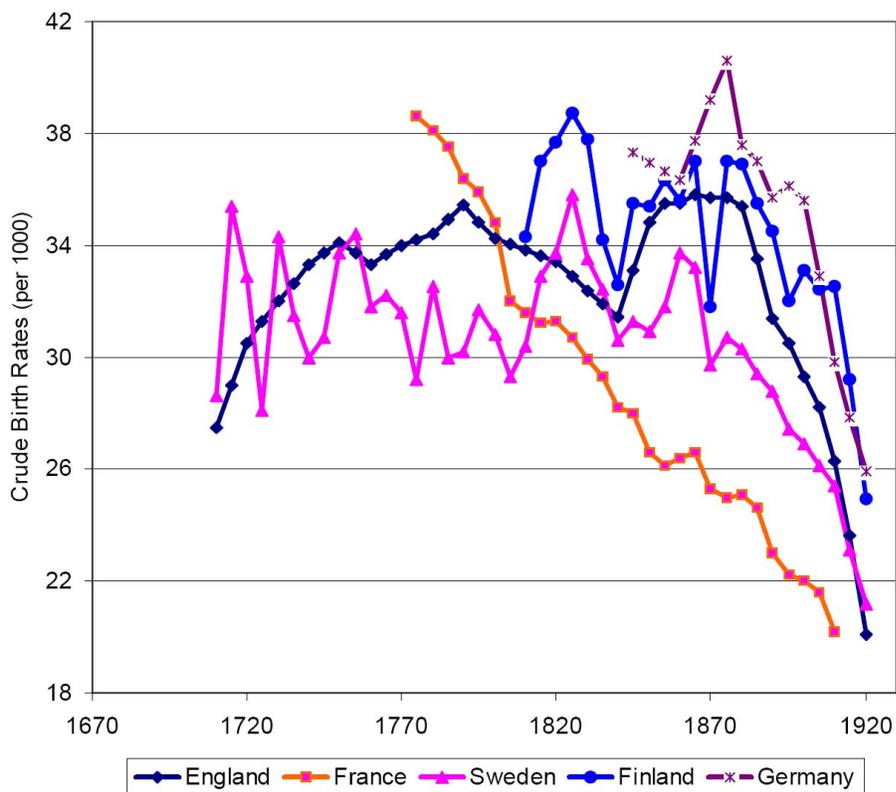
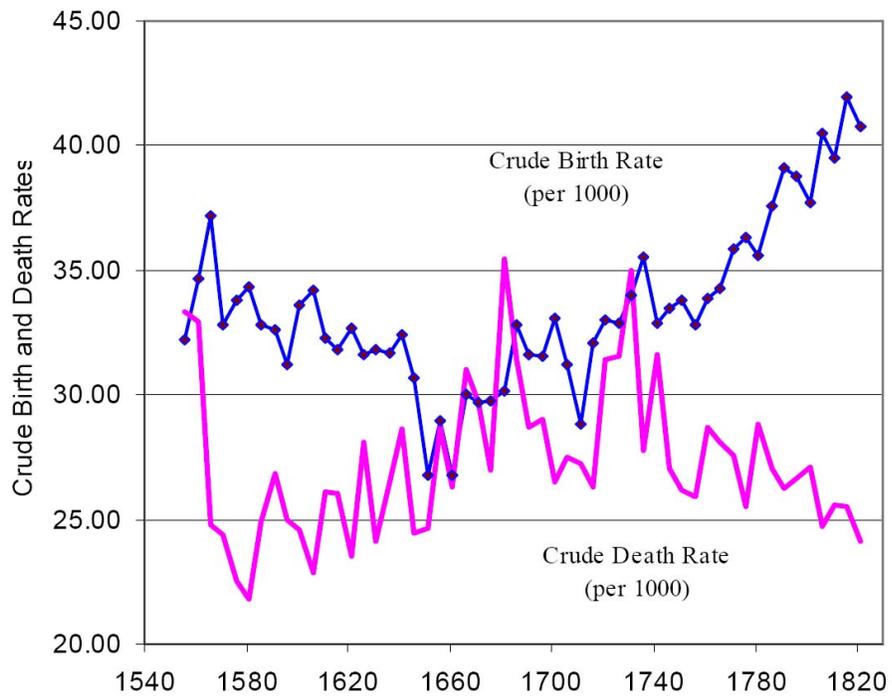


Figure 2: Literacy Rates in England (top) [source: Clark (2005), based on Schofield (1973), Houston (1982), and Cressy (1997)]; Primary School Enrollment Rates in England and France (bottom) [source: Galor (2005), based on Flora et al. (1983)]

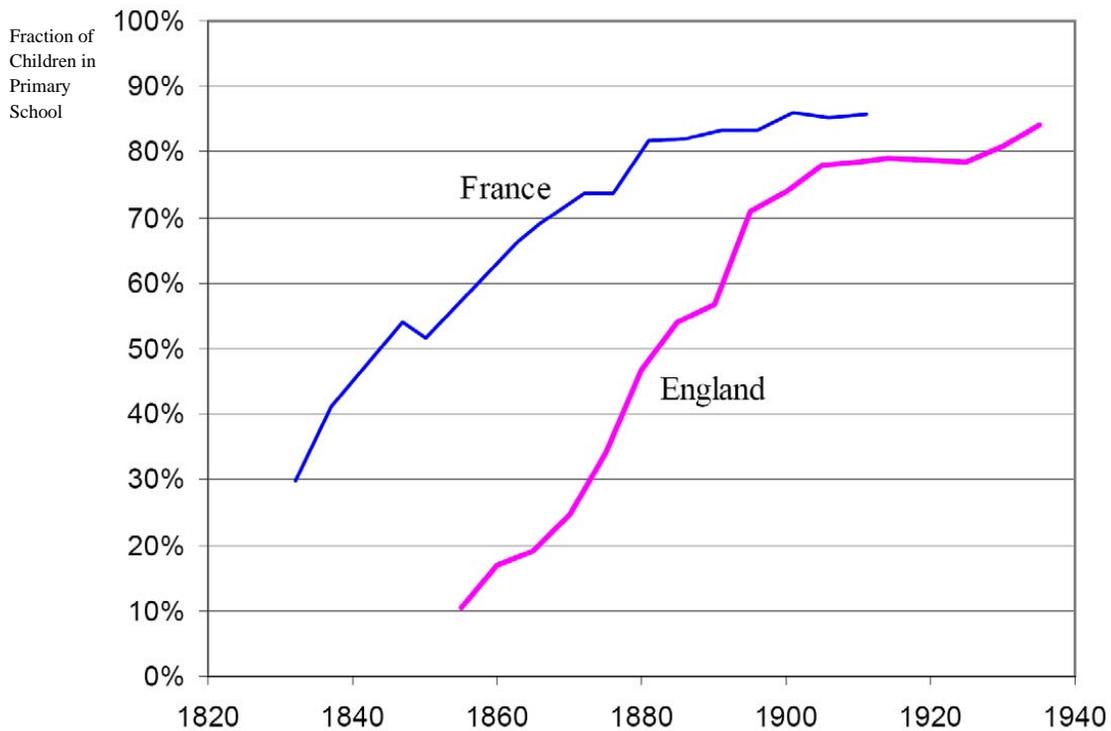
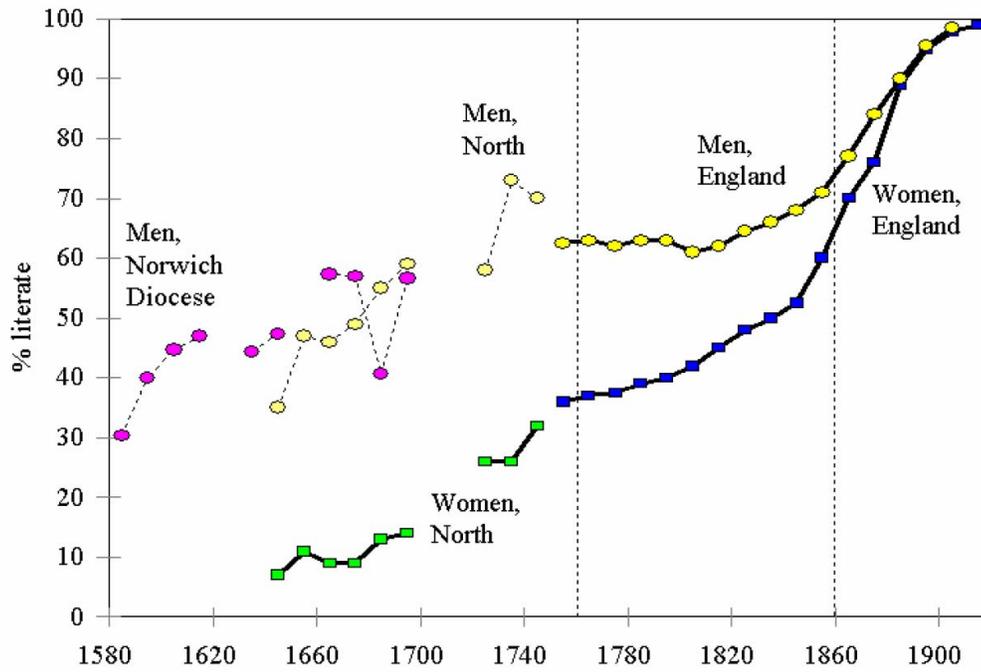


Figure 3: Fertility and Education Rates in the United States [source: eh.net and Goldin and Katz (2008)]

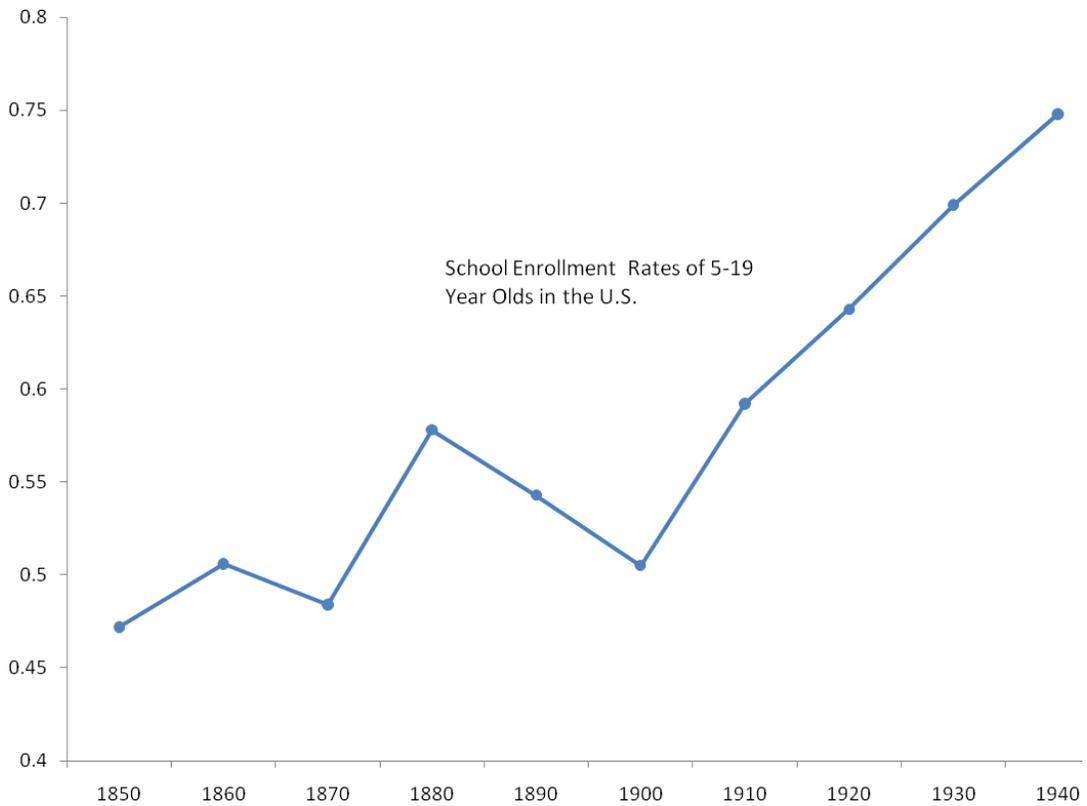
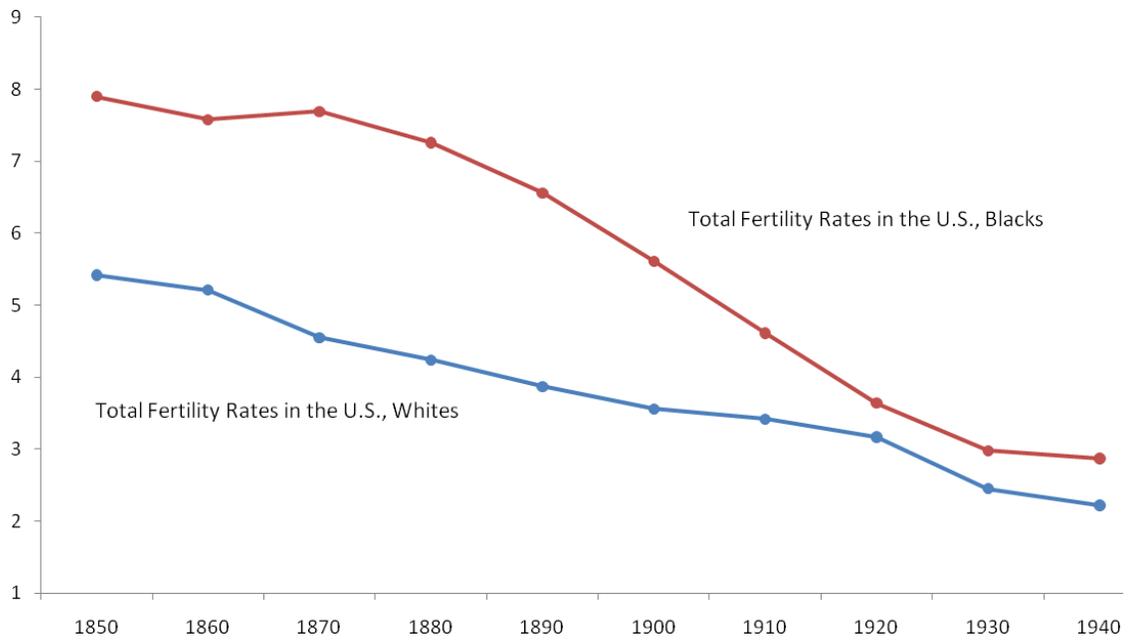


Figure 4: Skill Premium in English Construction (top) [source: Clark (2007)]; Skill Premium in England for Variety of Industries (bottom) [source: Williamson (1982)]

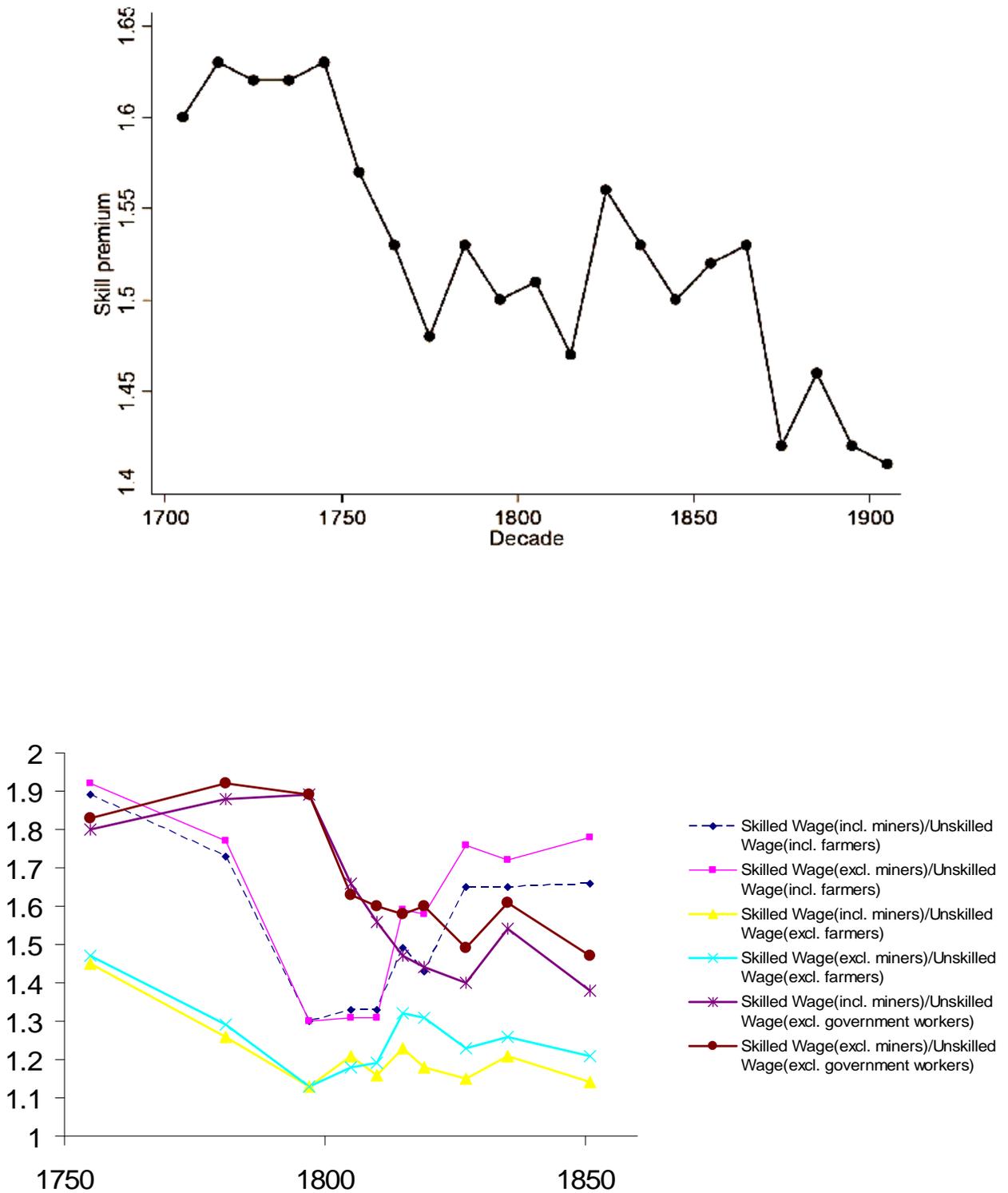


Figure 5: Skill Premium in the U.S. (top) [source: Goldin and Katz (2008)]; Skill Premium in 1750/1820 and GDP in 1913 for Many Countries (bottom) [source: van Zanden (2006)]

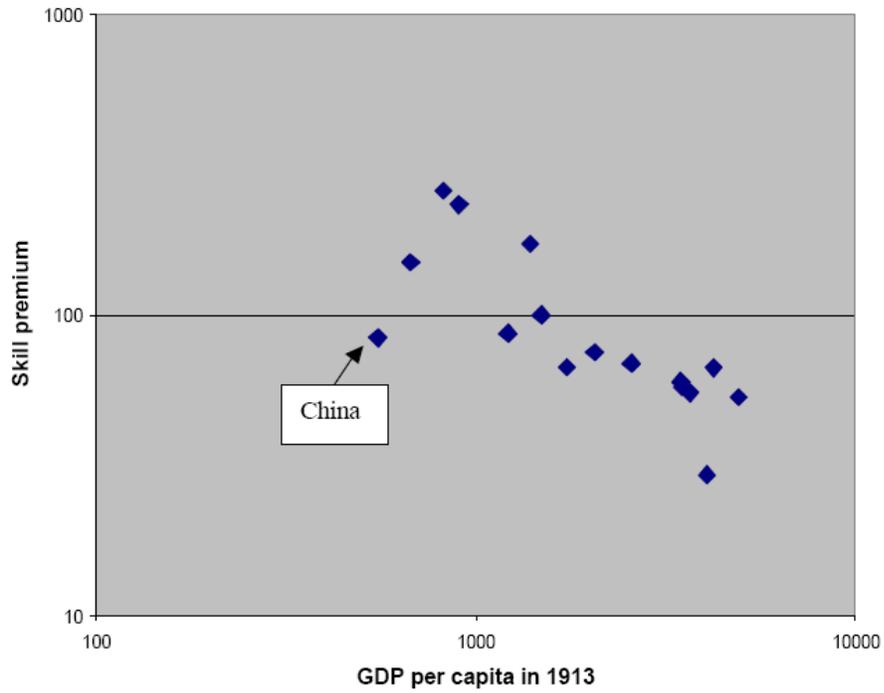
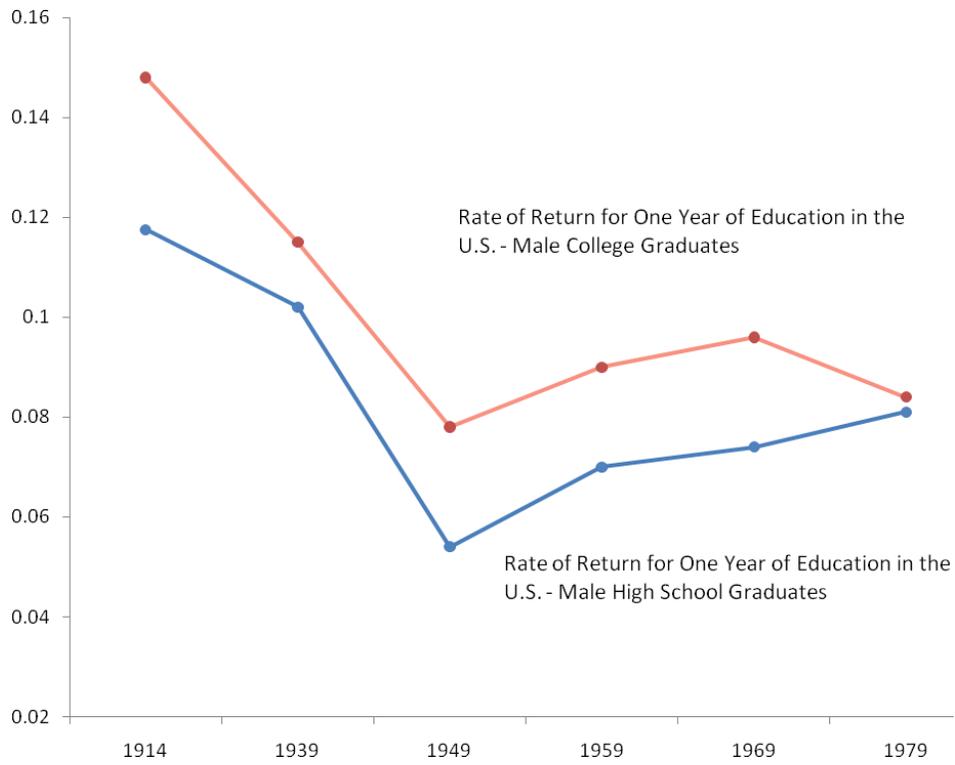


Figure 6: Simulated Values of Fertility and Education - baseline

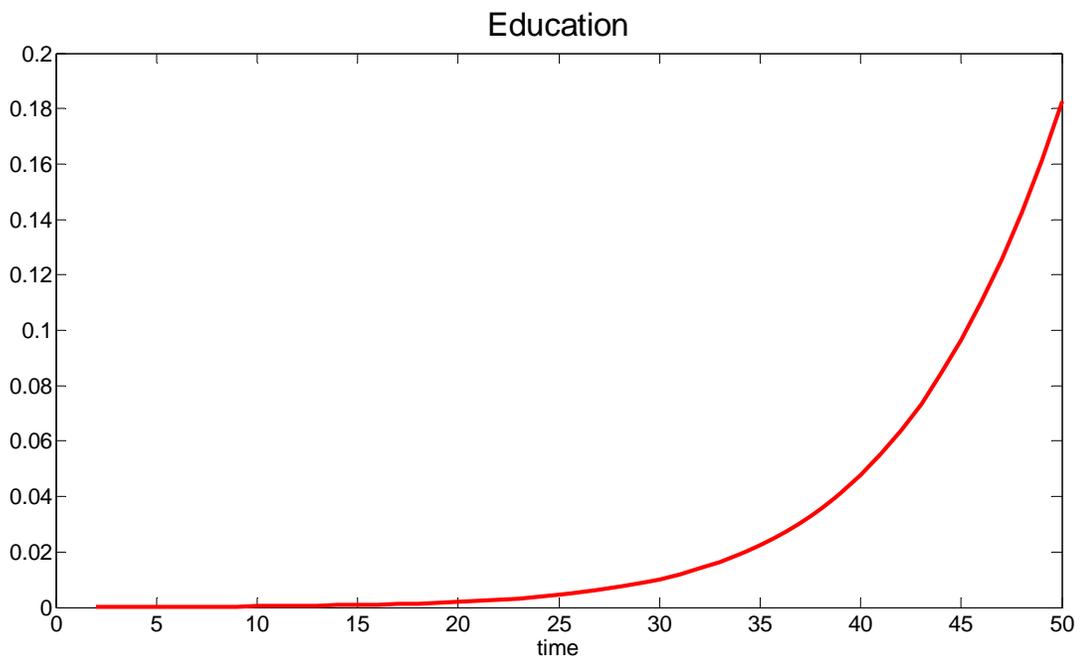
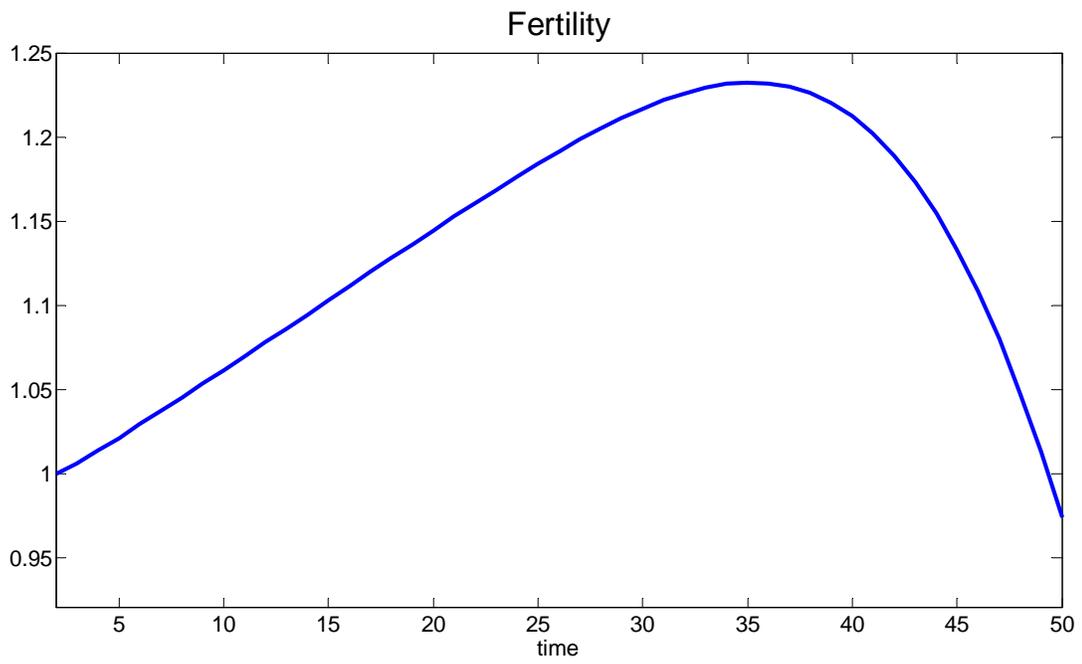


Figure 7: Simulated Values of Skill Premia and Relative Labor Supplies - baseline

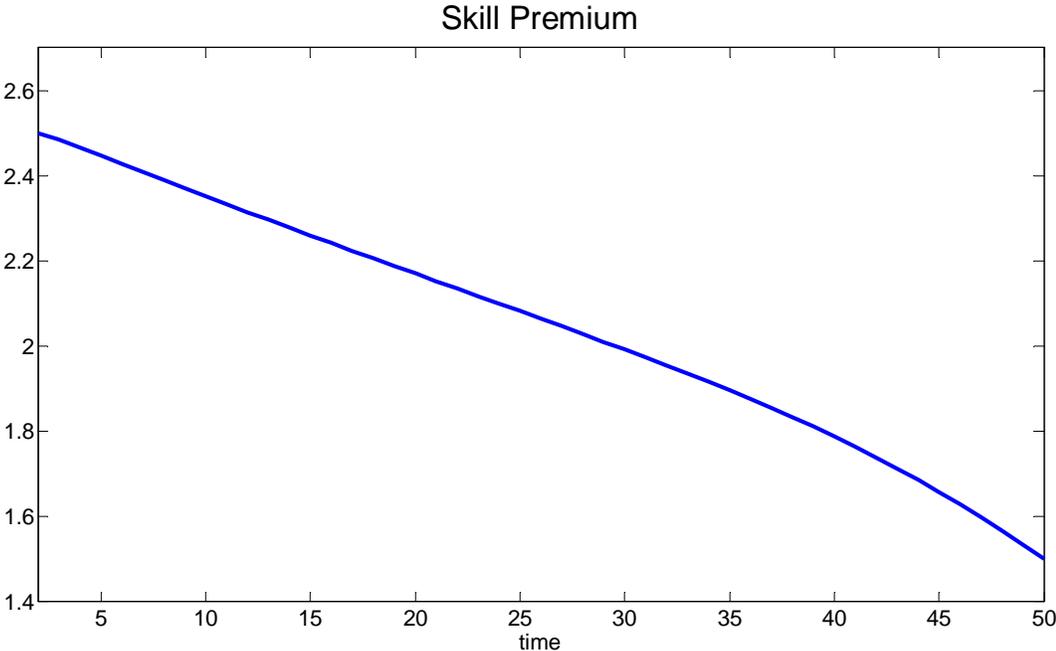


Figure 8: Simulated Values of Fertility, Education, Skill Premia and Relative Factors: $\uparrow \sigma = 3$

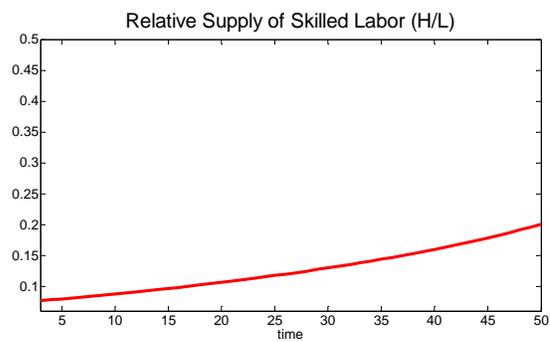
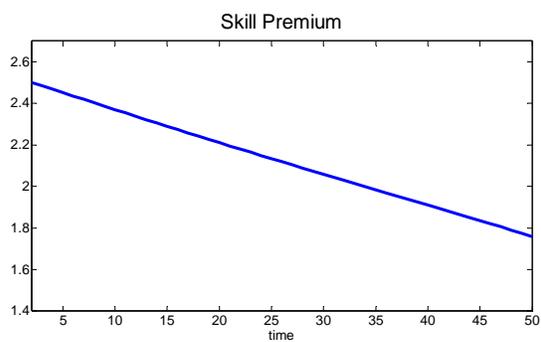
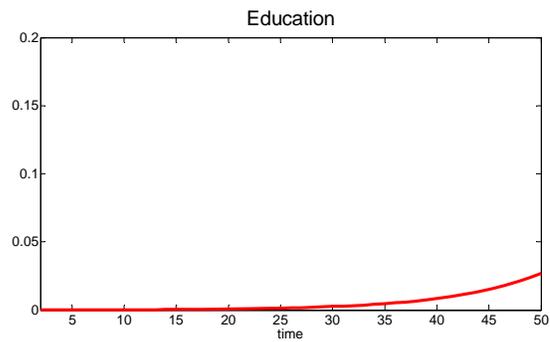
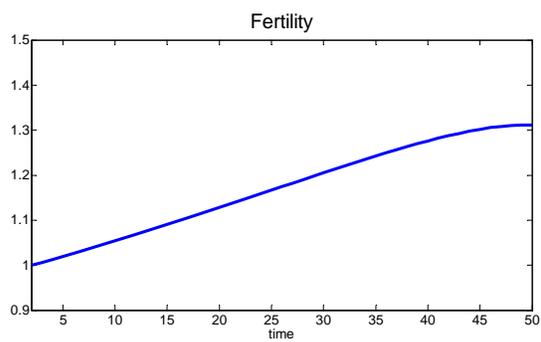


Figure 9: Simulated Values of Fertility, Education, Skill Premia and Relative Factors: $\uparrow \varepsilon = -1$

