Trade, Technology and the Great Divergence*

(or an Economic History of the World in Thirty-Two Equations)

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Abstract

This paper develops a model that captures the key features of the Industrial Revolution and the Great Divergence between the industrializing "North" and the lagging "South." Industrialization in a few core countries produced beneficial spill-overs around the world, but still led to massive income divergence. A model is needed that combines both features to create a convincing story of 18th-19th and 19th-20th century growth. To this end we construct a trade/growth model that includes both endogenous biased technologies and intercontinental trade. The Industrial Revolution was a sequence of unskilled-labor intensive innovations that initially incited fertility increases and limited human capital formation in both the North and the South. But subsequent skill-intensive growth and globalization forces in the late 19th century triggered the northern demographic transition and the subsequent Great Divergence.

- *Keywords*: Industrial Revolution, unified growth theory, endogenous growth, demography, skill premium, Great Divergence
- *JEL Codes*: O, F, N

*Preliminary sketch. Please do not quote.

1 Introduction

The last two centuries have witnessed dramatic changes in the global distribution of income and population. At the dawn of the Industrial Revolution, living standards between the richest and poorest economies of the world were roughly 2 to 1. With industrialization came both income and population growth within a few core countries. But massive divergence in living standards across the globe did not take place until the latter half of the 19th century, the time when the first great era of globalization started to take shape. Today the gap between material living standards in the richest and poorest economies of the world is 30 or 40 to 1, in large part due to the events of the 19th century. It is an interesting coincidence then that unprecedented growth in inter-continental commerce (conceivably a great force of convergence) coincided so precisely with unprecedented divergence in living standards across the world. These phenomena beg an explanation. This paper argues that the interactions between biased technological developments of the North and burgeoning commerce between the North and the South sowed the seeds for divergence, contributing enormously to today's great wealth disparity.

A number of the "stylized facts" of economic history motivate our theory. One concerns the nature of industrialization itself - technological change was unskilled labor-intensive during the early Industrial Revolution but became relatively skill-intensive during the latter nineteenth century. Indeed, England's early industrialization was in many respects largely a revolution in the cotton textile industry, with the adoption of the factory system of production and its associated new machinery. The textile industry was revolutionary in its ability to employ large numbers of unskilled and uneducated workers with minimal supervision, thus diminishing the productive role for skilled labor and education. By the 1850's, however, two major changes in technological growth occurred - it became much more widespread, and it became far more complementary to skilled workers.

Another historical feature of great importance was the North's expanding influence over the South in two major ways - through flows of technology and flows of trade. Indeed, changes spawned by the Industrial Revolution in the nineteenth century all seemed to predict that it would soon transform most of the world in precisely the same way it was transforming England, northwestern Europe and the United States. By 1900 for example, the economic centers of the "South" such as Alexandria, Bombay and Shanghai were fully integrated into the British economy, both in terms of transport costs and capital markets (Clark 2007).

Concerning the flows of knowledge, the new technologies of the Industrial Revolution could be exported mechanically with relative ease to most of the world. After all, while developing new knowledge was an arduous task, copying this knowledge was much easier. And because the new technologies of early industrialization were not very sophisticated, they were quickly transmitted to, and easily adopted by, much of the world.

Concerning the flow of trade, intercontinental commerce (what we might call "North-South"

trade) did not grow particularly robust until the latter half of the 19th century. By the 1840s steam ships were faster and more reliable than sailing ships, but their high coal consumption limited how much cargo they could transport; consequently only very light and valuable freight was shipped (O'Rourke and Williamson 1999). But by 1870 a number of innovations dramatically reduced the cost of steam ocean transport, and real ocean freight rates fell by nearly 35% from 1870 to 1910. Thus, while a closed economy assumption would certainly be more appropriate for the first stages of the Industrial Revolution (1750-1850), a more open economy framework would better describe the latter stages of industrialization (1850-1910).

To analyze the intellectual puzzle of the Great Divergence, we develop a model that has a number of key features which mimic these historical realities. The first feature of our approach is that we endogenize the direction of technological change that occurs in the North. Technologies are sector specific, and sectors have different degrees of skill intensities. Following the endogenous growth literature, we allow potential innovators in the North to observe the employment of factors in different sectors, and tailor their research efforts towards particular sectors. Thus the direction of innovation will depend on Northern employment and demography. Further, these innovations have the potential to diffuse to the South. These technologies are not tailor made for the South, but nonetheless they can aid the South through knowledge transmission and adoption.

The second key feature is that we endogenize demography itself. More specifically, we allow households to make implicit education and fertility decisions based on the perceived wages of skilled and unskilled labor. The method is similar to traditional endogenous demography models where households face a quality/quantity tradeoff with respect to their children. Thus, when the premium for skilled labor rises families choose to have fewer but better educated children.

The final feature is that we allow for burgeoning trade between the North and the South. During the initial stages of industrialization, trade is not possible due to prohibitively high transport costs. These costs however exogenously decrease over time; at a certain point trade becomes feasible, at which point the South exchanges labor-intensive products for the North's skill-intensive products. At this stage development paths diverge - the North's specialization in skilled production techniques produces a demographic transition, while the South's specialization in unskilled production techniques produces more population.

With this basic setup, we simulate the model to roughly capture the main features of industrialization and divergence between the North and the South from roughly 1700 to 1900. Because of the great abundance of unskilled labor, innovators in the North first develop unskilled-intensive technologies, and these flow to and are adopted by the South. Thus early industrialization is characterized by unskilled intensive technological growth and population growth both in the North and the South; consequently living standards in the two regions do not diverge very greatly. Later on however trade becomes possible - at this point the North starts specializing in skill-intensive innovation and production. This induces a demographic transition of falling fertility

and rising education rates in the North. The South of the other hand specializes in unskilled-intensive production, inducing further population growth. Thus the South's static gains from trade become a dynamic impetus to prosperity, and living standards between the two regions diverge dramatically as a result.

We argue that analyzing the interactions between the North and the South, and between trade and technological flows, is critical to understanding both the Industrial Revolution and the Great Divergence. In this respect the paper relates most closely to Galor and Mountford (2005) (GM). They similarly argue that northern gains from trade translated into rising education and income, while southern gains from trade mainly translated into population increases. However, our model differs from theirs in several important respects. First, the GM approach employs a semi-Ricardian model with no technological diffusion; consequently the North always maintains a technological edge in skill-intensive production. But economic historians have suggested that technological diffusion from the North may have mitigated the adverse effects of trade on southern industrialization (Findlay 1996). Instead, we employ a Heckscher-Ohlin model where trade arises from factoral differences, allowing for *perfect* technological diffusion. Both features are relevant for this study - we know that Heckscher-Ohlin oriented trade was important during the 19th century since commodity price convergence induced factor price convergence during this time (O'Rourke and Williamson 1994; O'Rourke, Taylor and Williamson 1996; O'Rourke and Williamson 1999, Chapter 4), and we know that technologies readily diffused from the industrializing North to the rest of the world (Mokyr 1999; Clark 2007, Chapter 15).

Further, since no knowledge diffusion is allowed, GM cannot comment on the "appropriateness" of technological diffusion. From the endogenous growth literature, we know that the nature of technological progress depends on many local conditions, including prices and factor endowments (Acemoglu 2002). But the flow of these technologies to other regions may not benefit the receiving countries if they do not have appropriate factor endowments (Basu and Weil 1998; Acemoglu and Zilibotti 2001). Very different from GM, our model uses the tools of endogenous growth theory to analyze this additional channel of divergence between the North and the South.

Finally, rather than suddenly open up the North and South to trade, we allow for *gradual* increases in North-South commerce. The British economy (and other Western economies) presumably did not undergo a discontinuous switch from a closed to an open state, and thus we will impose continuously declining transport costs to achieve such a transition.

2 Static Equilibrium

Here we sketch out a model that we will use to describe both a northern economy and a southern economy (subscripts denoting region are suppressed for the time being).

Total production for a region is given by:

$$Y = \left(\frac{\alpha}{2} y_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha) y_2^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} y_3^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\alpha \in [0,1]$ and $\sigma \geq 0$. σ is the elasticity of substitution among intermediate goods y_1, y_2, and y_3. The production of these goods in turn are given by:

$$y_1 = A_1 L_1 \tag{2}$$

$$y_1 = A_2 L_2^{\gamma} H_2^{1-\gamma} \tag{3}$$

$$y_3 = A_3 H_3 \tag{4}$$

where A_1, A_2 and A_3 are the technological levels of sectors 1, 2, and 3, respectively. These technological levels in turn are represented by a series of *sector-specific* machines. Specifically,

$$A_1 = \int_0^{N_1} \left(\frac{x_1(j)}{L_1}\right)^{\alpha} dj \tag{5}$$

$$A_2 = \int_0^{N_2} \left(\frac{x_2(j)}{L_2^{\gamma} H_2^{1-\gamma}}\right)^{\alpha} dj \tag{6}$$

$$A_3 = \int_0^{N_3} \left(\frac{x_3(j)}{H_3}\right)^{\alpha} dj \tag{7}$$

where $x_i(j)$ is machine of type j that can be employed only in sector i. Technological progress in sector i can then be represented by growth in the number of machine-*types* that are extant for the sector, represented by N_i (we endogenize this growth in the next sections).

Treating the technological coefficients as exogenous for the time being, we can assume that markets for both the final good and intermediate goods are perfectly competitive. Thus, prices are equal to unit costs. Solving the cost minimization problems for productions, and normalizing the price of final output to one, yields the unit cost functions

$$1 = \left[\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{8}$$

$$p_1 = \frac{w_l}{A_1} \tag{9}$$

$$p_2 = \left(\frac{1}{A_2}\right) w_l^{\gamma} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{-\gamma} \tag{10}$$

$$p_3 = \frac{w_h}{A_3} \tag{11}$$

where naturally p_i denotes the price for intermediate good x_i.

Full employment of total unskilled labor and total skilled labor implies the following factor-market clearing conditions:

$$L = \frac{x_1}{A_1} + \frac{w_l^{\gamma-1} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{1-\gamma} x_2}{A_2} \tag{12}$$

$$H = \frac{w_l^{\gamma} w_h^{-\gamma} (1-\gamma)^{\gamma} \gamma^{-\gamma} x_2}{A_2} + \frac{x_3}{A_3} \tag{13}$$

Finally, the demands for intermediate goods from final producers can be derived from a standard C.E.S. objective function (so demands will be negatively related to own price, will be a function of a price index, and will be proportional to total product). Specifically, intermediate goods market clearing requires

$$x_i = \left(\frac{\Upsilon_i^{\sigma} p_i^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma}} \right) Y \tag{14}$$

for $i = 1, 2, 3$, $\Upsilon_1 = \Upsilon_3 = \alpha/2$, and $\Upsilon_2 = 1 - \alpha$.

Provided that we have values for L, H, A_1, A_2 and A_3, along with parameter values, this yields eight equations with eight unknowns: p_1, p_2, p_3, x_1, x_2, x_3, w_l and w_h. The solution for these variables constitutes the solution for the static model in the case of exogenously determined technological and demographic variables.

3 Endogenizing Technological Growth in the North

In this section we describe how innovators in the North endogenously develop new technologies. In general, modeling purposive research and development effort is difficult when prices and factors change over time. This is because it is typically assumed that the gains from innovation will flow to the innovator throughout his lifetime, and this flow will often depend on the price of the product being produced and the factors required for production at each moment in time.[1] If prices and factors are constantly changing (as they will in any economy where trade barriers fall gradually or factors evolve endogenously), a calculation of the true discounted profits from an invention may be impossibly complicated.

To avoid such needless complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last *one time period only.* More specifically, technological progress is sector-specific, and comes about though increases in the varieties of

[1]For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

machines employed in each sector. New varieties of machines are developed by profit-maximizing inventors, who *monopolistically* produce and sell the machines to competitive producers of the intermediate goods y_1, y_2 or y_3. However, we assume the blueprints to these machines become public knowledge the time period after the machine is invented, at which point these machines become old and are *competitively* produced and sold.[2] Thus while we need to distinguish between old and new sector-specific machines, we avoid complicated dynamic programming problems inherent in multiple-period profit streams.[3]

Thus, we can re-define sector-specific technological levels given by (5) - (7) as a series of both old and new machines at time t as:

$$A_{1,t} = \left(\int_0^{N_{1,t-1}} x_1(j)^\alpha dj + \int_{N_{1,t-1}}^{N_{1,t}} x_1(j)^\alpha dj \right) \left(\frac{1}{L_1} \right)^\alpha$$

$$A_{2,t} = \left(\int_0^{N_{2,t-1}} x_2(j)^\alpha dj + \int_{N_{2,t-1}}^{N_{2,t}} x_2(j)^\alpha dj \right) \left(\frac{1}{L_2^\gamma H_2^{1-\gamma}} \right)^\alpha$$

$$A_{3,t} = \left(\int_0^{N_{3,t-1}} x_3(j)^\alpha dj + \int_{N_{3,t-1}}^{N_{3,t}} x_3(j)^\alpha dj \right) \left(\frac{1}{H_3} \right)^\alpha$$

Thus in each sector i there are N_{t-1} varieties of old machines that are competitively produced, and there are $N_t - N_{t-1}$ varieties of new machines that are monopolistically produced (again, suppressing country subscripts).

Next, we must describe producers of intermediate goods in each region. These three different groups of producers each separately solve the following maximization problems:

Sector 1 producers: $\max_{[L_1, x_1(j)]} p_1 y_1 - w_l L_1 - \int_0^{N_1} \chi_1(j) x_1(j) dj$

Sector 2 producers: $\max_{[L_2, H_2, x_2(j)]} p_2 y_2 - w_l L_2 - w_h H_2 - \int_0^{N_2} \chi_2(j) x_2(j) dj$

Sector 3 producers: $\max_{[H_3, x_3(j)]} p_3 y_3 - w_h H_3 - \int_0^{N_3} \chi_3(j) x_3(j) dj$

where $\chi_i(j)$ is the price of machine j employed in sector i. For each type of producer, solving the maximization problem with respect to machine j yields a solution for machine demand:

$$x_1(j) = \chi_1(j)^{\frac{1}{\alpha-1}} (\alpha p_1)^{\frac{1}{1-\alpha}} L_1 \tag{15}$$

$$x_2(j) = \chi_2(j)^{\frac{1}{\alpha-1}} (\alpha p_2)^{\frac{1}{1-\alpha}} L_2^\gamma H_2^{1-\gamma} \tag{16}$$

[2]Here one can assume either that patent protection for intellectual property lasts one time period, or that it takes one time period for potential competitors to reverse-engineer the blueprints for new machines.

[3]See Rahman (2008) for more discussion of this simplifying assumption.

$$x_3(j) = \chi_3(j)^{\frac{1}{\alpha-1}} (\alpha p_3)^{\frac{1}{1-\alpha}} H_3 \tag{17}$$

New machine producers, having the sole right to produce the machine, set the price of their machines to maximize instantaneous profit. This price will be a constant markup over the marginal cost of producing a machine. Assuming that the cost of a machine is unitary, this implies that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1/\alpha$ for new machines. Thus, substituting in this mark-up price, and realizing that instantaneous profits are $(1/\alpha) - 1$ multiplied by the number of new machines sold, instantaneous revenues by new machine producers are given by:

$$\pi_1 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_1)^{\frac{1}{1-\alpha}} L_1 \tag{18}$$

$$\pi_2 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_2)^{\frac{1}{1-\alpha}} L_2^{\gamma} H_2^{1-\gamma} \tag{19}$$

$$\pi_3 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_3)^{\frac{1}{1-\alpha}} H_3 \tag{20}$$

Old machines, on the other hand, are competitively produced; competition will drive the price of all these machines down to marginal cost, so that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1$ for all old machines. Sectoral productivities can then be expressed simply as a combination of old and new machines demanded by producers. Plugging in the appropriate machine prices into our machine demand expressions (15) - (17), and plugging these machine demands into our sectoral productivities, we can express these productivities as:

$$A_1 = \left(N_{1,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{1,t} - N_{1,t-1})\right) (\alpha p_1)^{\frac{\alpha}{1-\alpha}} \tag{21}$$

$$A_2 = \left(N_{2,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{2,t} - N_{2,t-1})\right) (\alpha p_2)^{\frac{\alpha}{1-\alpha}} \tag{22}$$

$$A_3 = \left(N_{3,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{3,t} - N_{3,t-1})\right) (\alpha p_3)^{\frac{\alpha}{1-\alpha}} \tag{23}$$

Thus, given the number of old and new machines that can be used in each sector, we can simultaneously solve equations (8) - (14) and (21) - (23) to solve for prices, wages, intermediate goods and technological levels for a hypothetical economy. Our next goal then is to use this model to discuss the historic interactions of two economies.

4 Trade versus Technology

Which could have contributed more to the Great Divergence, the flow of "inappropriate" technologies to the South, or the South's increasing specialization of low-skilled goods. Inappropriate technological spillovers are bad from a static sense but perhaps good from a dynamic sense (in that they could promote healthy incentives to reduce fertility and increase education). Southern specialization is good from a static sense (due to static gains from trade) but bad from a dynamic sense (in that they could induce more fertility and less education).

To answer this question, we perform a thought experiment by simulating two economies. The above model describes a hypothetical country - now we will use it to describe both a "northern" economy (one that develops its own machine blueprints and is relatively more skill-endowed) and a "southern" economy (one that benefits from diffused northern machine blueprints and is relatively more unskilled labor-endowed).

Simulating these economies over time will require us to endogenize the time paths of technologies, trade volumes, and factors. We next turn to a brief discussion of each of these issues.

4.1 A Dynamic Model

1) How do technologies grow in the North?

For this, we can assume that resource costs to research are rising in N, and falling in "Baconian knowledge," or B. Thus, a no-arbitrage (free entry) condition for potential researchers in the North can be described as:

$$\pi_i \leq c\left(\frac{N_i}{B}\right) \tag{24}$$

Specifically, we can assume the following functional form for these research costs:

$$c\left(\frac{N_i}{B}\right) \leq \left(\frac{N_{i,t+1}}{B_t}\right)^{\phi} \tag{25}$$

for $i = 1, 3$ (for convenience we assume no research occurs in sector 2. This way technological growth is unambiguously factor-biased). Given some level of Baconian knowledge (which we can assume grows at some exogenous rate) and number of existing machines, we can determine the resource costs of research. When Baconian knowledge is low relative to the number of available machine-types used in sector i, the costs of inventing a new machine in sector i is high (see O'Rourke et al 2007 for a fuller discussion). The revenues from owning a new machine-blueprint, on the other hand, are given by equations (18) - (20). Thus from (24) we see that northern innovation in sector i becomes more attractive when Baconian knowledge is large, when the number of machine-types in sector i is low, when then price of good i is high, and when the employment in sector i is high.

Note that if $\pi_i > c(N_i/B)$, there are potential profits from research in sector i. However, this will induce research activity, increasing the number of new machines, and hence costs of research, up. We assume in fact that N_i adjusts upward such that costs of research just offset the revenues of new machine production. Thus increases in B are matched by increases in northern levels of N_i such that the no-arbitrage condition holds with equality whenever technological growth in the sector occurs.

2) How do technologies grow in the South?

At first pass, we can assume that the South has no ability to innovate on their own. Rather, they rely on the North to inherit new blueprints for machines; once these blueprints are received by someone in the South, they themselves become monopolists (again, for just one time period) who then earn profits that can be characterized by equations (18) - (20). Thus, while certainly the South possesses intellectual property protection (limited to one time period, same as the North), there is no ability by southerners to perform independent research.

Specifically, we assume that the number of machine blueprints in the South evolve according to the following relationship:

$$\Delta N_{i,t+1}^{S} = \rho \left(N_{i,t+1}^{N} - N_{i,t}^{S} \right) \tag{26}$$

where $0 \leq \rho \leq 1$ is the extent of the the diffusion of blueprints from the North to the South. Southern technological levels will then be similarly described by (21) - (23), but with southern-specific number of blueprints (potentially lower than the North) and southern-specific prices.

3) How do trade technologies evolve?

Here we use an amended version of (1), where production for each region is given by

$$Y^n = \left(\frac{\alpha}{2} \left(y_1^n + aZ_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(y_2^n \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(y_3^n - Z_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{27}$$

$$Y^s = \left(\frac{\alpha}{2} \left(y_1^s - Z_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(y_2^s \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(y_3^s + aZ_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{28}$$

Z_1 is the amount of good 1 that is exported by the South, Z_3 is the amount of good 3 that is exported by the North, and $0 < a < 1$ is an iceberg factor for traded goods (i.e. the proportion of exports not lost in transit). Thus the North imports only a fraction a of Southern exports, and the South imports only a fraction a of Northern exports. Intermediate goods production is still described by (2) - (4). Given this, it is straightforward to show

Proposition 1 *If* $\left(\frac{p_3^n}{p_1^n} \right) / \left(\frac{p_3^s}{p_1^s} \right) > a^2$, $Z_1 = Z_3 = 0$.

If transport costs are large (that is, if a is small) relative to cross-country price differences, no trade occurs. We will assume that early on transport technologies are indeed not advanced enough to permit trade. We will also assume that these transport technologies exogenously improve over time. A rising value of a will induce positive and rising levels of Z_1 and Z_3. Note that we assume that there is no trade in y_2 - because this is produced using both L and H, differences in p_2 are very small between the North and the South, and thus the assumption is not particularly restrictive.[4] Further, the limiting case of $a = 1$ produces goods and factor price convergence, and thus (in the case of identical technologies in the North and South) replicates the integrated equilibrium, even in the absence of trade in good 2.

General equilibrium in this case is a 28 equation system that solves for prices, wages, commodities, employment, trade, and sectoral productivity levels in each region. The equilibrium is described in more detail in the appendix.

4) How do factors evolve?

Let us assume for simplicity that with a constant skill premium, fertility is at its replacement rate (keeping population constant), and education is at some relatively low rate. Any skill-premium that rises will imply rising education and lower fertility; any skill-premium that falls will imply falling education and higher fertility. Thus we are assuming that the quality-quantity tradeoff of children faced by households is strictly a function of changes in the skill premium. Specifically,

$$\frac{L_{t+1}}{L_t} = \left(\frac{w_{h,t+1}}{w_{l,t+1}} / \frac{w_{h,t}}{w_{l,t}} \right)^{-\xi} \tag{29}$$

$$\frac{H_{t+1}}{H_t} = \left(\frac{w_{h,t+1}}{w_{l,t+1}} / \frac{w_{h,t}}{w_{l,t}} \right)^{\xi} \tag{30}$$

where $\xi > 0$ captures the demographic responsiveness to relative wage changes. Thus, we are implicitly assuming that fertility increases translate into more *unskilled* labor, while education increases translate into more *skilled* labor.

[4] Indeed, trade in all three goods would produce an analytical problem. It is well known among trade economists that when there are more traded goods than factors of production, country-specific production levels, and hence trade volumes, become indeterminate. See Melvin (1968) for a thorough discussion.

4.2 Simulations[5]

We consider 3 basic possibilities, a baseline case where northern technologies diffuse perfectly to the South and trade technologies improve linearly, another where no trade is permitted but technological diffusion is, and a third case where no technological diffusion is permitted but trade is. For each case we simulate the model for 50 time periods.

Case 1: **Baseline case** [$\frac{\Delta a}{a} > 0$, $\rho = 1$]. Here we assume that iceberg costs of trade decrease linearly such that a_1 and a_3 equals one near the end of the simulation (around t=45), and that knowledge diffuses perfectly from the North to the South. This produces four distinct phases of growth - we will call these phases unskilled growth, balanced growth, diversified trade, specialized trade.

The first phase is unskilled growth ($t = 4$ to 15). Because the North is endowed with more unskilled labor than skilled labor, technological growth occurs first in the unskilled-intensive sector.[6] Thus N_1 begins to grow in both the North and the South. This induces the skill premium to fall in both regions, and consequently for both the North and the South unskilled labor rises slightly and skilled labor falls slightly. Notice that this phase of growth produces a slight tendency for the South to *converge* to the North. This result, echoed by the appropriate technology literature, occurs because the South has a relatively heavy endowment of unskilled labor compared with the North.

The second phase is balanced growth ($t = 15$ to 27). At this point Baconian knowledge has grown sufficiently large so that sector 3 technologies can rise as well. While the skill premium continues to drift downward for both countries, this balanced growth reverses southern convergence; because the North has more skilled labor than the South, increases in sector 3 productivity gives a slight edge to the North in terms of growth rates.

The third phase is diversified trade ($t = 28$ to 41). Before this point, iceberg costs were continuously falling, but still had not reached the point where trade between the two regions would be feasible. After $t = 27$ however, trade becomes possible and the world equilibrium is characterized by Appendix A - while both regions continue to produce all three goods, the North specializes in and exports y_3 and the South specializes in and exports y_1. Changes in prices and wages that result from Stolper-Samuelson-like forces create wrenching changes in both societies. In the North, decreases in p_1 and L_1 forces innovators to abandon their development of unskill-intensive technologies; on the other hand, increases in p_3 and H_3 raises the revenues of producing new skill-intensive machines and thereby increases the rate of growth of skill-intensive

[5]The parameter values used in the simulations are as follows: $\sigma = 3$, $\alpha = 0.5$, $\gamma = 0.5$, $\phi = 2$, $\xi = 0.5$. Initial endowments are set to be $L_n = 2.5$, $L_s = 2.5$, $H_n = 1.6$, $H_s = 1.1$; initial machine blueprints for both countries are set to be $N_1 = 10$, $N_2 = 15$, $N_3 = 10$; initial trade technology is set to be $a = 0.8$; initial B is set high enough so that growth in at least one sector is possible early in the simulation; B grows 2 percent each time period.

[6]A further requirement here is that production between sectors is grossly substitutable, which implies that $\sigma > 1$. O'Rourke et al. 2007 describes this result in more detail.

technologies. In this way, trade and technology forces reinforce each other in the North to transform it into a highly skill-intensive environment. In the South, on the other hand, trade and technological forces countervail each other - while knowledge diffusion is skill-intensive, its rising comparative advantage is unskill-intensive. The latter effect strengthens by continually falling transport costs, lowering skilled labor in the South and thus weakening the former effect. Ultimately, trade effects dominate, creating a rapidly rising gap in living standards between the two regions. In essence, the South gets a double whammy - it specializes in the good that creates unskilled labor growth, and technology growth is "inappropriate" in the sense that it augments a shrinking sector.

The fourth and final phase is partially specialized trade (after $t = 41$). This equilibrium is described in Appendix B. At this point, the South abandons its production of y_3 altogether. The country now is essentially technologically stagnant - new machines are no longer produced in the South since only new varieties of N_3 are available, and the South has no more use for these. But once the South leaves its cone of diversification, Stolper-Samuelson effects are not as strong and so income per capita divergence does not rise as rapidly as before. Soon after, a converges to 1 and subsequent trade volumes do not rise very fast (although they do rise slightly afterwards due to continued skill-biased technological growth in the North). Divergence after this point arises mostly from technological growth in the North that the South cannot exploit.

Case 2: $[\frac{\Delta a}{a} = 0,\ \rho = 1]$. Here trade technologies do not improve, and so trade between the North and the South never occurs. Consequently there are only two growth phases - unskilled growth and balanced growth. As we saw in Case 1, both convergence and divergence are minuscule compared with the divergence that results from diversified and specialized trade. Any divergence that does occur is due to "inappropriate" technology diffusion - skill-biased growth is not as beneficial to the South due to a smaller skill endowment. But as we can see this source of divergence is very small compared with the forces of trade-induced specialization.

Case 3: $[\frac{\Delta a}{a} > 0,\ \rho = 0]$. Here trade costs exogenously fall as in Case 1, but now we have no technological diffusion from the North to the South. This case really has only three phases of growth - unskilled growth, balanced growth, and specialized trade. Because neither unskilled nor skilled technologies diffuse to the South, divergence naturally occurs between the two regions. Once trade becomes feasible (at around $t = 37$), divergence spikes up due to Stolper-Samuelson effects. Further, the South almost immediately abandons its production of y_3; this is because trade is now due to a combination of Heckscher-Ohlin and Ricardian effects, and so the North has an especially strong comparative advantage in producing and exporting y_3. Divergence continues to occur during the specialized trade phase from a combination of no technological growth in the South and increasing southern comparative advantage of unskilled-intensive production.

This final case provides an interesting contrast with our baseline case. Even though here we have absolutely no technological growth in the South, divergence is no higher here than in the

baseline case with *perfect* diffusion. The lack of knowledge diffusion in this case delays trade; this is because unskilled intensive growth makes the North lose its comparative advantage in skill-intensive production, and a prolonged period of balanced growth is required for the North to get it back. Thus the lack of technology growth in the South increases divergence, but the limited trade prospects this creates actually reduces divergence.

Thus our baseline case highlights the great irony of the Industrial Revolution - that while it was an equalizing force in incomes per capita across the world early on, it sowed the seeds for massive subsequent divergence. One could extrapolate forward from this case to suggest the unprecedented divergence of the 20th century. Technological progress in the North becomes strictly skilled-intensive. Not only does the South specialize in production that lowers education and fosters population growth, it no longer is able to benefit from any technological spillovers from the North! So perhaps the final answer to the question of which was the main force of divergence - biased technology spillovers or trade-induced specialization - is neither. Or rather, both - specialization patterns created a great deal of income divergence during the 19th century; on the other hand, the specialized trade phase creates divergence from "inappropriate" technology flows, and this might be more indicative of the 20th century. Both forces led to the wide disparity of incomes per capita we currently see.

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A Diversified Trade Equilibrium

With trade of goods y_1 and y_3 between the North and the South, productions in each region are given by (27) and (28).

For each region $c \in n, s$, the following conditions characterize the diversified trade equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{31}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{32}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{33}$$

$$\left(\frac{1}{A_1^c}\right)y_1^c + \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma-1}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^c = L^c \tag{34}$$

$$\left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^c + \left(\frac{1}{A_3^c}\right)y_3^c = H^c \tag{35}$$

$$y_1^n + a_1 Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{36}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{37}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}(p_2^c)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^c)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^c)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^c)^{1-\sigma}}\right)\cdot Y^c \tag{38}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{39}$$

$$y_3^s + a_3 Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{40}$$

$$A_1^n\left(A_1^n L_1^n + a_1 Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{41}$$

$$A_3^n\left(A_3^n H_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma-\sigma\gamma-1} \tag{42}$$

$$A_1^s \left(A_1^s L_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma-\sigma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{43}$$

$$A_3^s \left(A_3^s H_3^s + a_3 Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma-\sigma\gamma-1} \tag{44}$$

$$A_1^c = \left(N_{1,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t}^c - N_{1,t-1}^c\right)\right) \left(\alpha p_1^c\right)^{\frac{\alpha}{1-\alpha}} \tag{45}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t}^c - N_{2,t-1}^c\right)\right) \left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}} \tag{46}$$

$$A_3^c = \left(N_{3,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t}^c - N_{3,t-1}^c\right)\right) \left(\alpha p_3^c\right)^{\frac{\alpha}{1-\alpha}} \tag{47}$$

$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{48}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{49}$$

Equations (31) - (33) are unit cost functions, (34) and (35) are full employment conditions, (36) - (40) denote regional goods clearance conditions, (41) - (44) equate the marginal products of raw factors, (45) - (47) describe sector-specific technologies, and (48) and (49) describe the balance of payments for each region. Solving this system for the unknowns p_1^n, p_1^s, p_2^n, p_2^s, p_3^n, p_3^s, y_1^n, y_1^s, y_2^n, y_2^s, y_3^n, y_3^s, w_l^n, w_l^s, w_h^n, w_h^s, L_1^n, L_1^s, H_3^n, H_3^s, A_1^n, A_2^n, A_3^n, A_1^s, A_2^s, A_3^s, Z_1 and Z_3 constitutes the static partial trade equilibrium.

Each region will produce all three goods so long as factors and technologies are "similar enough." If factors of production or technological levels sufficiently differ, the North produces only goods 2 and 3, while the South produces only goods 1 and 2. No other specialization scenario is possible for the following reasons: first, given that both the North and South have positive levels of L and H, full employment of resources implies that they cannot specialize completely in good 1 or good 3. Second, specialization solely in good 2 is not possible either, since a region with a comparative advantage in this good would also have a comparative advantage in either of the other goods. This implies that each country must produce at least two goods. Further, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these goods, regardless of the technological differences between the two regions. See Cunat and Maffezzoli (2002) for a fuller discussion.

B Specialized Trade Equilibrium

The specialized equilibrium is one where the North does not produce any good 1 and the South does not produce any good 3. Productions in each region are then given by

$$Y^n = \left(\frac{\alpha}{2}\left(aZ_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^n\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(y_3^n - Z_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{50}$$

$$Y^s = \left(\frac{\alpha}{2}\left(y_1^s - Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^s\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(aZ_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{51}$$

Once again, we do not permit any trade of good 2. For each region $c \in n, s$, the following conditions characterize this equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{52}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{53}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{54}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma-1}\left(w_h^n\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^n = L^n \tag{55}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma}\left(w_h^n\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^n + \left(\frac{1}{A_3^n}\right)y_3^n = H^n \tag{56}$$

$$\left(\frac{1}{A_1^s}\right)y_1^s + \left(\frac{1}{A_2^s}\right)\left(w_l^s\right)^{\gamma-1}\left(w_h^s\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^s = L^s \tag{57}$$

$$\left(\frac{1}{A_2^s}\right)\left(w_l^s\right)^{\gamma}\left(w_h^s\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^s = H^s \tag{58}$$

$$a_1 Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^n\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^n\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^n\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^n\right)^{1-\sigma}}\right)\cdot Y^n \tag{59}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^s\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^s\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^s\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^s\right)^{1-\sigma}}\right)\cdot Y^s \tag{60}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}\left(p_2^c\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^c\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^c\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^c\right)^{1-\sigma}}\right)\cdot Y^c \tag{61}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^n\right)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}\left(p_1^n\right)^{1-\sigma} + (1-\alpha)^{\sigma}\left(p_2^n\right)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}\left(p_3^n\right)^{1-\sigma}}\right)\cdot Y^n \tag{62}$$

$$a_3 Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma} (p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma} (p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3^s)^{1-\sigma}} \right) \cdot Y^s \tag{63}$$

$$A_3^n \left(A_3^n H_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha} \right) A_2^{n\,\frac{\sigma-1}{\sigma}} \left(L^n\right)^{-\gamma+\sigma\gamma} \left(H^n - H_3^n\right)^{\gamma-\sigma\gamma-1} \tag{64}$$

$$A_1^s \left(A_1^s L_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha} \right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma-\sigma+\sigma\gamma} \left(H^s\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{65}$$

$$A_1^s = \left(N_{1,t-1}^s + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t}^s - N_{1,t-1}^s \right) \right) \left(\alpha p_1^s\right)^{\frac{\alpha}{1-\alpha}} \tag{66}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t}^c - N_{2,t-1}^c \right) \right) \left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}} \tag{67}$$

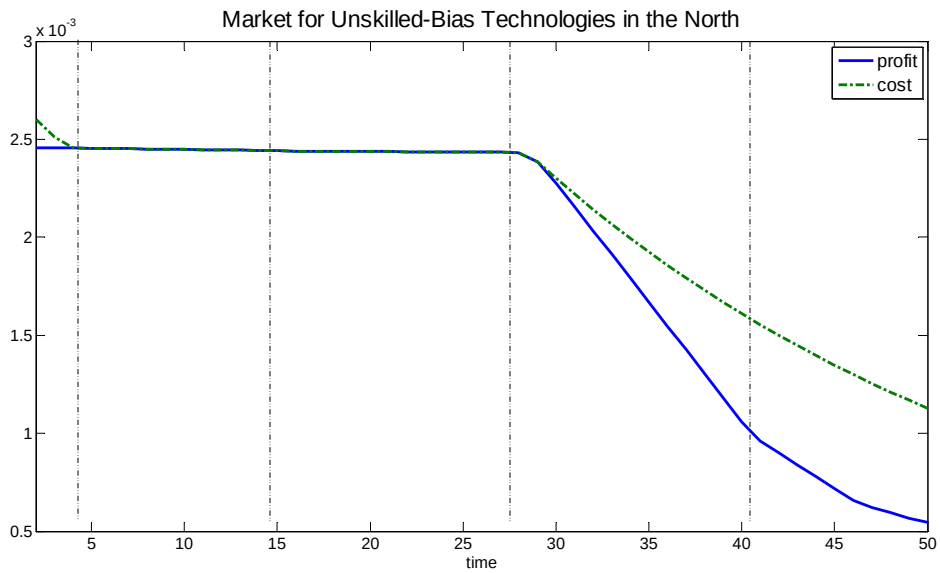
$$A_3^n = \left(N_{3,t-1}^n + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t}^n - N_{3,t-1}^n \right) \right) \left(\alpha p_n^c\right)^{\frac{\alpha}{1-\alpha}} \tag{68}$$

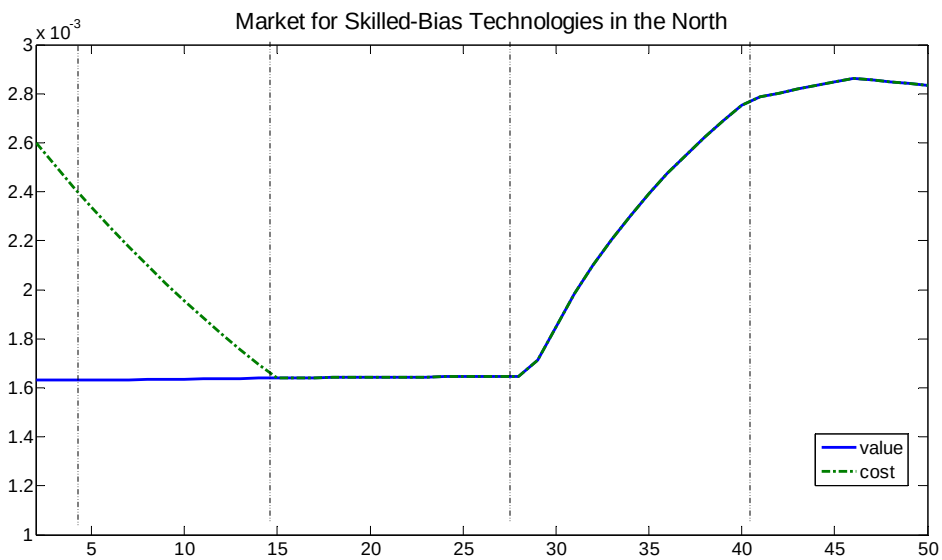
$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{69}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{70}$$

Figures

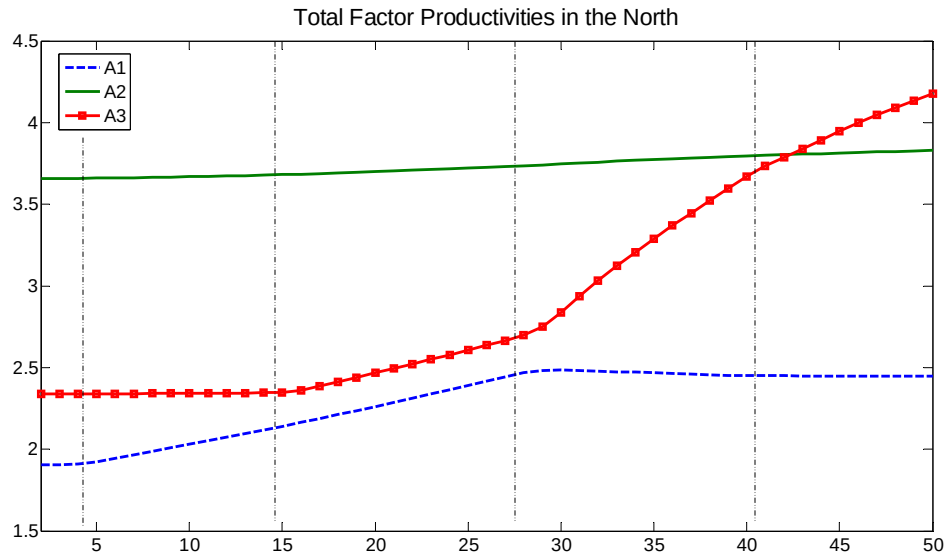
Case I – Perfect Knowledge Diffusion and Falling Trade Costs

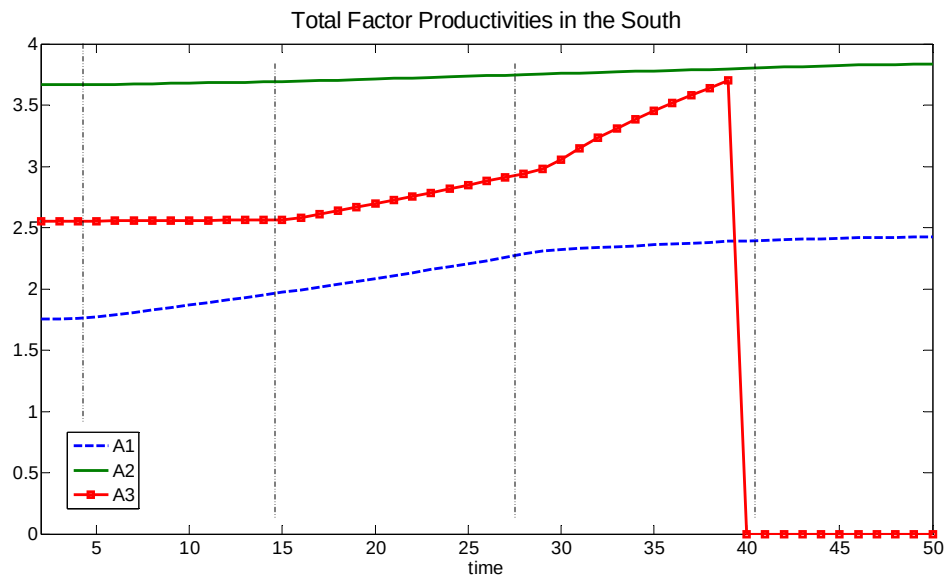




- The dotted lines indicate the start of each "phase" of development – unskilled labor-biased technological growth, balanced technological growth, diversified trade, and specialized trade.

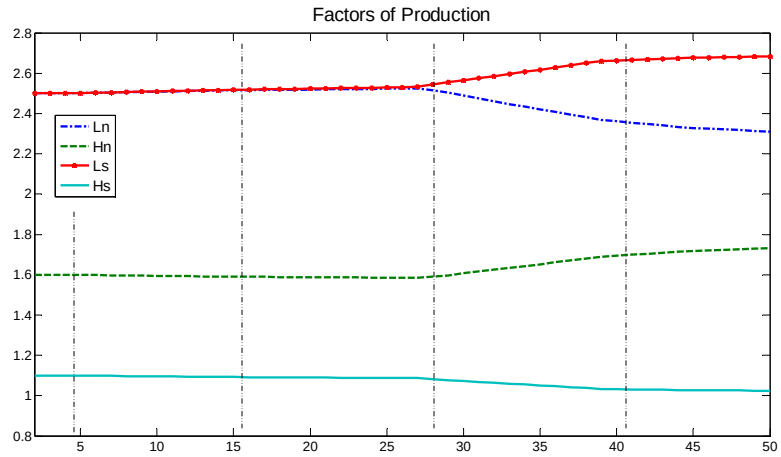
- Note that because $L_n > H_n$ (combined with the fact that $\sigma>1$) means that the revenues of new research in the unskilled sector is initially larger than those for the skilled sector. Consequently, unskilled knowledge is the first to grow.

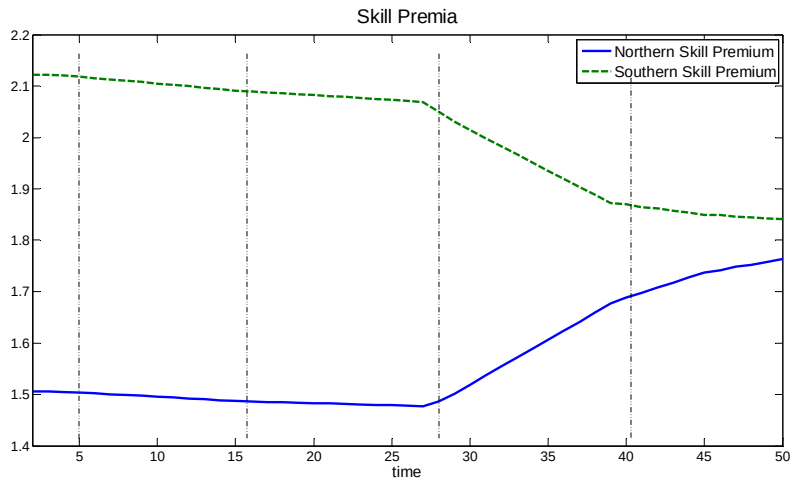


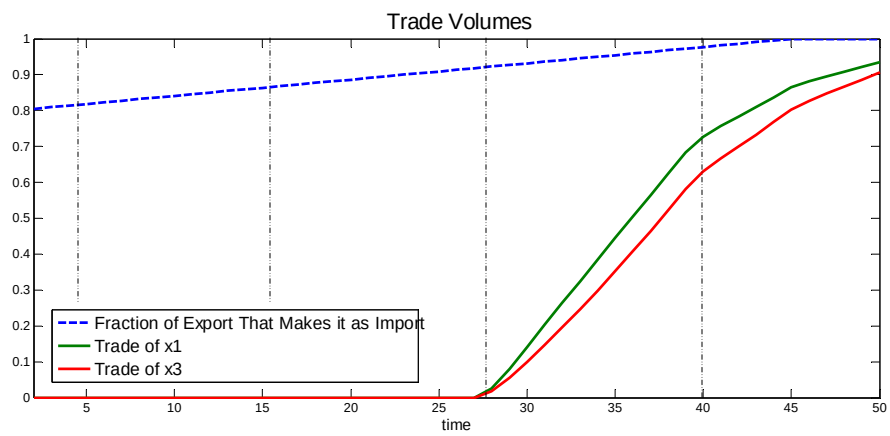


- From the cost-benefit analysis of potential research illustrated above, we see that unskilled technologies grow only during the first two phases; skilled technologies on the other hand grow during the last three phases. Note however that because sectoral productivities (given by equations 21-23) are functions of both number of blueprints and prices, productivities can slightly change even with technological stagnation.

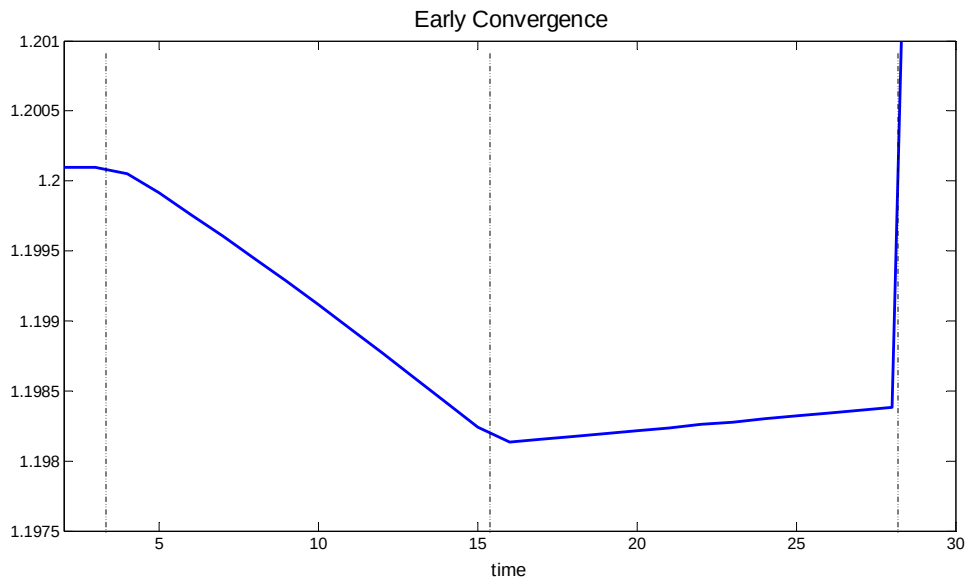
- Because in the specialized phase the South does not produce any y_3, it's productivity in this sector falls to zero.

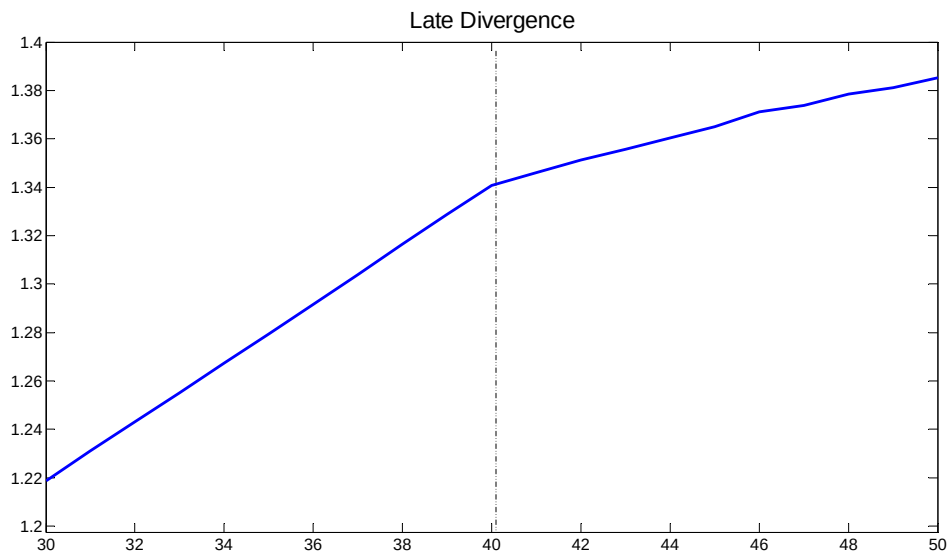






- The top two lines in the top figure are unskilled labor for the North and the South. During the first phase of unskill-intensive growth, the skill premia for both regions fall, and this induces an upward drift of unskilled labor and downward drift of skilled labor for both regions. The diversified trade phase produces the most rapid changes to relative wages (through a combination of Stolper-Samuelson effects and northern skill-intensive growth, each effect reinforcing the other), which in turn produces the greatest changes to factors of production.

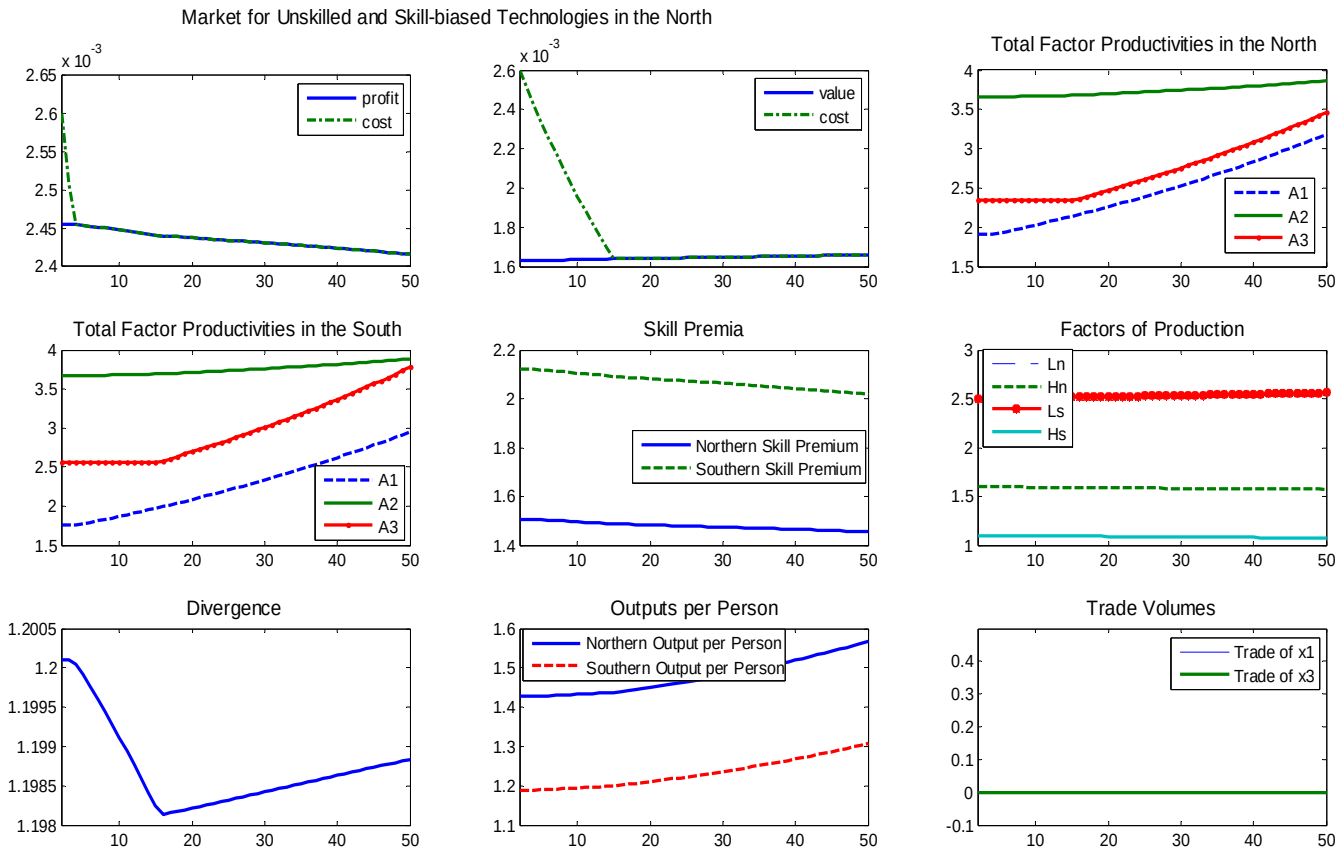




- Income per capita is defined as Y/L, and so these graphs chart the evolution of $(Y_n/ L_n)/(Y_s/ L_s)$.

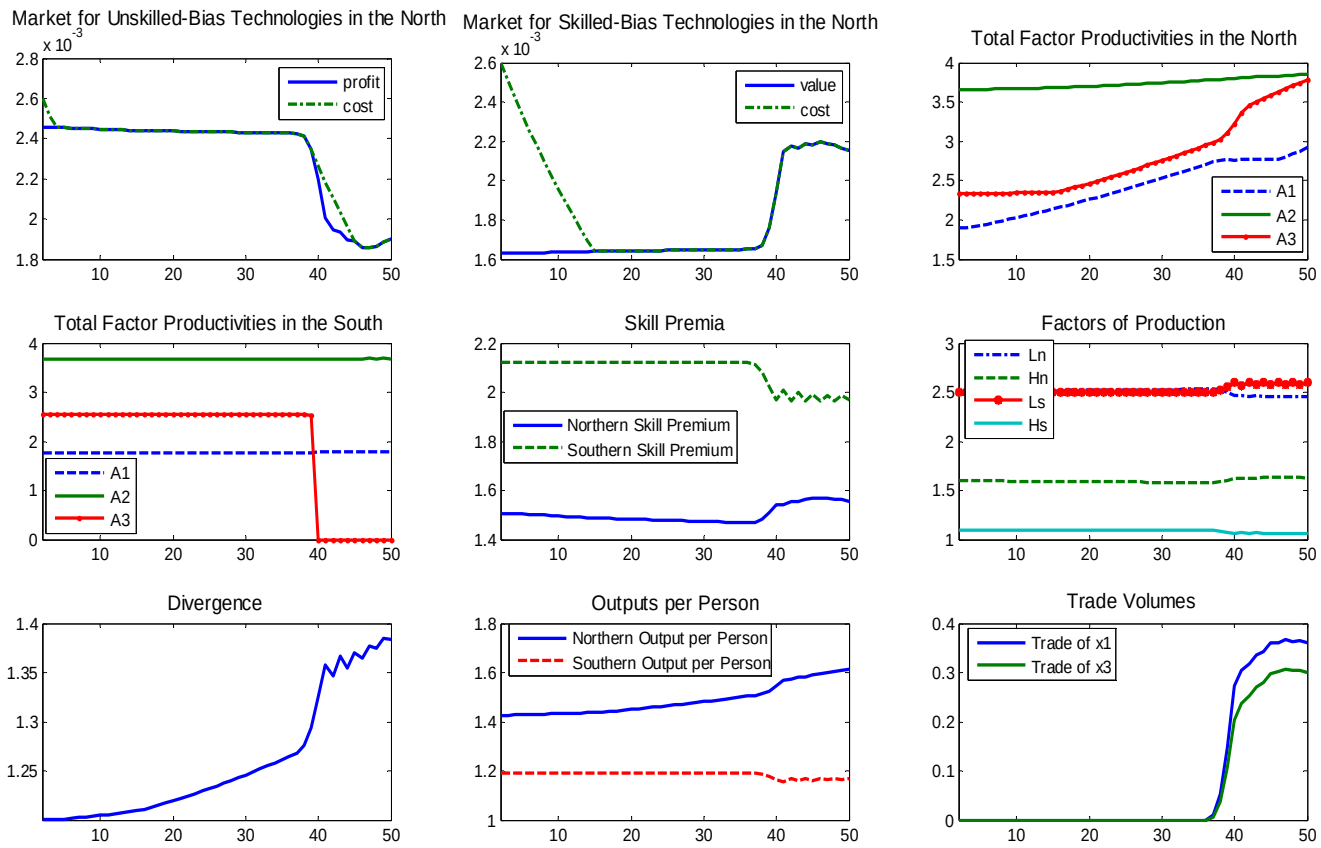
- The first stage of unskilled-intensive growth produces very slight convergence (about -0.15 % for 12 periods), while balanced growth essentially leads to a constant income per capita gap between the North and the South. Divergence truly takes off with diversified trade – this stage produces the fastest rising income gap (about a 10% increase over 10 periods). Finally, the specialized trade phase produces moderate divergence (primarily from "inappropriate" knowledge diffusion) of an additional 3% over 10 more periods.

Case II – Perfect Knowledge Diffusion and Constant Trade Costs



- With no trade permitted, there are no diversified or specialized trade phases. Any divergence that results is then strictly due to "inappropriate" technology flows from the North to the South. As we can see, such effects are miniscule without any accompanying trade effects.

Case III – No Knowledge Diffusion and Falling Trade Costs



- With no technological diffusion permitted, divergence early on is due strictly to the lack of technological growth in the South. Once trade becomes feasible, there is a burst of trade that results in an extra burst of divergence. Even though the South does not have any technological growth, cumulative divergence is no greater in this case than in Case I with *perfect* technological diffusion.