Trade, Technology and the Great Divergence*

(or An Economic History of the World in Just Thirty-Six Equations)

Kevin H. O'Rourke, Ahmed S. Rahman, and Alan M. Taylor

November 2009

Abstract

This paper develops a model that captures the key features of the Industrial Revolution and the Great Divergence between the industrializing "North" and the lagging "South." Industrialization in a few core northern regions produced beneficial spill-overs around the world, but still led to massive income divergence. A model is needed that combines both features to create a convincing story of 18th-19th and 19th-20th century growth. To this end we construct a trade/growth model that includes both endogenous biased technologies and intercontinental trade. The Industrial Revolution began as a sequence of unskilled-labor intensive innovations which initially incited fertility increases and limited human capital formation in both the North and the South. Diffusion of these technologies from the North to the South hastened trade between the two; this in turn fostered a divergence in incomes per capita between the North and the South, for two main reasons. One, the South increasingly specialized in technologically-stagnating areas of production. Two, the South increasingly specialized in production that fostered even greater fertility increases and educational decreases.

- *Keywords*: Industrial Revolution, unified growth theory, endogenous growth, demography, skill premium, Great Divergence
- *JEL Codes*: O, F, N

---

*Prepared for the French Alps FRESH meetings, September 8-9, 2009. Preliminary sketch. Please do not quote.

1 Introduction

The last two centuries have witnessed dramatic changes in the global distribution of income and population. At the dawn of the Industrial Revolution, living standards between the richest and poorest economies of the world were roughly 2 to 1. With industrialization came both income and population growth within a few core countries. But massive divergence in living standards across the globe did not take place until the latter half of the 19th century, the time when the first great era of globalization started to take shape. Today the gap between material living standards in the richest and poorest economies of the world is 30 or 40 to 1, in large part due to the events of the 19th century. It is an interesting coincidence then that unprecedented growth in inter-continental commerce (conceivably a great force of convergence) coincided so precisely with unprecedented divergence in living standards across the world. These phenomena beg an explanation. This paper argues that the diffusion of knowledge from core to peripheral regions spurred subsequent commerce between these regions and sowed the seeds for divergence, contributing enormously to today's great wealth disparity.

A number of the "stylized facts" of economic history motivate our theory. One concerns the nature of industrialization itself - technological change was unskilled labor-intensive during the early Industrial Revolution but became relatively skill-intensive during the latter nineteenth century. Indeed, England's early industrialization was in many respects largely a revolution in the cotton textile industry, with the adoption of the factory system of production and its associated new machinery. The textile industry was revolutionary in its ability to employ large numbers of unskilled and uneducated workers with minimal supervision, thus diminishing the productive role for skilled labor and education. By the 1850's, however, two major changes in technological growth occurred - it became much more widespread, and it became far more complementary to skilled workers.

Another historical feature of great importance was the North's expanding influence over the South in two major ways - through flows of technology and flows of trade. Indeed, changes spawned by the Industrial Revolution in the nineteenth century all seemed to predict that it would soon transform most of the world in precisely the same way it was transforming England, northwestern Europe and the United States. By 1900 for example, the economic centers of the "South" such as Alexandria, Bombay and Shanghai were fully integrated into the British economy, both in terms of transport costs and capital markets (Clark 2007).

Concerning the flows of knowledge, the new technologies of the Industrial Revolution could be exported mechanically with relative ease to most of the world. After all, while developing new knowledge was an arduous task, copying this knowledge was much easier. This was particularly true of the technologies of early industrialization; since they were not very sophisticated, they were quickly transmitted to, and easily adopted by, much of the world.

Concerning the flow of trade, intercontinental commerce between "western" economies and the rest of the world (what we might mildly mislabel as "North-South" trade) was not particularly robust until the latter half of the 19th century. By the 1840s steam ships were faster and more reliable than sailing ships, but their high coal consumption limited how much cargo they could transport; consequently only very light and valuable freight was shipped (O'Rourke and Williamson 1999). But by 1870 a number of innovations dramatically reduced the cost of steam ocean transport, and real ocean freight rates fell by nearly 35% from 1870 to 1910. Thus, while a closed economy model would be more appropriate to describe the first stages of the Industrial Revolution (1750-1850), a more open economy framework would better describe the latter stages of industrialization (1850-1910).

To analyze the intellectual puzzle of the Great Divergence, we develop a model that has a number of key features which mimic these historical realities. The first feature of our approach is that we endogenize the direction of technological change that occurs in the North. Technologies are sector specific, and sectors have different degrees of skill intensities. Following the endogenous growth literature, we allow potential innovators in the North to observe the employment of factors in different sectors, and tailor their research efforts towards particular sectors. Thus the direction of innovation will depend on Northern employment and demography. Further, these innovations have the potential to diffuse to the South. These technologies are not tailor made for the South, but nonetheless can aid the South through knowledge transmission and adoption.

The second key feature is that we endogenize demography itself. More specifically, we allow households to make education and fertility decisions based on market wages for skilled and unskilled labor. The method is similar to other endogenous demography models where households face a quality/quantity tradeoff with respect to their children. Thus, when the premium for skilled labor rises families choose to have fewer but better educated children.

The final feature is that we allow for burgeoning trade between the North and the South. During the initial stages of industrialization, trade is not possible due to prohibitively high transport costs. These costs however exogenously decrease over time; at a certain point trade becomes feasible, at which point the South exchanges labor-intensive products for the North's skill-intensive products. At this stage development paths begin to diverge - the North's specialization in skilled production produces a demographic transition, while the South's specialization in unskilled production stifles technological growth and produces more population.

With this basic setup, we simulate the model to roughly capture the main features of industrialization and divergence between the North and the South from roughly 1750 to 1900. Because of the great abundance of unskilled labor, innovators in the North first develop unskilled-intensive technologies, and these flow to and are adopted by the South. Thus early industrialization is characterized by unskilled intensive technological growth and population growth both in the North and the South; consequently living standards in the two regions do not diverge. This diffusion of knowledge however hastens the possibility of trade - at this point the North starts specializing in skill-intensive innovation and production. This induces a demographic transition of falling fertility and rising education rates in the North. The South of the other hand specializes in unskilled-intensive production, inducing both technological stagnation and further population growth. Thus the South's static gains from trade become a dynamic impetus to prosperity, and living standards between the two regions diverge dramatically as a result.

We argue that analyzing the interactions between the North and the South, and between trade and technological flows, is critical to understanding both the Industrial Revolution and the Great Divergence. In this respect the paper relates most closely to Galor and Mountford (2006, 2008) (GM). They similarly argue that northern gains from trade translated into rising education and income, while southern gains from trade mainly translated into population increases. However, our model differs from theirs in several important respects. First, the GM approach employs a semi-Ricardian model with no technological diffusion; consequently the North always maintains a technological edge in skill-intensive production. Instead, we employ a Heckscher-Ohlin model where trade arises from factoral differences, allowing for the possibility of *perfect* technological diffusion. Both features are relevant for this study - we know that Heckscher-Ohlin oriented trade was important during the 19th century since commodity price convergence induced factor price convergence during this time (O'Rourke and Williamson 1994; O'Rourke, Taylor and Williamson 1996; O'Rourke and Williamson 1999, Chapter 4), and we know that technologies readily diffused from the industrializing North to the rest of the world (Mokyr 1999; Clark 2007, Chapter 15).

Further, since no knowledge diffusion is allowed, GM cannot comment on the "appropriateness" of technological diffusion. From the endogenous growth literature, we know that the nature of technological progress depends on many local conditions, including prices and factor endowments (Acemoglu 2002). But the flow of these technologies to other regions may not benefit the receiving countries if they do not have appropriate factor endowments (Basu and Weil 1998; Acemoglu and Zilibotti 2001). Very different from GM, our model uses the tools of endogenous growth theory to analyze this additional channel of divergence between the North and the South.

Finally, rather than suddenly open up the North and South to trade, we allow for *gradual* increases in North-South commerce. The British economy (and other Western economies) presumably did not undergo a discontinuous switch from a closed to an open state, and thus we will impose continuously declining transport costs to achieve such a transition.

2 Production with Given Technologies and Factors

Here we sketch out a model that we will use to describe both a northern economy and a southern economy (subscripts denoting region are suppressed for the time being).

Total production for a region is given by:

$$Y = \left(\frac{\alpha}{2} y_1^{\frac{\sigma-1}{\sigma}} + (1-\alpha) y_2^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} y_3^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\alpha \in [0,1]$ and $\sigma \geq 0$. σ is the elasticity of substitution among intermediate goods y_1, y_2, and y_3. The production of these goods in turn are given by:

$$y_1 = A_1 L_1 \tag{2}$$

$$y_1 = A_2 L_2^{\gamma} H_2^{1-\gamma} \tag{3}$$

$$y_3 = A_3 H_3 \tag{4}$$

where A_1, A_2 and A_3 are the technological levels of sectors 1, 2, and 3, respectively. These technological levels in turn are represented by a series of *sector-specific* machines. Specifically,

$$A_1 = \int_0^{N_1} \left(\frac{x_1(j)}{L_1} \right)^{\alpha} dj \tag{5}$$

$$A_2 = \int_0^{N_2} \left(\frac{x_2(j)}{L_2^{\gamma} H_2^{1-\gamma}} \right)^{\alpha} dj \tag{6}$$

$$A_3 = \int_0^{N_3} \left(\frac{x_3(j)}{H_3} \right)^{\alpha} dj \tag{7}$$

where $x_i(j)$ is machine of type j that can be employed only in sector i. Technological progress in sector i can then be represented by growth in the number of machine-*types* that are extant for the sector, represented by N_i (we endogenize the growth of these in the next sections).

Treating technological coefficients as exogenous for the time being, we can assume that markets for both the final good and intermediate goods are perfectly competitive. Thus, prices are equal to unit costs. Solving the cost minimization problems for productions, and normalizing the price of final output to one, yields the unit cost functions

$$1 = \left[\left(\frac{\alpha}{2} \right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2} \right)^{\sigma} (p_3)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

$$p_1 = \frac{w_l}{A_1} \tag{9}$$

$$p_2 = \left(\frac{1}{A_2} \right) w_l^{\gamma} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{-\gamma} \tag{10}$$

$$p_3 = \frac{w_h}{A_3} \tag{11}$$

where p_i denotes the price for intermediate good y_i, w_l is the wage paid to L and w_h is the wage paid to H.

Full employment of total unskilled labor and total skilled labor implies the following factor-market clearing conditions:

$$L = \frac{y_1}{A_1} + \frac{w_l^{\gamma-1} w_h^{1-\gamma} (1-\gamma)^{\gamma-1} \gamma^{1-\gamma} y_2}{A_2} \tag{12}$$

$$H = \frac{w_l^{\gamma} w_h^{-\gamma} (1-\gamma)^{\gamma} \gamma^{-\gamma} y_2}{A_2} + \frac{y_3}{A_3} \tag{13}$$

Finally, the demands for intermediate goods from final producers can be derived from a standard C.E.S. objective function.[1] Specifically, intermediate goods market clearing requires

$$y_i = \left(\frac{\Upsilon_i^{\sigma} p_i^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma} (p_1)^{1-\sigma} + (1-\alpha)^{\sigma} (p_2)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma} (p_3)^{1-\sigma}} \right) Y \tag{14}$$

for $i = 1, 2, 3$, $\Upsilon_1 = \Upsilon_3 = \alpha/2$, and $\Upsilon_2 = 1 - \alpha$.

Provided that we have values for L, H, A_1, A_2 and A_3, along with parameter values, this yields eight equations with eight unknowns: p_1, p_2, p_3, x_1, x_2, x_3, w_l and w_h. The solution for these variables constitutes the solution for the *static* model in the case of exogenously determined technological and demographic variables.

3 Endogenizing Technologies in the North

In this section we describe how innovators in the North endogenously develop new technologies. In general, modeling purposive research and development effort is difficult when prices and factors change over time. This is because it is typically assumed that the gains from innovation will flow to the innovator throughout his lifetime, and this flow will often depend on the price of the product being produced and the factors required for production at each moment in time.[2] If prices and factors are constantly changing (as they will in any economy where trade barriers fall gradually or factors evolve endogenously), a calculation of the true discounted profits from an invention may be impossibly complicated.

To avoid such needless complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last *one time period only.* More specifically, technological progress is sector-specific, and comes about though increases in the varieties of machines employed in each sector. New varieties of machines are developed by profit-maximizing

[1] Here demands will be negatively related to own price, will be a function of a price index, and will be proportional to total product.

[2] For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

inventors, who *monopolistically* produce and sell the machines to competitive producers of the intermediate goods y_1, y_2 or y_3. However, we assume the blueprints to these machines become public knowledge the time period after the machine is invented, at which point these machines become old and are *competitively* produced and sold.[3] Thus while we need to distinguish between old and new sector-specific machines, we avoid complicated dynamic programming problems inherent in multiple-period profit streams.[4]

Thus, we can re-define sector-specific technological levels given by (5) - (7) as a series of both old and new machines at time t as:

$$A_{1,t} = \left(\int_0^{N_{1,t-1}} x_{1,old}(j)^{\alpha} dj + \int_{N_{1,t-1}}^{N_{1,t}} x_{1,new}(j)^{\alpha} dj \right) \left(\frac{1}{L_1} \right)^{\alpha}$$

$$A_{2,t} = \left(\int_0^{N_{2,t-1}} x_{2,old}(j)^{\alpha} dj + \int_{N_{2,t-1}}^{N_{2,t}} x_{2,new}(j)^{\alpha} dj \right) \left(\frac{1}{L_2^{\gamma} H_2^{1-\gamma}} \right)^{\alpha}$$

$$A_{3,t} = \left(\int_0^{N_{3,t-1}} x_{3,old}(j)^{\alpha} dj + \int_{N_{3,t-1}}^{N_{3,t}} x_{3,new}(j)^{\alpha} dj \right) \left(\frac{1}{H_3} \right)^{\alpha}$$

where $x_{i,old}$ are machines invented before t, and $x_{i,new}$ are machines invented at t. Thus in each sector i there are N_{t-1} varieties of old machines that are competitively produced, and there are $N_t - N_{t-1}$ varieties of new machines that are monopolistically produced (again, suppressing country subscripts).

Next, we must describe producers of intermediate goods in each region. These three different groups of producers each separately solve the following maximization problems:

Sector 1 producers: $\max_{[L_1, x_1(j)]} p_1 y_1 - w_l L_1 - \int_0^{N_1} \chi_1(j) x_1(j) dj$

Sector 2 producers: $\max_{[L_2, H_2, x_2(j)]} p_2 y_2 - w_l L_2 - w_h H_2 - \int_0^{N_2} \chi_2(j) x_2(j) dj$

Sector 3 producers: $\max_{[H_3, x_3(j)]} p_3 y_3 - w_h H_3 - \int_0^{N_3} \chi_3(j) x_3(j) dj$

where $\chi_i(j)$ is the price of machine j employed in sector i. For each type of producer, solving the maximization problem with respect to machine j yields a solution for machine demand:

$$x_1(j) = \chi_1(j)^{\frac{1}{\alpha-1}} (\alpha p_1)^{\frac{1}{1-\alpha}} L_1 \tag{15}$$

$$x_2(j) = \chi_2(j)^{\frac{1}{\alpha-1}} (\alpha p_2)^{\frac{1}{1-\alpha}} L_2^{\gamma} H_2^{1-\gamma} \tag{16}$$

[3] Here one can assume either that patent protection for intellectual property lasts one time period, or that it takes one time period for potential competitors to reverse-engineer the blueprints for new machines.

[4] See Rahman (2008) for more discussion of this simplifying assumption.

$$x_3(j) = \chi_3(j)^{\frac{1}{\alpha-1}} (\alpha p_3)^{\frac{1}{1-\alpha}} H_3 \tag{17}$$

New machine producers, having the sole right to produce the machine, set the price of their machines to maximize instantaneous profit. This price will be a constant markup over the marginal cost of producing a machine. Assuming that the cost of making a machine is unitary implies that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1/\alpha$ for new machines. Thus, substituting in this mark-up price, and realizing that instantaneous profits are $(1/\alpha) - 1$ multiplied by the number of new machines sold, instantaneous revenues by new machine producers are given by:

$$\pi_1 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_1)^{\frac{1}{1-\alpha}} L_1 \tag{18}$$

$$\pi_2 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_2)^{\frac{1}{1-\alpha}} L_2^{\gamma} H_2^{1-\gamma} \tag{19}$$

$$\pi_3 = \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} (p_3)^{\frac{1}{1-\alpha}} H_3 \tag{20}$$

Old machines, on the other hand, are competitively produced; competition will drive the price of all these machines down to marginal cost, so that $\chi_1(j) = \chi_2(j) = \chi_3(j) = \chi = 1$ for all old machines. Sectoral productivities can then be expressed simply as a combination of old and new machines demanded by producers. Plugging in the appropriate machine prices into our machine demand expressions (15) - (17), and plugging these machine demands into our sectoral productivities, we can express these productivities as:

$$A_1 = \left(N_{1,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{1,t} - N_{1,t-1})\right) (\alpha p_1)^{\frac{\alpha}{1-\alpha}} \tag{21}$$

$$A_2 = \left(N_{2,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{2,t} - N_{2,t-1})\right) (\alpha p_2)^{\frac{\alpha}{1-\alpha}} \tag{22}$$

$$A_3 = \left(N_{3,t-1} + \alpha^{\frac{\alpha}{1-\alpha}} (N_{3,t} - N_{3,t-1})\right) (\alpha p_3)^{\frac{\alpha}{1-\alpha}} \tag{23}$$

Thus, given the number of old and new machines that can be used in each sector, we can simultaneously solve equations (8) - (14) and (21) - (23) to solve for prices, wages, intermediate goods and technological levels for a hypothetical economy. Our next goal then is to also endogenize the levels of skilled and unskilled labor in this hypothetical economy.

4 Endogenizing Population and Labor-Types in Both Regions

We now introduce an over-lapping generations framework, where individuals live for two time periods. In their youths individuals work as unskilled workers; this income is consumed by their parents. When they become adults, individuals decide whether or not to expend a fixed resource cost to become a skilled worker. Adults also decide how many of their own children to have, who earn unskilled income for the adults. Adults however are required to forgo some income for child-rearing.

Specifically, an adult i's objective is to maximize current-period income. If an adult chooses to remain an unskilled worker (L), she aims to maximize I_l with respect to her number of children, where

$$I_l = w_l + n_l w_l - w_l \lambda n_l^{\phi} \tag{24}$$

w_l is the unskilled labor wage, n_l is the number of children that the unskilled adult has, and $\lambda > 0$ and $\phi > 1$ are constant parameters that affect the opportunity costs to child-rearing..

If an adult chooses to spend resources to become a skilled worker, she instead maximizes I_h with respect to her number of children, where

$$I_h = w_h + n_h w_l - w_h \lambda n_h^{\phi} - \tau_i \tag{25}$$

w_h is the skilled labor wage, n_h is the number of children that the skilled adult has, and τ_i is the resources she must spend to become skilled.

The first order conditions for each of these groups give us the optimal fertility for each group, n_l^* and n_h^*:

$$n_l^* = (\phi\lambda)^{\frac{1}{1-\phi}} \tag{26}$$

$$n_h^* = \left(\frac{w_h}{w_l}\phi\lambda\right)^{\frac{1}{1-\phi}} \tag{27}$$

Note that with $w_h > w_l$, the optimal fertility for a skilled worker is always smaller than that for an unskilled worker (this is simply because the opportunity costs of child-rearing are larger for skilled workers). Also note that the fertility for unskilled workers is constant, while the fertility for skilled workers falls with increases in the skill premium.

Finally, assume that τ varies across adults. The resource costs necessary to acquire an education can vary across individuals for many reasons, including differing incomes, access to schooling, or innate abilities. Say τ_i is uniformly distributed across $[0, b]$, where $b > 0$. An individual i who draws a particular τ_i will choose to become a skilled worker only if her optimized income as a skilled worker will be larger than her optimized income as an unskilled worker. Let us call τ^* the *threshold* cost to education; this is the education cost where the adult is indifferent between becoming a skilled worker or remaining an unskilled worker. Solving for this, we get

$$\tau^* = w_h + n_h^* w_l - w_h \lambda n_h^{*^\phi} - w_l - n_l^* w_l + w_l \lambda n_l^{*^\phi} \tag{28}$$

Only individuals whose τ_i fall below this level will opt to become skilled.

Figures 1 and 2 illustrate optimal fertility rates for two individuals - one with a relatively high τ and one with a relatively low τ. The straight lines illustrate how earnings increase as adults have more children; the slope of these lines is simply the unskilled wage w_l. The earnings line for a skilled worker is shifted up to show that she earns a premium. Cost curves get steeper with more children since $\phi > 1$. For skilled individuals, the cost curve is both higher (to illustrate the resource costs τ necessary to become skilled) and steeper (to illustrate the higher opportunity cost to have children). Notice then that the only difference between the high-τ individual and the low-τ individual is that the latter individual has a lower cost curve. The optimal fertility rates however are the same for both types of adults. Given these differences in the fixed costs of education, we can see that the high-τ individual will opt to remain an unskilled worker (and so have a fertility rate of n_l^*), while the low-τ individual will choose to become skilled (and have a fertility rate of n_h^*).

With this we can describe aggregate levels of skilled and unskilled labor (also described by full employment conditions (12) and (13)), fertility and education. Given a total adult population equal to *pop*, we can describe these variables as:

$$H = \left(\frac{\tau^*}{b}\right) pop \tag{29}$$

$$L = \left(1 - \frac{\tau^*}{b}\right) pop + npop \tag{30}$$

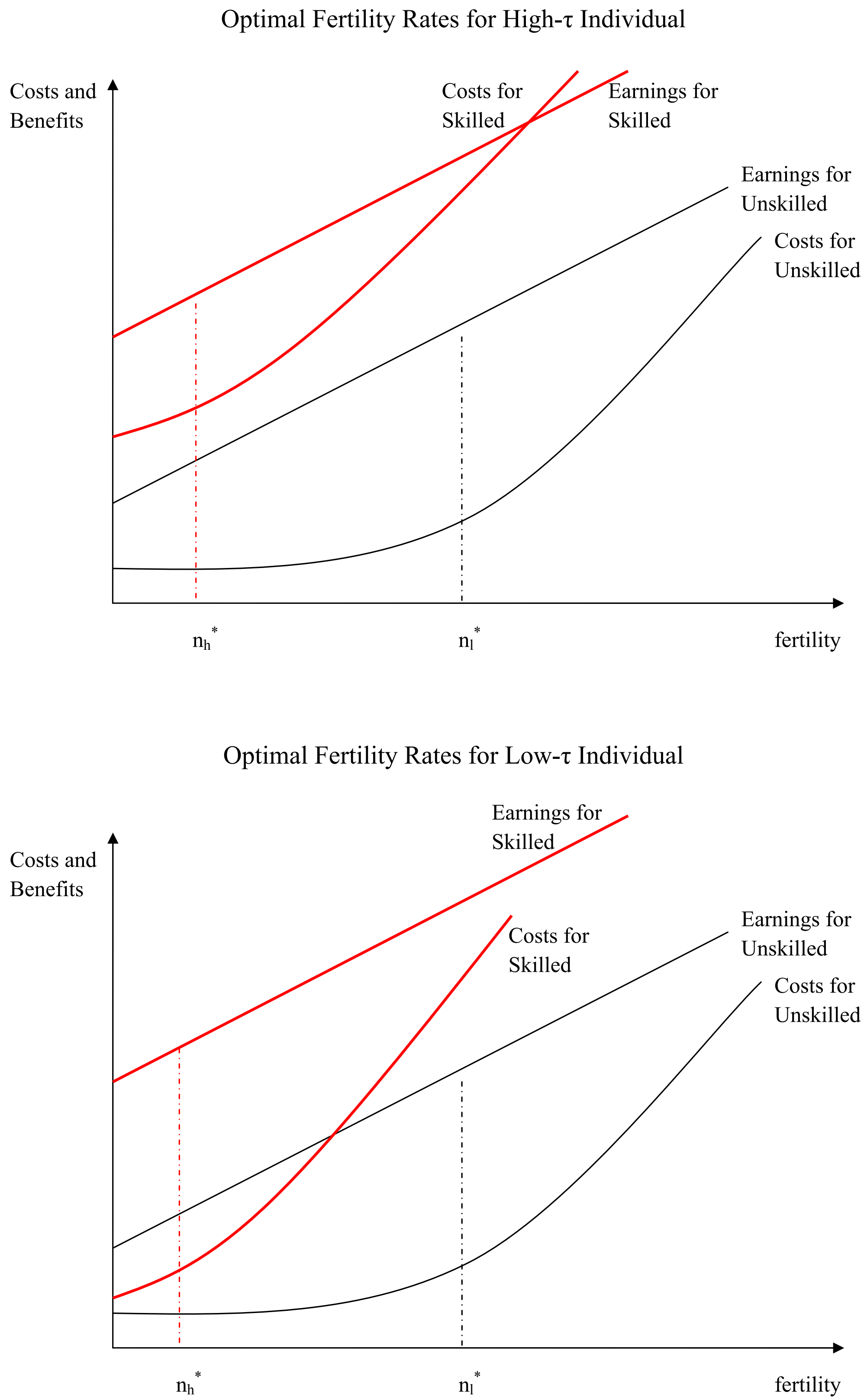
$$n = \left(1 - \frac{\tau^*}{b}\right) n_l^* + \left(\frac{\tau^*}{b}\right) n_h^* \tag{31}$$

$$e = \frac{\tau^*}{b} \tag{32}$$

where H is the number of skilled workers (comprised strictly of old workers), L is the number of unskilled workers (comprised of both old and young workers), n is aggregate fertility, and e is the fraction of the workforce that gets an education.

This completes the description of the static one-country model. The next section uses the theory to hypothetically interact two economies in history.

Figure 1: Optimal Fertility Rates for High and Low τ Individuals (for given w_l and w_h)



5 The Roles of Trade and Technological Diffusion in the Great Divergence

Technological diffusion to, and adoption by, under-developed regions is typically considered a source of income convergence between economies (Findlay 1996). In this section we will suggest that diffusion of knowledge from the North to the South actually sowed the seeds for eventual and inevitable divergence. To show this, we perform a thought experiment by simulating two economies. The above model describes a hypothetical country - now we will use it to describe both a "northern" economy (one that develops its own machine blueprints and is relatively more skill-endowed) and a "southern" economy (one that benefits from diffused northern machine blueprints and is relatively more unskilled labor-endowed).

The simulations demonstrate a number of things. Early industrialization came from the North and was unskilled labor intensive (O'Rourke et al 2008). The more these techniques were able to diffuse to the South, the *earlier* was trade able to occur between the regions (this is because more unskilled-intensive knowledge growth in the South allowed the South to retain its comparative advantage in unskilled-intensive production). Trade however fostered divergence for two reasons. One, the South increasingly specialized in technologically-stagnant production (the North stopped innovating for unskilled labor, for it was no longer profitable). Two, the South increasingly specialized in production that fostered even greater fertility increases and educational stagnation.

To simulate this tale however, we will first need to endogenize the time paths of technologies and trade volumes.

5.1 A Dynamic Model - The Growth of Technologies

How do technologies grow in the North? Recall that equations (18) - (20) describe one-period revenues for innovation. There must also be some resource costs to research. For this, we can assume that these costs are rising in N (applied knowledge), and falling in some measure of "general" knowledge, given by B (basic knowledge). Thus, a no-arbitrage (free entry) condition for potential researchers in the North can be described as:

$$\pi_i \leq c\left(\frac{N_i}{B}\right) \tag{33}$$

Specifically, we can assume the following functional form for these research costs:

$$c\left(\frac{N_i}{B}\right) \leq \left(\frac{N_{i,t+1}}{B_t}\right)^{\nu} \tag{34}$$

for $i = 1, 3$ (for convenience we assume no research occurs in sector 2. This way technological growth is unambiguously factor-biased), and $\nu > 0$. Given some level of basic knowledge (which we can assume grows at some exogenous rate) and number of existing machines, we can determine the resource costs of research. When basic knowledge is low relative to the number of available machine-types used in sector i, the costs of inventing a new machine in sector i is high (see O'Rourke et al. 2008 for a fuller discussion). Thus from (33) and (18) - (20) we see that northern innovation in sector i becomes more attractive when basic knowledge is large, when the number of machine-types in sector i is low, when then price of good i is high, and when the employment in sector i is high.

Note that if $\pi_i > c(N_i/B)$, there are potential profits from research in sector i. However, this will induce research activity, increasing the number of new machines, and hence costs of research, up. We assume in fact that N_i adjusts upward such that costs of research just offset the revenues of new machine production. Thus increases in B are matched by increases in northern levels of N_i such that the no-arbitrage condition holds with equality whenever technological growth in the sector occurs.

So how do technologies grow in the South? We can reasonably assume for this period that the South has no ability to innovate on their own. Rather, they rely on the North to inherit new blueprints for machines; once these blueprints are received by someone in the South, they themselves become monopolists (again, for just one time period) who then earn profits that can be characterized by equations (18) - (20). Thus, while certainly the South possesses intellectual property protection (limited to one time period, same as the North), there is no ability by southerners to perform independent research.

Specifically, we assume that the number of machine blueprints in the South evolve according to the following relationship:

$$\Delta N^S_{i,t+1} = \rho \left(N^N_{i,t+1} - N^S_{i,t} \right) \tag{35}$$

where $0 \leq \rho \leq 1$ is the extent of the the diffusion of blueprints from the North to the South. Southern technological levels will then be similarly described by (21) - (23), but with southern-specific number of blueprints (potentially lower than the North) and southern-specific prices.

Finally, we must specify how trade technologies evolve. Here we use an amended version of (1), where production for each region is given by

$$Y^n = \left(\frac{\alpha}{2} \left(y^n_1 + aZ_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(y^n_2 \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(y^n_3 - Z_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{36}$$

$$Y^s = \left(\frac{\alpha}{2} \left(y^s_1 - Z_1 \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(y^s_2 \right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2} \left(y^s_3 + aZ_3 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{37}$$

Z_1 is the amount of good 1 that is exported by the South, Z_3 is the amount of good 3 that is exported by the North, and $0 < a < 1$ is an iceberg factor for traded goods (i.e. the proportion of exports not lost in transit). Thus the North imports only fraction a of Southern exports, and the South imports only fraction a of Northern exports. Intermediate goods production is still described by (2) - (4).

5.2 Evolution of the World Economy

General equilibrium is a thirty-six equation system that solves for prices, wages, fertility, education, labor-types, intermediate goods, employment, trade, and sectoral productivity levels for both the North and the South. There are two primary differences between the two regions - the South has no capability to innovate themselves, and $b^n < b^s$ (this means that $\tau^{*^n} > \tau^{*^s}$, so that the South begins with relatively more unskilled labor than skilled labor). All other parameters are the same in both regions. Fertility is normalized to one in the beginning for each country, so that population is stable. The equilibrium is described in more detail in the appendix.

Because the model contains so many moving parts, we can only solve for general equilibrium numerically. Specifically, we assume that both basic technology (from eq 34) and trade technology (from eqs 36 and 37) start low enough so that neither technological progress nor trade are possible. We allow however for exogenous growth in basic knowledge and trade technologies, and solve for the endogenous variables each period. Let us first summarize the evolution of these two economies with a few propositions, starting with the nature of early industrialization.

Proposition 1 *If $N_1 = N_3$ and $L > H$ in the North, and $\sigma > 1$, initial technological growth will be unskilled-labor biased.*

From (18)-(20) we can see that revenues from innovation rise both in the price of the intermediate good (the "price effect") and in the scale of sectoral employment (the "market-size effect). If intermediate goods are grossly substitutable, market-size effects will outweigh price effects (see Acemoglu 2002 for more discussion of this).

Thus as basic knowledge exogenously grows, sector 1 will be the first to modernize. The logical implication of this is that early industrialization around the world (provided there is some technological diffusion) will be unskilled labor intensive (O'Rourke et al. 2008).

Proposition 2 *If $\left(\frac{p_3^n}{p_1^n}\right) / \left(\frac{p_3^s}{p_1^s}\right) > a^2$, $Z_1 = Z_3 = 0$.*

If transport costs are large (that is, if a is small) relative to cross-country price differences, no trade occurs. As mentioned above, we will assume that early on transport technologies are not advanced enough to permit trade, but as they rise they will allow Z_1 and Z_3 to rise as well. Note that we assume that there is no trade in y_2 - because this is produced using both L and

H, differences in p_2 are very small between the North and the South, and thus the assumption is not very restrictive or important.[5] Further, the limiting case of $a = 1$ produces goods and factor price convergence, and thus (in the case of identical technologies in the North and South) replicates the integrated equilibrium, even in the absence of trade in good 2.

Proposition 3 *The larger ρ is, the smaller will trade technology a need to be to allow trade to occur between the North and the South.*

This statement implies that greater technological diffusion hastens the arrival of the point where the North and South can trade. We can see from (9) - (11) that if A_l/A_3 rises in the North (as implied by Proposition 1), p_3^n/p_1^n rises as well. By Proposition 2, this means that trade will be possible for only a smaller higher range of a-values. With unskilled-biased technological diffusion however, the price ratio in the South mimics those in the North. Such diffusion allows the South to retain its comparative advantage in unskilled labor intensive production, so that the iceberg cost threshold where trade is feasible is lowered.

Proposition 4 *For given factors and technologies, there is a threshold level of trade technology a, where all values of a above this point imply that $y_3^s = 0$. There is another threshold level of trade technology a, where all values of a above this point imply that $y_1^n = 0$.*

As trade technologies improve, economies specialize more and more. There is indeed a point where they improve such that the North no longer needs to produce any y_1 (they just import it from the South), and the South no longer needs to produce any y_3 (they just import it from the North). This case we will call the "specialized trade equilibrium" (described in more detail in the Appendix).

Both trade and technological changes will change factor payments. The final proposition states how these changes can affect the factors of production themselves.

Proposition 5 *If $\phi > 1$, any increase in w_l (keeping w_h constant) will induce a decrease in e and an increase in n; furthermore, so long as ϕ is "big enough," any increase in w_h (keeping w_l constant) will induce an increase in e and a decrease in n.*

Proof.
Substituting our expressions for n_l^* and n_h^*, given by (26) and (27), into our expression for τ^*, given by (28), and rearranging terms a bit, we get the following expression:

[5] Indeed, trade in all three goods would produce an analytical problem. It is well known among trade economists that when there are more traded goods than factors of production, country-specific production levels, and hence trade volumes, are indeterminate. See Melvin (1968) for a thorough discussion.

$$\tau^* = (w_h - w_l) \,-\, w_l \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right) \,+\, wl^{\frac{\phi}{\phi-1}} wh^{\frac{1}{1-\phi}} \lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right)$$

First we must have the condition $\frac{\partial \tau^*}{\partial w_l} < 0$ hold. Solving for this and rearranging yields

$$\left(\frac{w_l}{w_h} \right)^{\frac{1}{\phi-1}} < 1 + \frac{1}{\lambda^{\frac{1}{1-\phi}} \left(\phi^{\frac{1}{1-\phi}} - \phi^{\frac{\phi}{1-\phi}} \right)}$$

Since the inverse of the skill-premium is always less than one, this expression always holds for any $\phi > 1$. Next we show what condition must hold in order to have the expression $\frac{\partial \tau^*}{\partial w_h} > 0$ be true. Solving and rearranging gives us

$$\lambda^{\frac{1}{\phi}} \phi > \frac{w_l}{w_h}$$

Thus for a given value of λ, ϕ needs to be large enough for this condition to hold. Finally, our expression for total fertility, (31), can be slightly rearranged as

$$n_l^* + (n_h^* - n_l^*) \left(\frac{\tau^*}{b} \right)$$

From (26) and (27) we know that the second term is always negative. So any increase in education from wage changes will lower aggregate fertility, and any decrease in education from wage changes will increase aggregate fertility.

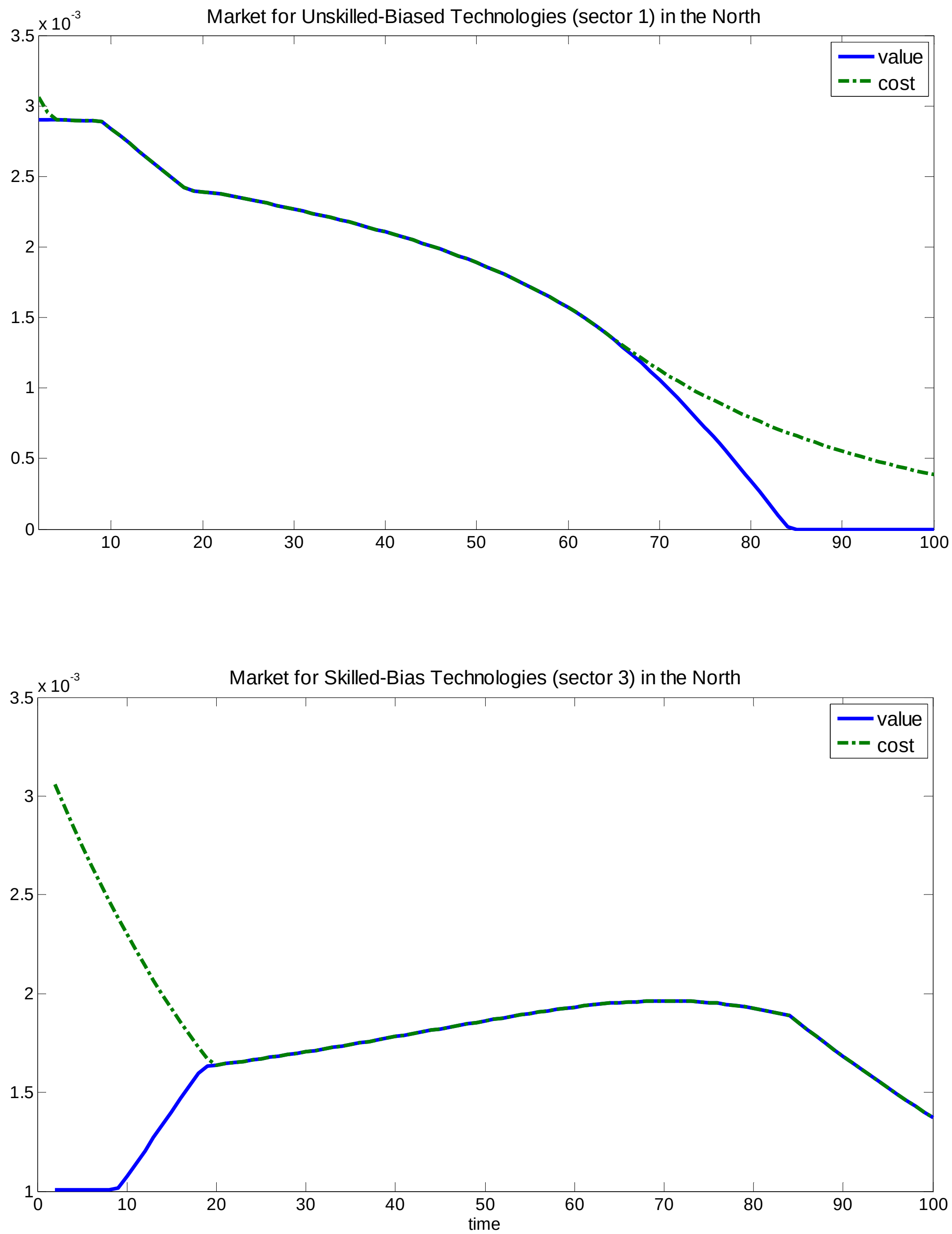
5.3 Simulations[6]

The simulation we run is where we set $\rho = 1$; that is, we allow for *perfect* diffusion of new machine blueprints from the North to the South; we are interesting in how income-per-capita divergence is still possible even when knowledge diffusion is perfect. Basic knowledge B and trade technology a are set such that neither technological growth nor trade is possible at first; each however exogenously rises over time. We run the simulation for 100 time periods to roughly capture major economic trends from around 1750 to the turn of the twentieth century.

Figures 2 and 3 summarize the evolution of technologies. In the beginning the costs of research are prohibitively high, so technologies are stagnant. But growth in basic knowledge allows us to

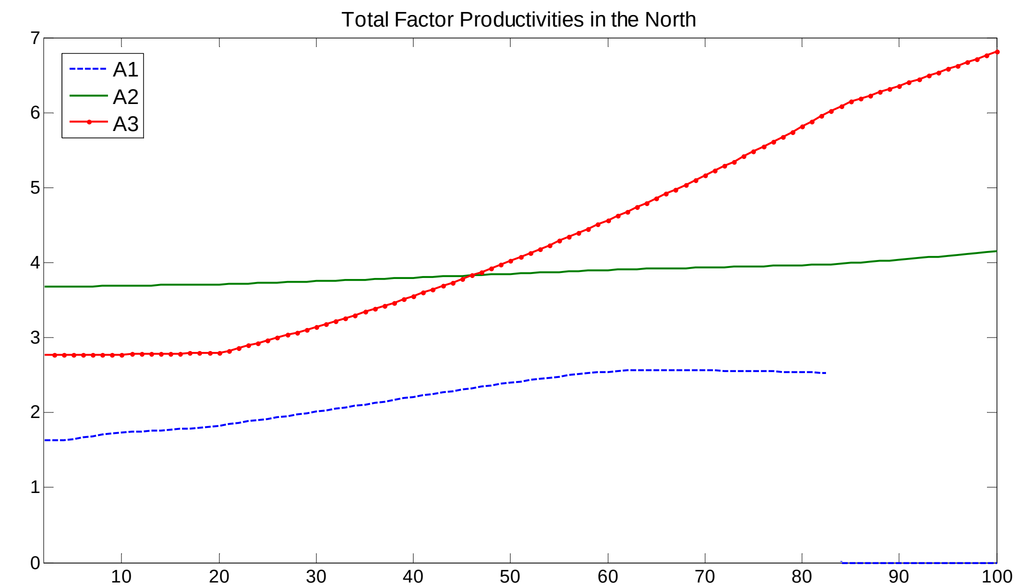
[6]The parameter values used in the simulations are as follows: $\sigma = 3$, $\alpha = 0.5$, $\gamma = 0.5$, $\lambda = 0.5$, $\phi = 10$, $\nu = 2$. We also set $b^n = 2$, $b^s = 6$, and *pop* $= 2$; this gives us initial factor endowments of $L_n = 3.14$, $L_s = 3.48$, $H_n = 0.86$, $H_s = 0.52$. Initial machine blueprints for both countries are set to be $N_1 = 10$, $N_2 = 15$, $N_3 = 10$; initial trade technology is set to be $a = 0.85$, and grows linearly such that $a = 1$ 100 periods later; initial B is set high enough so that growth in at least one sector is possible early in the simulation; B grows 2 percent each time period.

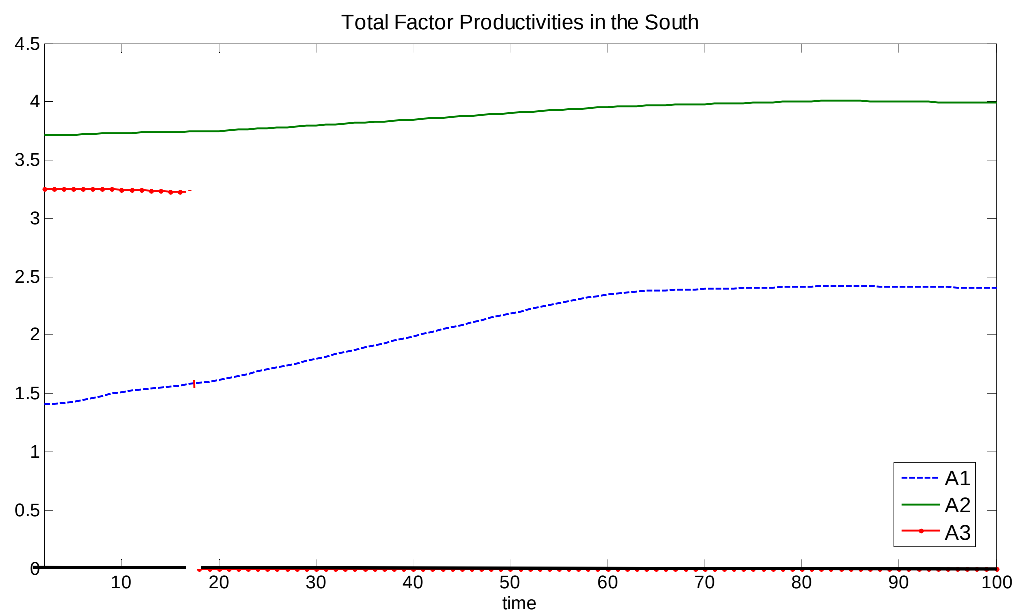
Figure 2: The Market for Technologies



Note that because $L_n > H_n$ (combined with the fact that $\sigma>1$), the revenues of new research in the unskilled sector is initially larger than those for the skilled sector. Consequently, unskilled knowledge is the first to grow (at t = 3). Trade (which becomes possible at t = 9) spurs innovation in the skilled sector since it raises both the price and employment level in sector 3, pulling research effort away from sector 1.

Figure 3: Factor Productivities





see the implications of Proposition 1 - because there is a greater abundance of unskilled labor relative to skilled labor in the North, the costs of research first catch up to revenues in sector 1. Due to perfect blueprint diffusion, A_1 grows in both regions.

This growth in unskilled labor intensive technologies lowers the relative returns to unskilled labor in both regions, inciting fertility increases and educational decreases in both regions (Proposition 5). We can see this manifest itself in the North by the increases in the revenues generated by innovation - as population rises in the North, the market-size effects caused by fertility increases raises the value of such innovation. Still, because skilled labor remains in relative scarce supply, the cost of innovation exceeds the benefits in sector 3 for the beginning of the simulation.

But evidently, looking at these figures, some event happens at $t = 10$ that in the North seems to pull resources away from sector 1 (so that revenues to research in the sector fall) and push resources into sector 3 (so that revenues in the sector rise). Soon after, two things occur. The North begins to develop new machine blueprints for skilled labor intensive sector 3. And the South abandons its production of the sector 3 good altogether. What happened?

The answer is that at $t = 10$ the trade technology parameter a becomes large enough so that commerce between the two regions becomes possible (Proposition 2). At this point the South starts exporting some of its production of y_1 and the North starts exporting some of its production of y_3. Trade happens early here because of the diffusion of unskilled intensive technologies from the North to the South; this helps the South retain its comparative advantage in unskilled-intensive production. Indeed, any limitations of this diffusion would have actually delayed the start of international commerce (Proposition 3). Once such trade occurs, both price and market-size effects rise in sector 3, and innovation in the sector in the North begins.

The opposite happens in the South. Producing very little of good 3 even in autarky, the South finds itself importing all of the good from the North once the North raises its productivity in the sector (Proposition 4). Of course, this ultimately means that it can no longer benefit from any further skill-intensive technological growth, and technological paths diverge between the two regions even with perfect technological diffusion. Thereafter, while trade allows the South to specialize in its comparative advantage and thereby raise its living standards, its inability to exploit the skilled labor intensive technological gains from the North inhibits its ability to grow further. Worse still for the South, the North ultimately abandons its innovations in unskilled-intensive production as it further specializes in good 3; at this point the South is technologically stagnant.

Figures 4 and 5 chart the evolution of fertility, education, trade, and incomes per person in both regions. Early industrialization without any trade is characterized by rising fertility and falling education. This period also implies a period of convergence in incomes per capita between the two regions, in large part because the South has a relatively larger endowment of unskilled labor, so that the unskilled intensive technologies from the North augment a larger share of the

Figure 4: Rates of Fertility and Education

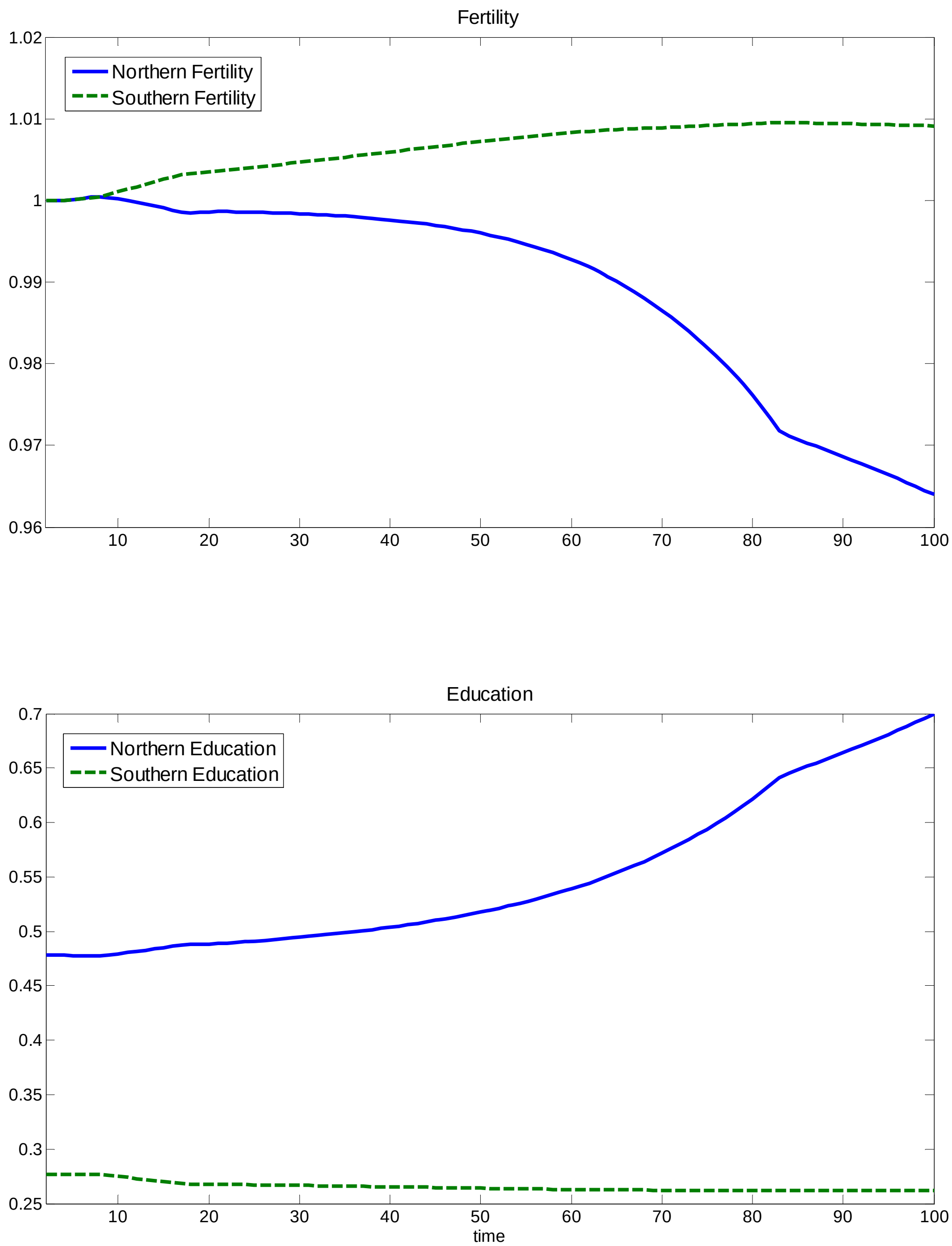
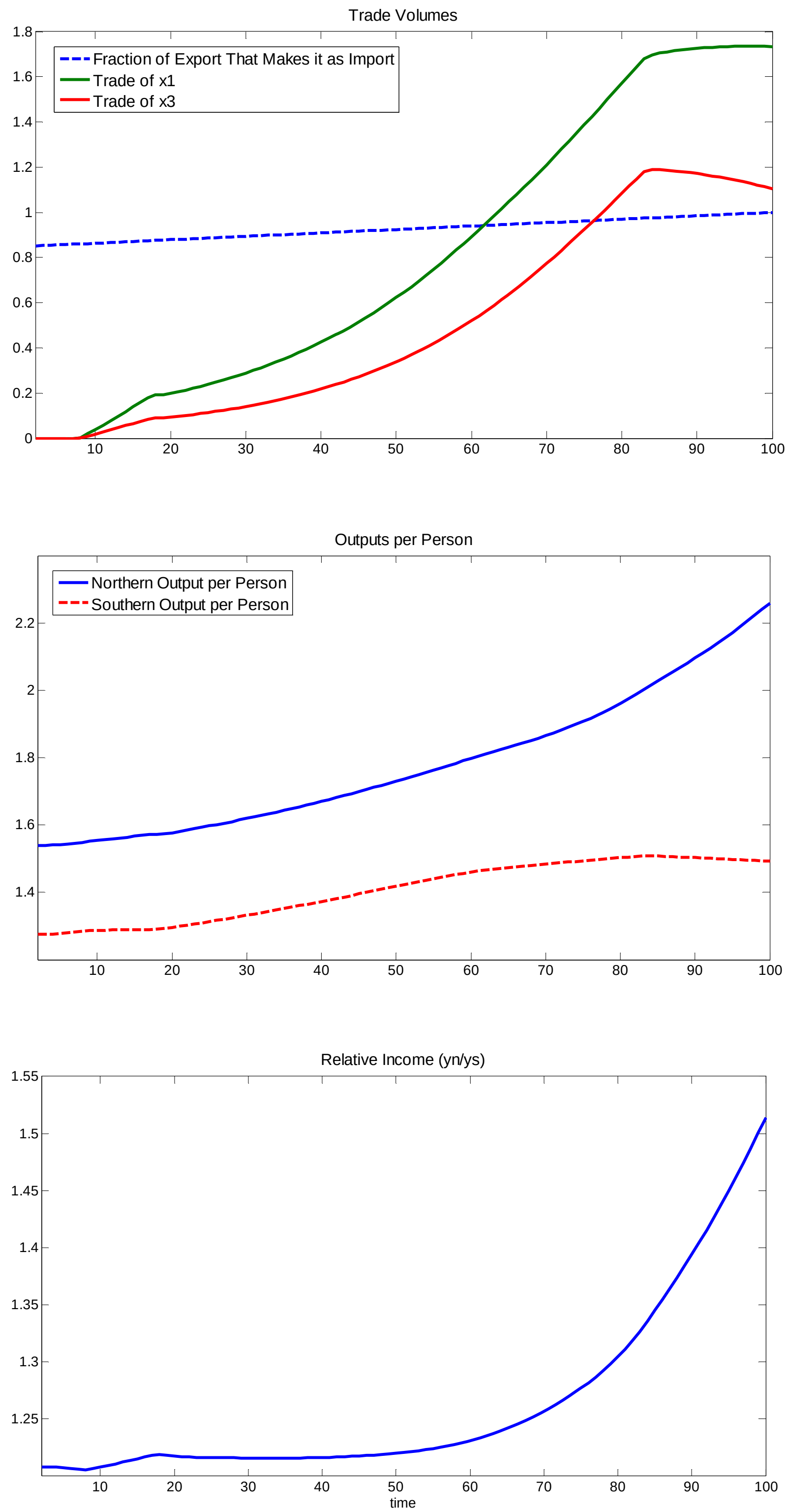


Figure 5: Trade and Output



economy in the South (see Appendix C for these cases).

With trade however, any trend towards convergence ultimately reverses. Two forces work against the South here. As we can see in Figure 3, southern technologies ultimately flatline - the South no longer produces the good that is technologically improving (good 3), while soon after this the North abandons improving the good the South does specialize in (good 1). The result is overall technological stagnation for the South.

The other force for divergence is the rapid fertility declines that happen in the North due to its specialization of good 3. The South on the other hand retains its high fertility due to the low skill premia generated by its specialization of good 1. The demographic transition of the North, and the lack thereof in the South, ultimately creates a great divergence in output per person between the two regions.[7]

Thus the model and simulation highlight the great irony of the Industrial Revolution - that while it *could have been* an equalizing force for incomes per capita across the world, it sowed the seeds for massive subsequent divergence by fostering trade and specialization. One could extrapolate forward from this case to suggest the unprecedented divergence of the 20th century. Technological progress in the North becomes strictly skilled-intensive. Not only does the South specialize in production that lowers education and fosters population growth, it no longer is able to benefit from any technological spillovers from the North! So perhaps the final answer to the question of which was the main force of divergence - biased technology spillovers or trade-induced specialization - is both, although one could argue that the primary culprit was trade. Specialization patterns created a great deal of income divergence during the 19th century by increasing the fertility of southern economies; such specialization also fostered "inappropriate" technology flows characteristic of the 20th century. Both forces led to the wide disparity of incomes per capita we currently see.

[7]In fact, the demographic transition is so dramatic in the North that the shrinking population induces a dramatic drop in the scale of the market for inventors, ultimately producing technological stagnation. In reality, immigration and other factors kept populations in northern countries from dropping so dramatically.

References

Acemoglu, Daron. (2002) "Directed Technical Change," *Review of Economic Studies* 69, pp. 781-809.

Acemoglu, Daron and Fabrizio Zilibotti (2001) "Productivity Differences," *Quarterly Journal of Economics* 116, pp. 563-606.

Basu, Susanto and David N. Weil (1998) "Appropriate Technology and Growth," *Quarterly Journal of Economics* 113, pp. 1025-1054.

Clark, Gregory. (2007) *A Farewell to Alms: A Brief Economic History of the World.* Princeton and Oxford: Princeton University Press.

Findlay, Ronald. (1996) "Modeling Global Interdependence: Centers, Peripheries, and Frontiers." *American Economic Review* 86(2), pp. 47-51.

Galor, Oded and Andrew Mountford (2006), "Trade and the Great Divergence: The Family Connection," *American Economic Review Papers and Proceedings*, Vol. 96, pp. 299-303.

Galor, Oded and Andrew Mountford. (2008) "Trade and the Great Divergence: Theory and Evidence,"*Review of Economic Studies*, Vol. 75, pp. 1143-1179.

Melvin, James R., "Production and Trade with Two Factors and Three Goods," *American Economic Review* 58, 1249-1268.

Mokyr, Joel. (1999) *The British Industrial Revolution: An Economic Perspective*, 2nd edition. Westview Press: Boulder, CO.

ORourke, Kevin H., Alan M. Taylor and J. G. Williamson. (1996) "Factor Price Convergence in the Late Nineteenth Century," *International Economic Review* 37, pp. 499-530.

O'Rourke, Kevin H. and Jeffrey G. Williamson. (1994) "Late 19th Century Anglo-American Factor Price Convergence: Were Heckscher and Ohlin Right?" *Journal of Economic History* 54, pp. 892-916.

O'Rourke, Kevin H., Ahmed S. Rahman and Alan M. Taylor. (2008) "Luddites and the Demographic Transition," NBER paper 14484.

O'Rourke, Kevin H. and Jeffrey G. Williamson. (1999) *Globalization and History.* Cambridge, MA.: MIT Press.

Rahman, Ahmed S. (2008) "The Appropriate Technological Frontier - Lessons for the Developing World," United States Naval Academy Working Paper No. 16.

Romer, Paul M. (1990) "Endogenous Technological Change," *Journal of Political Economy* 98(5), pp. S71-S102.

A Diversified Trade Equilibrium

With trade of goods y_1 and y_3 between the North and the South, productions in each region are given by (36) and (37).

For each region $c \in n, s$, the following conditions characterize the diversified trade equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{38}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{39}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{40}$$

$$\left(\frac{1}{A_1^c}\right)y_1^c + \left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma-1}(w_h^c)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^c = L^c \tag{41}$$

$$\left(\frac{1}{A_2^c}\right)(w_l^c)^{\gamma}(w_h^c)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^c + \left(\frac{1}{A_3^c}\right)y_3^c = H^c \tag{42}$$

$$y_1^n + a_1 Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{43}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{44}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}(p_2^c)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^c)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^c)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^c)^{1-\sigma}}\right)\cdot Y^c \tag{45}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^n)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{46}$$

$$y_3^s + a_3 Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma} + (1-\alpha)^{\sigma}(p_2^s)^{1-\sigma} + \left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{47}$$

$$A_1^n\left(A_1^n L_1^n + a_1 Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma-\sigma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{48}$$

$$A_3^n\left(A_3^n H_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}\left(L^n - L_1^n\right)^{-\gamma+\sigma\gamma}\left(H^n - H_3^n\right)^{\gamma-\sigma\gamma-1} \tag{49}$$

$$A_1^s \left(A_1^s L_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma-\sigma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma+\sigma-\sigma\gamma-1} \tag{50}$$

$$A_3^s \left(A_3^s H_3^s + a_3 Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right) A_2^{s\,\frac{\sigma-1}{\sigma}} \left(L^s - L_1^s\right)^{-\gamma+\sigma\gamma} \left(H^s - H_3^s\right)^{\gamma-\sigma\gamma-1} \tag{51}$$

$$A_1^c = \left(N_{1,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{1,t}^c - N_{1,t-1}^c\right)\right) \left(\alpha p_1^c\right)^{\frac{\alpha}{1-\alpha}} \tag{52}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{2,t}^c - N_{2,t-1}^c\right)\right) \left(\alpha p_2^c\right)^{\frac{\alpha}{1-\alpha}} \tag{53}$$

$$A_3^c = \left(N_{3,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}} \left(N_{3,t}^c - N_{3,t-1}^c\right)\right) \left(\alpha p_3^c\right)^{\frac{\alpha}{1-\alpha}} \tag{54}$$

$$H^c = \left(\frac{\tau^{*c}}{b^c}\right) pop^c \tag{55}$$

$$L^c = \left(1 - \frac{\tau^{*c}}{b^c}\right) pop^c + n^c pop^c \tag{56}$$

$$n^c = \left(1 - \frac{\tau^{*c}}{b^c}\right) n_l^{*c} + \left(\frac{\tau^{*c}}{b^c}\right) n_h^{*c} \tag{57}$$

$$e^c = \frac{\tau^{*c}}{b^c} \tag{58}$$

$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{59}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{60}$$

Equations (38) - (40) are unit cost functions, (41) and (42) are full employment conditions, (43) - (47) denote regional goods clearance conditions, (48) - (51) equate the marginal products of raw factors, (52) - (54) describe sector-specific technologies, , (55) - (64) describe fertility, education and labor-types for each region, and (65) and (66) describe the balance of payments for each region. Solving this system for the unknowns p_1^n, p_1^s, p_2^n, p_2^s, p_3^n, p_3^s, y_1^n, y_1^s, y_2^n, y_2^s, y_3^n, y_3^s, w_l^n, w_l^s, w_h^n, w_h^s, L_1^n, L_1^s, H_3^n, H_3^s, A_1^n, A_2^n, A_3^n, A_1^s, A_2^s, A_3^s, L^n, L^s, H^n, H^s, n^n, n^s, e^n, e^s, Z_1 and Z_3 constitutes the static partial trade equilibrium.

Population growth for each region is given simply by

$$pop_t^c = n_{t-1}^c pop_{t-1}^c$$

Each region will produce all three goods so long as factors and technologies are "similar enough." If factors of production or technological levels sufficiently differ, the North produces only goods 2 and 3, while the South produces only goods 1 and 2. No other specialization scenario is possible for the following reasons: first, given that both the North and South have positive levels of L and H, full employment of resources implies that they cannot specialize completely in good 1 or good 3. Second, specialization solely in good 2 is not possible either, since a region with a comparative advantage in this good would also have a comparative advantage in either of the other goods. This implies that each country must produce at least two goods. Further, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these goods, regardless of the technological differences between the two regions. See Cunat and Maffezzoli (2002) for a fuller discussion.

B Specialized Trade Equilibrium

The specialized equilibrium is one where the North does not produce any good 1 and the South does not produce any good 3. Productions in each region are then given by

$$Y^n = \left(\frac{\alpha}{2}\left(aZ_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^n\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(y_3^n - Z_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{61}$$

$$Y^s = \left(\frac{\alpha}{2}\left(y_1^s - Z_1\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(y_2^s\right)^{\frac{\sigma-1}{\sigma}} + \frac{\alpha}{2}\left(aZ_3\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{62}$$

Once again, we do not permit any trade of good 2. For each region $c \in n, s$, the following conditions characterize this equilibrium.

$$p_1^s = \frac{w_l^s}{A_1^s} \tag{63}$$

$$p_2^c = \left(\frac{1}{A_2^c}\right)\left(w_l^c\right)^{\gamma}\left(w_h^c\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{-\gamma} \tag{64}$$

$$p_3^n = \frac{w_h^n}{A_3^n} \tag{65}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma-1}\left(w_h^n\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^n = L^n \tag{66}$$

$$\left(\frac{1}{A_2^n}\right)\left(w_l^n\right)^{\gamma}\left(w_h^n\right)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^n + \left(\frac{1}{A_3^n}\right)y_3^n = H^n \tag{67}$$

$$\left(\frac{1}{A_1^s}\right)y_1^s + \left(\frac{1}{A_2^s}\right)\left(w_l^s\right)^{\gamma-1}\left(w_h^s\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\gamma^{1-\gamma}y_2^s = L^s \tag{68}$$

$$\left(\frac{1}{A_2^s}\right)(w_l^s)^{\gamma}(w_h^s)^{-\gamma}(1-\gamma)^{\gamma}\gamma^{-\gamma}y_2^s = H^s \tag{69}$$

$$a_1 Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma}+(1-\alpha)^{\sigma}(p_2^n)^{1-\sigma}+\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{70}$$

$$y_1^s - Z_1 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma}+(1-\alpha)^{\sigma}(p_2^s)^{1-\sigma}+\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{71}$$

$$y_2^c = \left(\frac{(1-\alpha)^{\sigma}(p_2^c)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^c)^{1-\sigma}+(1-\alpha)^{\sigma}(p_2^c)^{1-\sigma}+\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^c)^{1-\sigma}}\right)\cdot Y^c \tag{72}$$

$$y_3^n - Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^n)^{1-\sigma}+(1-\alpha)^{\sigma}(p_2^n)^{1-\sigma}+\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^n)^{1-\sigma}}\right)\cdot Y^n \tag{73}$$

$$a_3 Z_3 = \left(\frac{\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{-\sigma}}{\left(\frac{\alpha}{2}\right)^{\sigma}(p_1^s)^{1-\sigma}+(1-\alpha)^{\sigma}(p_2^s)^{1-\sigma}+\left(\frac{\alpha}{2}\right)^{\sigma}(p_3^s)^{1-\sigma}}\right)\cdot Y^s \tag{74}$$

$$A_3^n\left(A_3^n H_3^n - Z_3\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)(1-\gamma)}{\alpha}\right)A_2^{n\,\frac{\sigma-1}{\sigma}}(L^n)^{-\gamma+\sigma\gamma}(H^n - H_3^n)^{\gamma-\sigma\gamma-1} \tag{75}$$

$$A_1^s\left(A_1^s L_1^s - Z_1\right)^{-\frac{1}{\sigma}} = \left(\frac{2(1-\alpha)\gamma}{\alpha}\right)A_2^{s\,\frac{\sigma-1}{\sigma}}(L^s - L_1^s)^{-\gamma-\sigma+\sigma\gamma}(H^s)^{\gamma+\sigma-\sigma\gamma-1} \tag{76}$$

$$A_1^s = \left(N_{1,t-1}^s + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{1,t}^s - N_{1,t-1}^s\right)\right)(\alpha p_1^s)^{\frac{\alpha}{1-\alpha}} \tag{77}$$

$$A_2^c = \left(N_{2,t-1}^c + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{2,t}^c - N_{2,t-1}^c\right)\right)(\alpha p_2^c)^{\frac{\alpha}{1-\alpha}} \tag{78}$$

$$A_3^n = \left(N_{3,t-1}^n + \alpha^{\frac{\alpha}{1-\alpha}}\left(N_{3,t}^n - N_{3,t-1}^n\right)\right)(\alpha p_n^c)^{\frac{\alpha}{1-\alpha}} \tag{79}$$

$$H^c = \left(\frac{\tau^{*c}}{b^c}\right)pop^c \tag{80}$$

$$L^c = \left(1-\frac{\tau^{*c}}{b^c}\right)pop^c + n^c pop^c \tag{81}$$

$$n^c = \left(1-\frac{\tau^{*c}}{b^c}\right)n_l^{*c} + \left(\frac{\tau^{*c}}{b^c}\right)n_h^{*c} \tag{82}$$

$$e^c = \frac{\tau^{*^c}}{b^c} \tag{83}$$

$$\frac{p_1^n}{p_3^n} = \frac{Z_3}{aZ_1} \tag{84}$$

$$\frac{p_1^s}{p_3^s} = \frac{aZ_3}{Z_1} \tag{85}$$

C How Trade Creates Divergence - Cases Where Trade Costs Are Higher

Here we present two alternative simulations where we have more prohibitive trade costs. Figure 6 illustrate these simulations. In the first case, we do not allow for any trade at all. In this case the North begins to develop unskilled-intensive technologies (same as our original case), but here it never specializes in skill-intensive production. As a result, its technological path is always relatively unskilled-intensive, even when skill-intensive technological growth becomes feasible (at roughly $t = 30$).

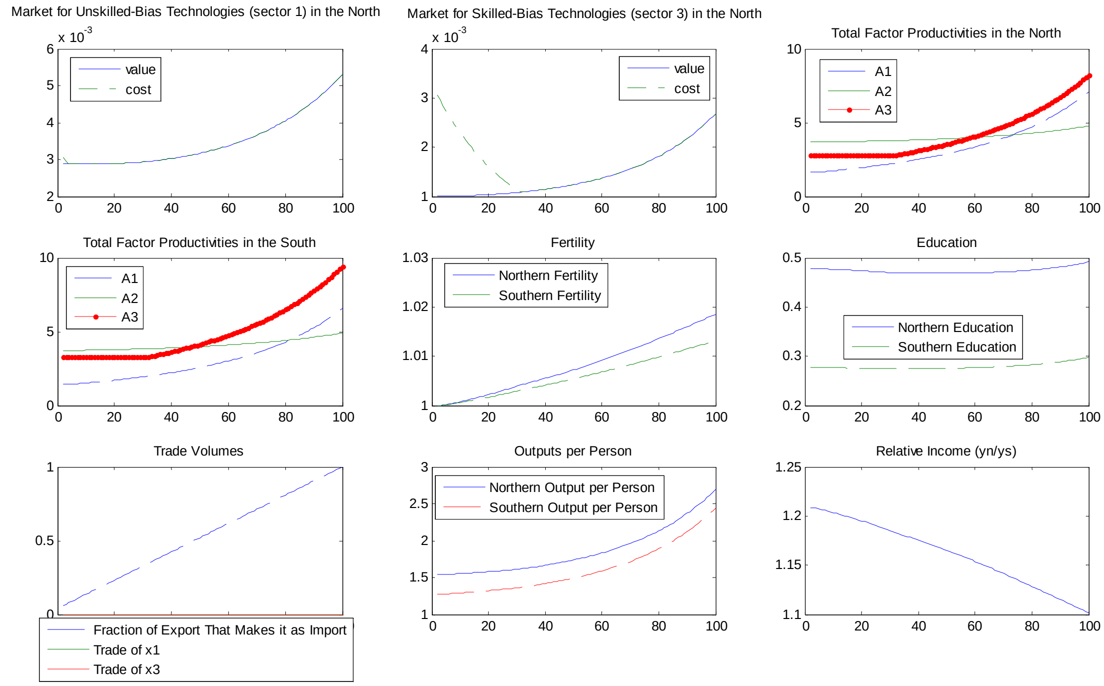
Ironically, even though these innovations are catered for Northern producers, the South is the greater beneficiary, for it has a relatively larger pool of unskilled labor that can become more productive. The result is *convergence* in incomes per capita between the two regions. Another irony is that the lack of a demographic transition in the North creates greater incentives for northern innovators to generate even more technological growth (since the market-size effects of population growth make innovation more profitable). Thus without any trade at all, technological growth around the world flourishes.

The other case is where we set the initial value of $a = 0.75$, and have it linearly grow so that it reaches the value of one after 100 time periods. Trade in this case is delayed until roughly half of the simulation. The pattern here is one where there is initial income per capita convergence followed by divergence. This divergence is somewhat muted by the North's continued innovative efforts in sector 1 for a longer period of time; because trade is not so dramatic here as in our main case, the North continues to produce good 1 throughout.

The lesson here is that without intercontinental trade, there is convergence in incomes. Trade not only erases this force for convergence, it contributes to the divergence in incomes between the regions.

Figure 6: Simulations Where Trade Costs Are Higher

No trade (initial a = 0.05)



Delayed trade (initial a = 0.75)