**Practice Problem 5.1, first part:** In the schematic below, find the closed-loop gain $v_o/v_s$. (The second part, omitted here, asks for $i_o$ when $v_s = 1$ V.)

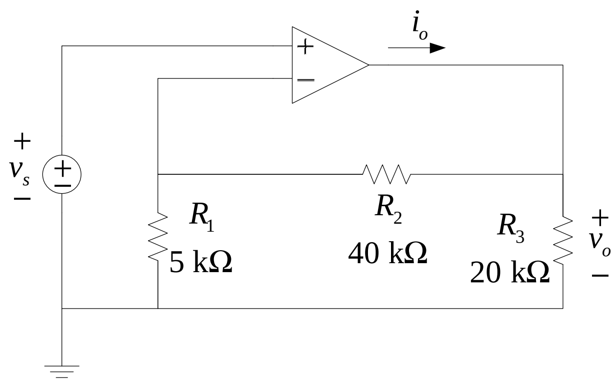


Figure 1: Schematic Diagram for Practice Problem 5.1

Solution

We can use nodal analysis to solve this problem if we replace the op amp with a non-ideal model in which the gain $A = 2 \times 10^5$, the input resistance $R_{\text{in}} = 2\ \text{M}\Omega$, and the output resistance is $R_0 = 50\ \Omega$. The revised schematic is shown in Figure 2. At Node A, $V_A = v_s$ and at Node C, $V_C = Av_{R_{\text{in}}} = A\left(V_A - V_B\right)/R_{\text{in}}$.

Using Kirchoff's current law at nodes B and D yields

$$i_{R_{\text{in}}} - i_{R_1} - i_{R_2} = 0$$
$$i_{R_2} + i_{R_0} - i_{R_3} = 0.$$

Using Ohm's law, these equations become

$$\frac{v_{R_{\text{in}}}}{R_{\text{in}}} - \frac{v_{R_1}}{R_1} - \frac{v_{R_2}}{R_2} = 0$$
$$\frac{v_{R_2}}{R_2} + \frac{v_{R_0}}{R_0} - \frac{v_{R_3}}{R_3} = 0.$$

Replacing resistor voltages with equivalent expressions using nodal voltages yields

$$\frac{v_s - v_B}{R_{\text{in}}} - \frac{v_B}{R_1} - \frac{v_B - v_D}{R_2} = 0$$
$$\frac{v_B - v_D}{R_2} + \frac{A\left(v_s - v_B\right) - v_D}{R_0} - \frac{v_D}{R_3} = 0.$$

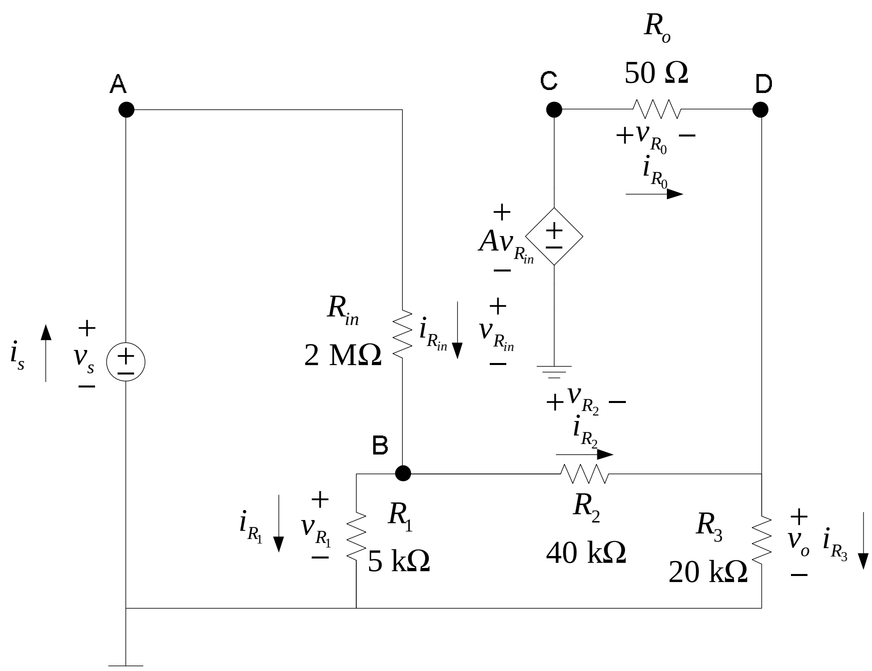


Figure 2: In this schematic the op amp has been replaced with a non-ideal model, which makes the analysis straightforward.

Regarding $v_B$ and $v_D$ as the unknowns, we can write these equations as

$$\left(-\frac{1}{R_{\text{in}}}-\frac{1}{R_1}-\frac{1}{R_2}\right)v_B+\frac{1}{R_2}v_D=-\frac{v_s}{R_{\text{in}}}$$
$$\left(\frac{1}{R_2}-\frac{A}{R_0}\right)v_B-\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)v_D=-A\frac{v_s}{R_0}$$

Multiplying the first of these equations by $\frac{1}{R_2}-\frac{A}{R_0}$ and subtracting this from $\left(-\frac{1}{R_{\text{in}}}-\frac{1}{R_1}-\frac{1}{R_2}\right)$ times the second equation gives

$$\left(\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)-\frac{1}{R_2}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)\right)v_D=$$
$$=A\frac{v_s}{R_0}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{v_s}{R_{\text{in}}}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)$$

Solving for $v_D$ gives

$$v_D=\frac{A\frac{v_s}{R_0}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{v_s}{R_{\text{in}}}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)}{\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)-\frac{1}{R_2}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)}.$$

Now $v_o=v_D$, so

$$\frac{v_o}{v_s}=\frac{\frac{A}{R_0}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_{\text{in}}}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)}{\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)-\frac{1}{R_2}\left(\frac{1}{R_2}-\frac{A}{R_0}\right)}.$$

Dividing numerator and denominator by $A$, we get

$$\frac{v_o}{v_s}=\frac{\frac{1}{R_0}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_{\text{in}}}\left(\frac{1}{AR_2}-\frac{1}{R_0}\right)}{\frac{1}{A}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)-\frac{1}{R_2}\left(\frac{1}{AR_2}-\frac{1}{R_0}\right)}$$
$$=\frac{\frac{1}{R_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_{\text{in}}}\left(\frac{1}{AR_2}\right)}{\frac{1}{A}\left(\frac{1}{R_{\text{in}}}+\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_0}+\frac{1}{R_3}\right)-\frac{1}{R_2}\left(\frac{1}{AR_2}-\frac{1}{R_0}\right)}.$$

When the given values of $A$ and the resistors are substituted into this equation, we find $v_o/v_s=8.99959$, although the book gives the answer 9.00041.