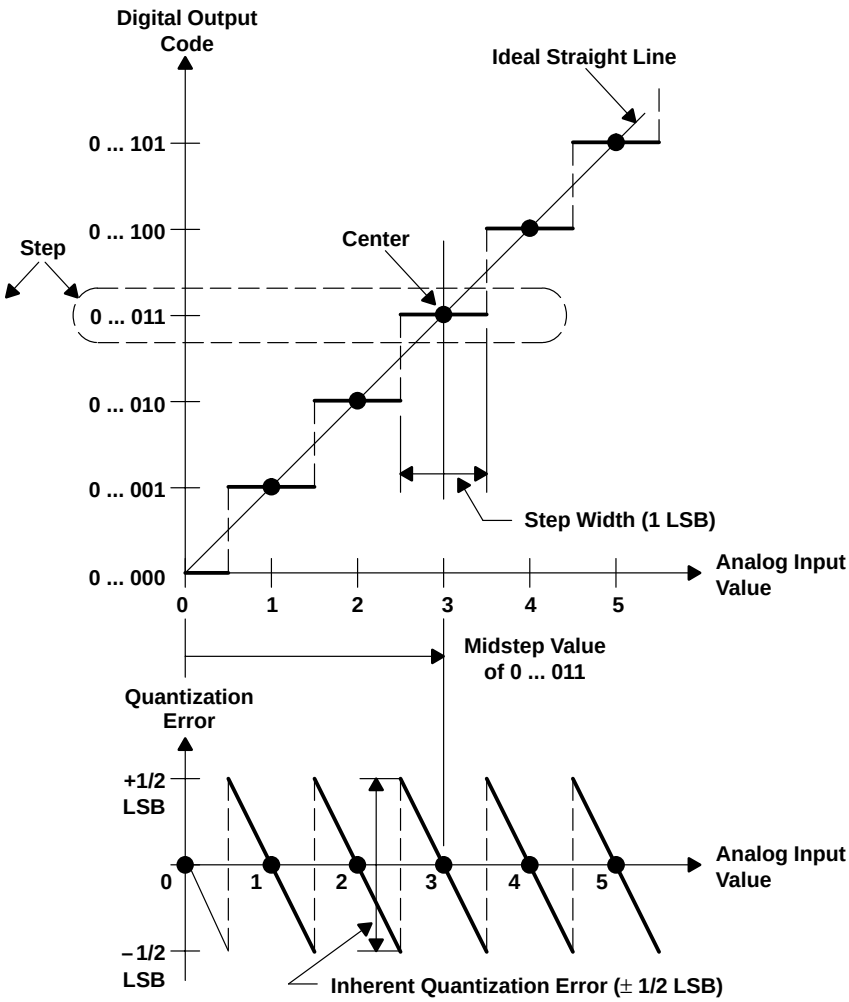
CONVERSION CODE

RANGE OF ANALOG INPUT VALUES	DIGITAL OUTPUT CODE
4.5 • 5.5	0 ... 101
3.5 • 4.5	0 ... 100
2.5 • 3.5	0 ... 011
1.5 • 2.5	0 ... 010
0.5 • 1.5	0 ... 001
0 • 0.5	0 ... 000



Elements of Transfer Diagram for an Ideal Linear ADC

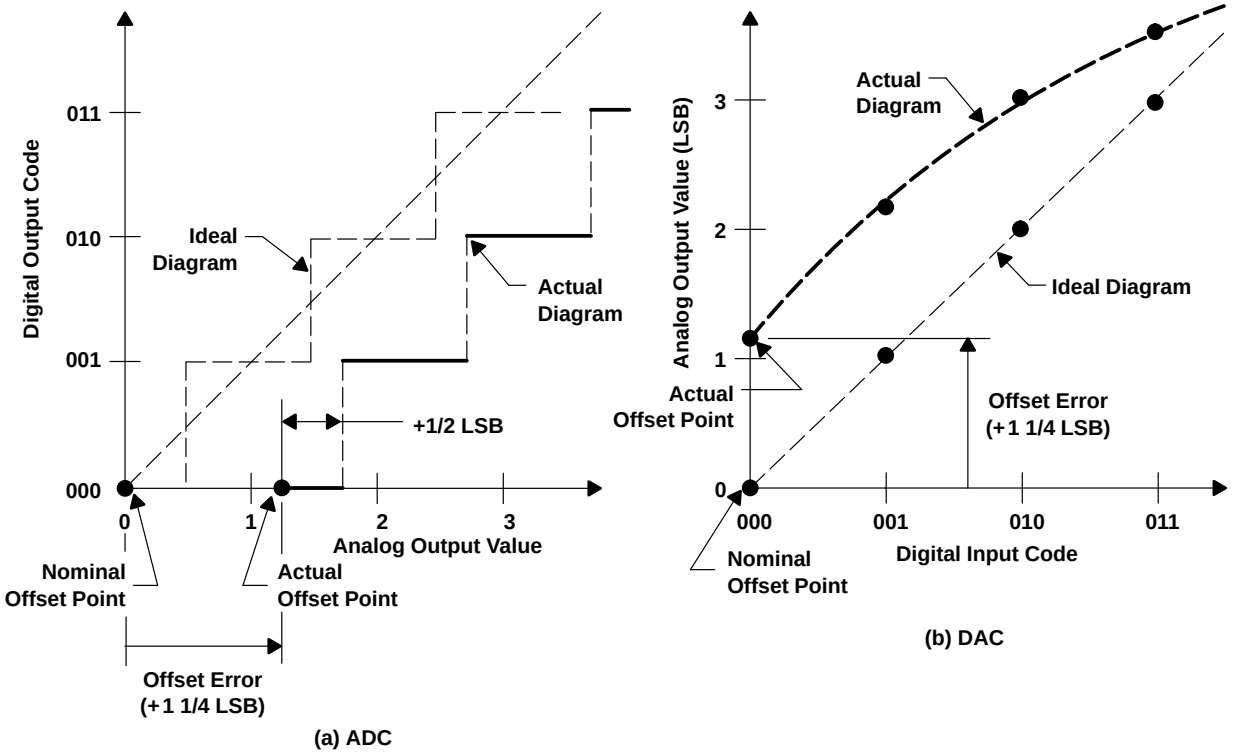
Figure 1. The Ideal Transfer Function (ADC)

3 SOURCES OF STATIC ERROR

Static errors, that is those errors that affect the accuracy of the converter when it is converting static (dc) signals, can be completely described by just four terms. These are offset error, gain error, integral nonlinearity and differential nonlinearity. Each can be expressed in LSB units or sometimes as a percentage of the FSR. For example, an error of 1/2 LSB for an 8-bit converter corresponds to 0.2% FSR.

3.1 Offset Error

The offset error as shown in Figure 3 is defined as the difference between the nominal and actual offset points. For an ADC, the offset point is the midstep value when the digital output is zero, and for a DAC it is the step value when the digital input is zero. This error affects all codes by the same amount and can usually be compensated for by a trimming process. If trimming is not possible, this error is referred to as the zero-scale error.

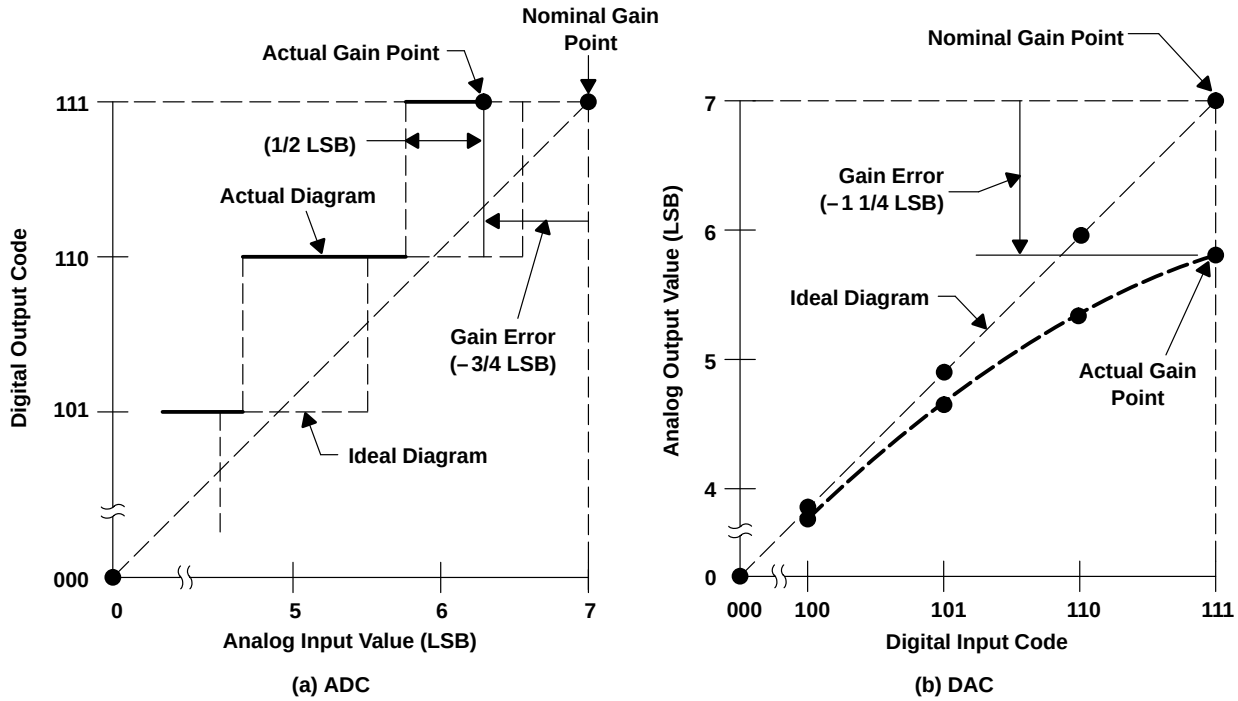


Offset error of a Linear 3-Bit Natural Binary Code Converter (Specified at Step 000)

Figure 3. Offset Error

3.2 Gain Error

The gain error shown in Figure 4 is defined as the difference between the nominal and actual gain points on the transfer function after the offset error has been corrected to zero. For an ADC, the gain point is the midstep value when the digital output is full scale, and for a DAC it is the step value when the digital input is full scale. This error represents a difference in the slope of the actual and ideal transfer functions and as such corresponds to the same percentage error in each step. This error can also usually be adjusted to zero by trimming.

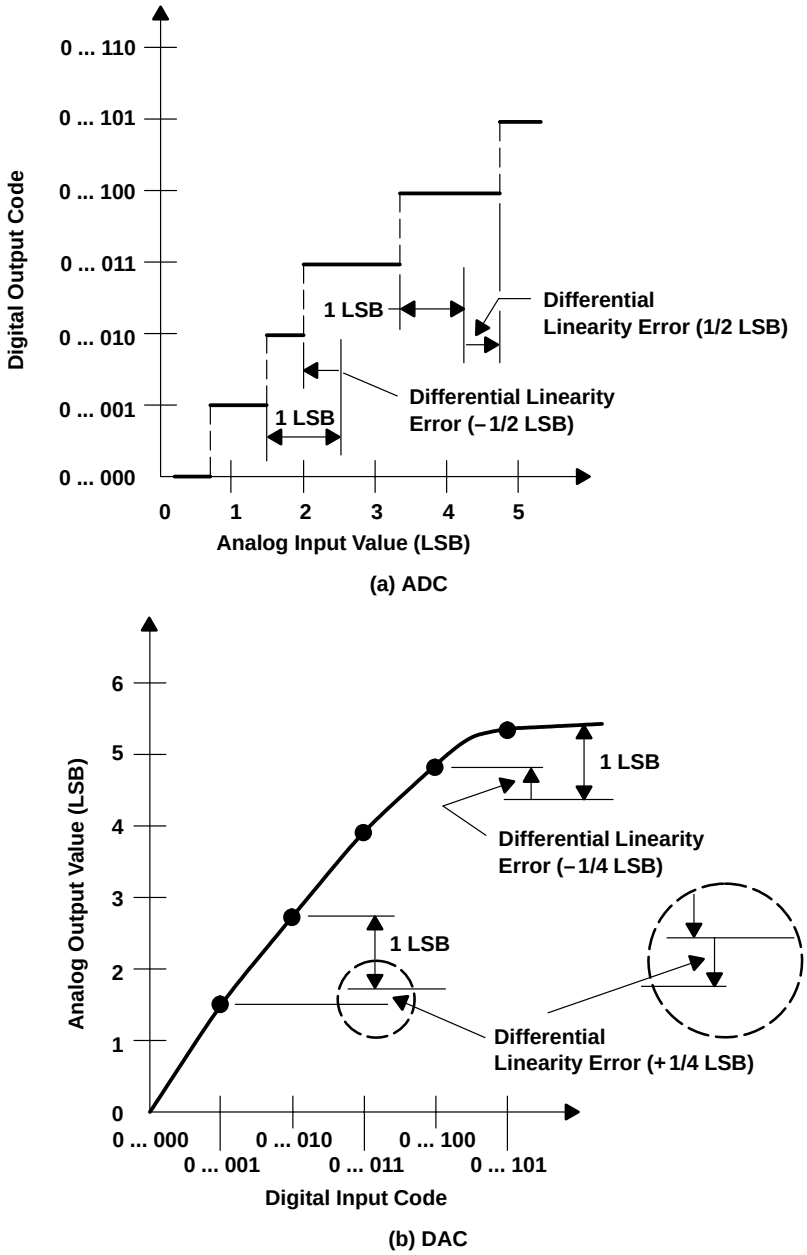


Gain Error of a Linear 3-Bit Natural Binary Code Converter (Specified at Step 111), After Correction of the Offset Error

Figure 4. Gain Error

3.3 Differential Nonlinearity (DNL) Error

The differential nonlinearity error shown in Figure 5 (sometimes seen as simply differential linearity) is the difference between an actual step width (for an ADC) or step height (for a DAC) and the ideal value of 1 LSB. Therefore if the step width or height is exactly 1 LSB, then the differential nonlinearity error is zero. If the DNL exceeds 1 LSB, there is a possibility that the converter can become nonmonotonic. This means that the magnitude of the output gets smaller for an increase in the magnitude of the input. In an ADC there is also a possibility that there can be missing codes i.e., one or more of the possible 2^n binary codes are never output.



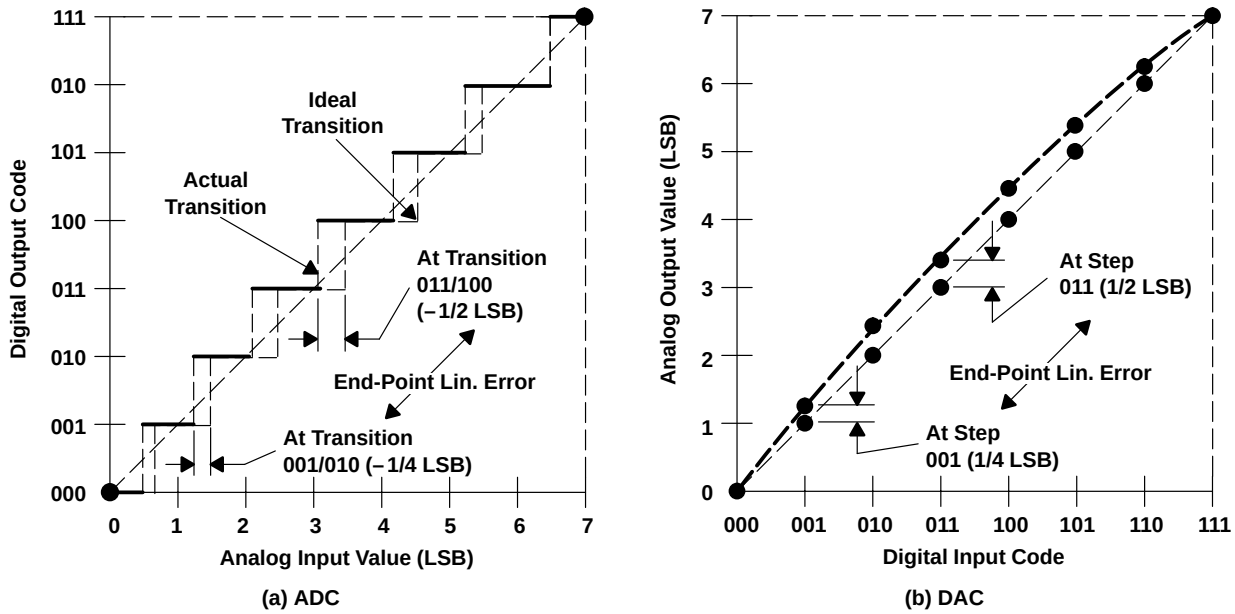
Differential Linearity Error of a Linear ADC or DAC

Figure 5. Differential Nonlinearity (DNL)

3.4 Integral Nonlinearity (INL) Error

The integral nonlinearity error shown in Figure 6 (sometimes seen as simply linearity error) is the deviation of the values on the actual transfer function from a straight line. This straight line can be either a best straight line which is drawn so as to minimize these deviations or it can be a line drawn between the end points of the transfer function once the gain and offset errors have been nullified. The second method is called end-point linearity and is the usual definition adopted since it can be verified more directly.

For an ADC the deviations are measured at the transitions from one step to the next, and for the DAC they are measured at each step. The name integral nonlinearity derives from the fact that the summation of the differential nonlinearities from the bottom up to a particular step, determines the value of the integral nonlinearity at that step.

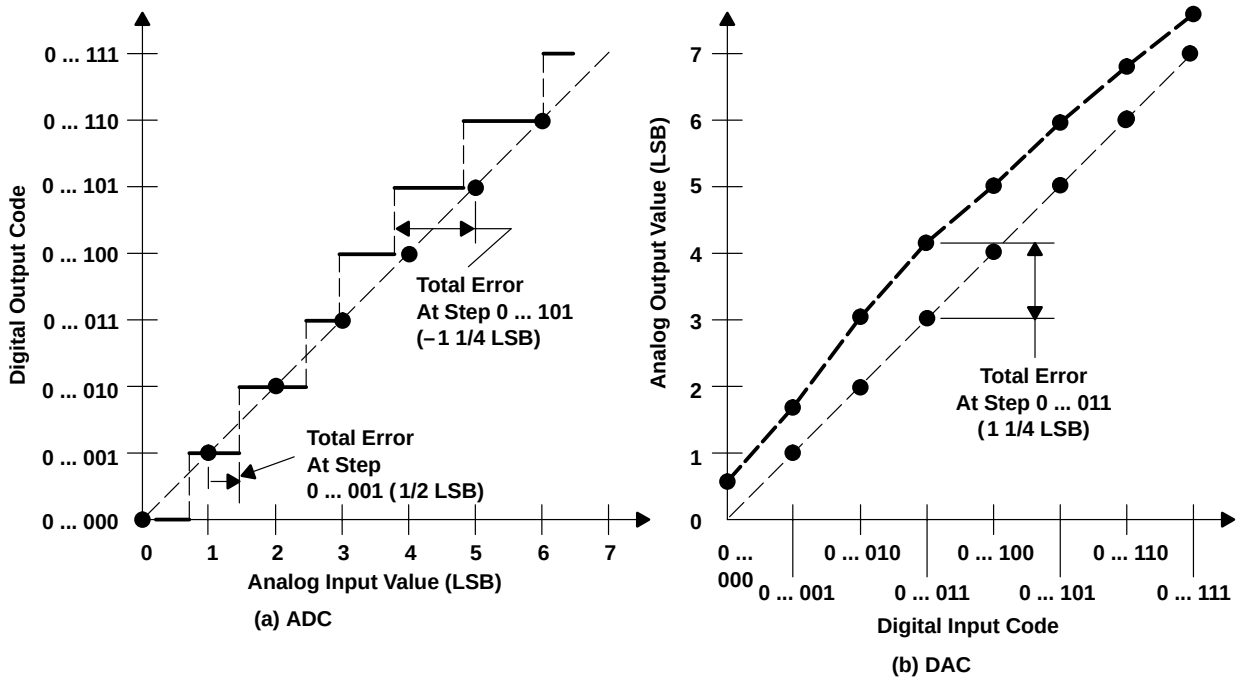


End-Point Linearity Error of a Linear 3-Bit Natural Binary-Coded ADC or DAC (Offset Error and Gain Error are Adjusted to the Value Zero)

Figure 6. Integral Nonlinearity (INL) Error

3.5 Absolute Accuracy (Total) Error

The absolute accuracy or total error of an ADC as shown in Figure 7 is the maximum value of the difference between an analog value and the ideal midstep value. It includes offset, gain, and integral linearity errors and also the quantization error in the case of an ADC.



Absolute Accuracy or Total Error of a Linear ADC or DAC

Figure 7. Absolute Accuracy (Total) Error

4 APERTURE ERROR

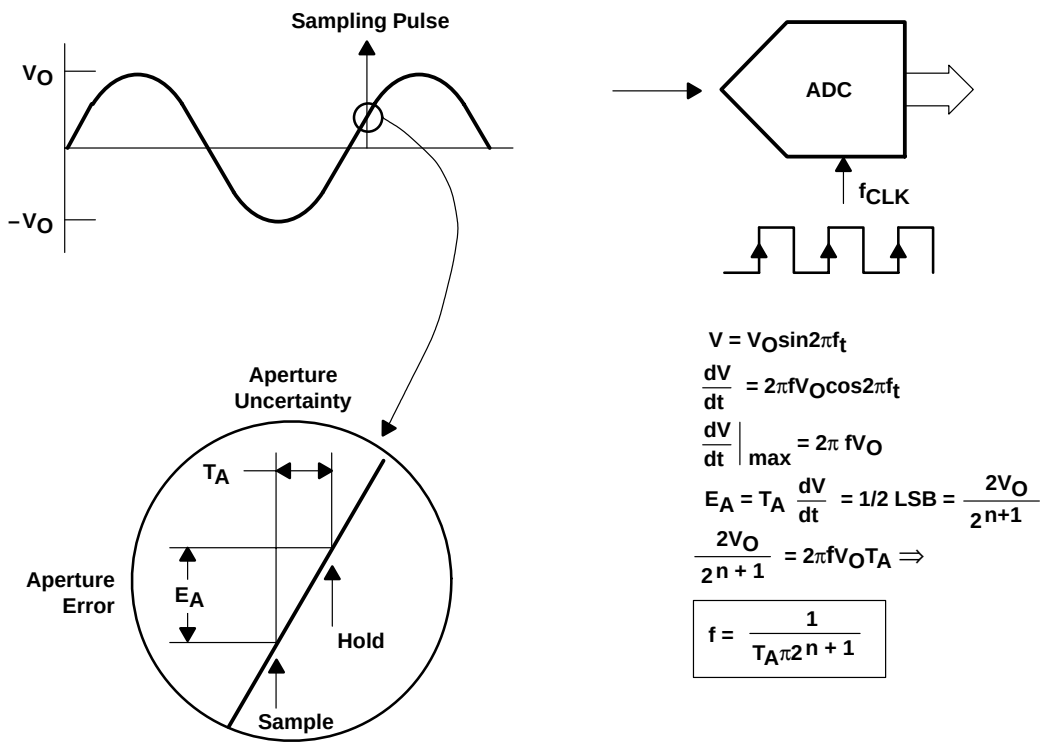


Figure 8. Aperture Error

Aperture error is caused by the uncertainty in the time at which the sample/hold goes from sample mode to hold mode as shown in Figure 8. This variation is caused by noise on the clock or the input signal. The effect of the aperture error is to set another limitation on the maximum frequency of the input sine wave because it defines the maximum slew rate of that signal. For a sine wave input as shown, the value of the input V is defined as:

$$V = V_O \sin 2\pi ft$$

The maximum slew rate occurs at the zero crossing point and is given by:

$$\left.\frac{dV}{dt}\right|_{max} = 2\pi f V_O$$

If the aperture error is not to affect the accuracy of the converter, it must be less than 1/2 LSB at the point of maximum slew rate. For an n bit converter therefore:

$$E_A = T_A \frac{dV}{dt} = 1/2 \text{ LSB} = \frac{2\ V_O}{2^{n+1}}$$

Substituting into this gives

$$\frac{2\ V_O}{2^{n+1}} = 2\pi f V_O T_A$$

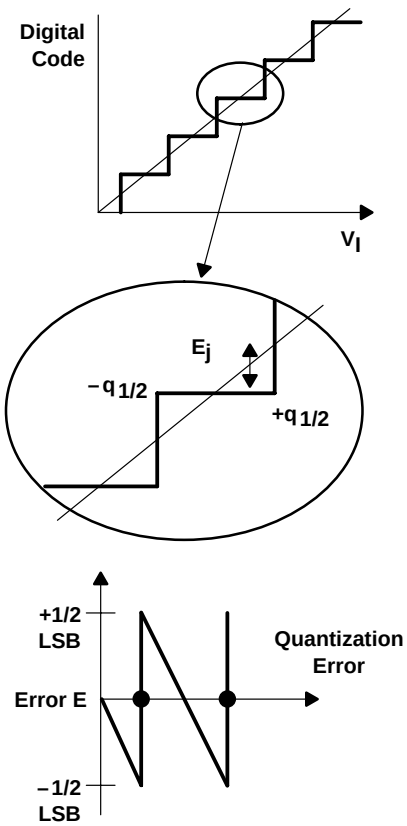
So that the maximum frequency is given by

$$f_{MAX} = \frac{1}{T_A \pi 2^{n+1}}$$

5 QUANTIZATION EFFECTS

The real world analog input to an ADC is a continuous signal with an infinite number of possible states, whereas the digital output is by its nature a discrete function with a number of different states determined by the resolution of the device. It follows from this therefore, that in converting from one form to the other, certain parts of the analog signal that were represented by a different voltage on the input are represented by the same digital code at the output. Some information has been lost and distortion has been introduced into the signal. This is quantization noise.

For the ideal staircase transfer function of an ADC, the error between the actual input and its digital form has a uniform probability density function if the input signal is assumed to be random. It can vary in the range ±1/2 LSB or ±q/2 where q is the width of one step as shown in Figure 9.



Error at the jth step

$E_j = (V_j - V_I)$

The mean square error over the step

$$\overline{E}_j^2 = \frac{1}{q_1} \int_{-q/2}^{+q/2} E_j^2 \, dE = \frac{q^2}{12}$$

Assuming equal steps, the total error is

$\overline{N}^2 = q^2/12$ **(Mean square quantization noise)**

For an input sine wave F(t) = A sinωt, the signal power

$$\overline{F}^2(t) = \frac{1}{2\pi} \int_0^{2\pi} A^2 \sin^2 \omega t \, d\omega t = \frac{A^2}{2}$$

$$\text{and } q = \frac{2A}{2^n} = \frac{A}{2^{n-1}}$$

$$SNR = 10 \text{ Log} \left(\frac{F^2}{n^2}\right) = 10 \text{ Log} \left(\frac{A^2/2}{A^2/3 \times 2^n}\right)$$

$$SNR = 6.02n + 1.76 \text{ dB}$$

Figure 9. Quantization Effects

Where

$$p(\epsilon) = \frac{1}{q} \text{ for } \left(-\frac{q}{2} \le \epsilon \le +\frac{q}{2}\right)$$

Otherwise

$$p(\epsilon) = 0$$

The average noise power (mean square) of the error over a step is given by

$$\overline{N}^2 = \frac{1}{q} \int_{-q/2}^{q/2} \epsilon^2 d\epsilon$$

which gives

$$\overline{N}^2 = \frac{q^2}{12}$$

6 IDEAL SAMPLING

In converting a continuous time signal into a discrete digital representation, the process of sampling is a fundamental requirement. In an ideal case, sampling takes the form of a pulse train of impulses which are infinitesimally narrow yet have unit area. The reciprocal of the time between each impulse is called the sampling rate. The input signal is also idealized by being truly bandlimited, containing no components in its spectrum above a certain value (see Figure 10).

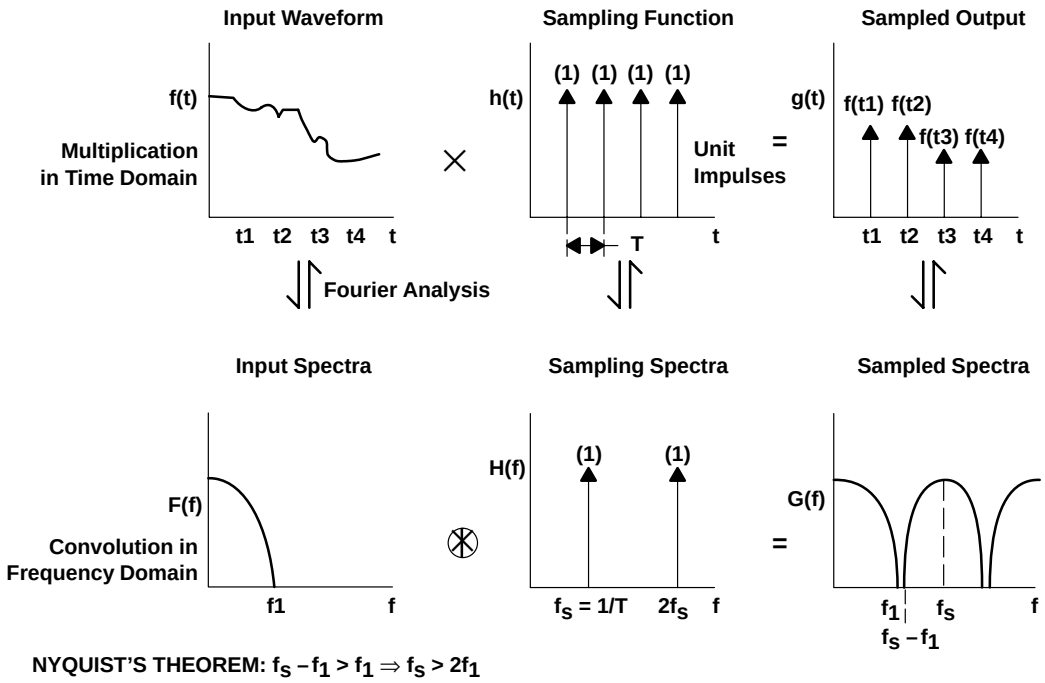


Figure 10. Ideal Sampling

The ideal sampling condition shown is represented in both the frequency and time domains. The effect of sampling in the time domain is to produce an amplitude modulated train of impulses representing the value of the input signal at the instant of sampling. In the frequency domain, the spectrum of the pulse train is a series of discrete frequencies at multiples of the sampling rate. Sampling convolves the spectra of the input signal with that of the pulse train to produce the combined spectrum shown, with double sidebands around each discrete frequency which are produced by the amplitude modulation. In effect some of the higher frequencies are folded back so that they produce interference at lower frequencies. This interference causes distortion which is called aliasing.

If the input signal is bandlimited to a frequency f1 and is sampled at frequency f_S, as shown in the figure, overlap (and hence aliasing) does not occur if

$$f1 < f_S - f1 \qquad \text{i.e.,} \quad 2f1 < f_S$$

Therefore if sampling is performed at a frequency at least twice as great as the maximum frequency of the input signal, no aliasing occurs and all of the signal information can be extracted. This is Nyquist's Sampling Theorem, and it provides the basic criteria for the selection of the sampling rate required by the converter to process an input signal of a given bandwidth.

7 REAL SAMPLING

The concept of an impulse is a useful one to simplify the analysis of sampling. However, it is a theoretical ideal which can be approached but never reached in practice. Instead the real signal is a series of pulses with the period equalling the reciprocal of the sampling frequency. The result of sampling with this pulse train is a series of amplitude modulated pulses (see Figure 11).

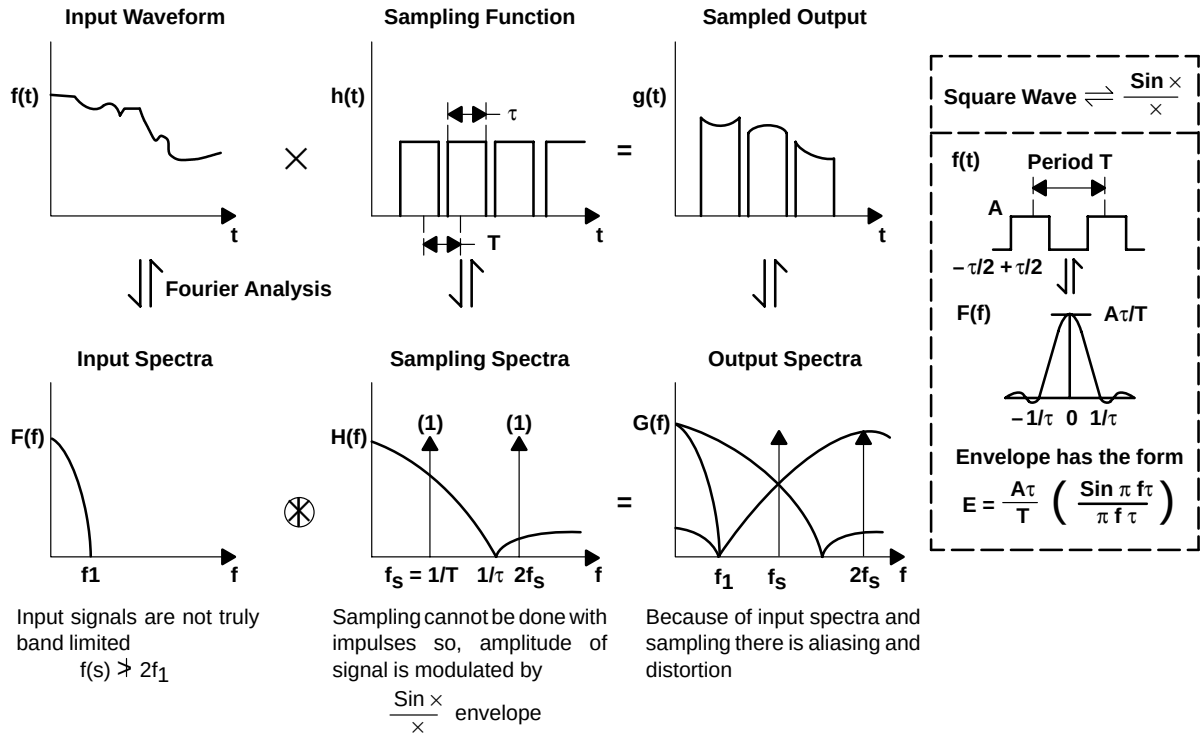


Figure 11. Real Sampling

Examining the spectrum of the square wave pulse train shows a series of discrete frequencies, as with the impulse train, but the amplitude of these frequencies is modified by an envelope which is defined by (sin x)/x [sometimes written sinc(x)] where x in this case is πf_S. For a square wave of amplitude A, the envelope of the spectrum is defined as

$$\text{Envelope} = A\left(\frac{\tau}{T}\right)\left[\sin(\pi f_S \tau)\right] \;/\; \pi f_S \tau$$

The error resulting from this can be controlled with a filter which compensates for the sinc envelope. This can be implemented as a digital filter, in a DSP, or using conventional analog techniques.

8 ALIASING EFFECTS AND CONSIDERATIONS

No signal is truly deterministic and therefore in practice has infinite bandwidth. However, the energy of higher frequency components becomes increasingly smaller so that at a certain value it can be considered to be irrelevant. This value is a choice that must be made by the system designer.

As shown, the amount of aliasing is affected by the sampling frequency and by the relevant bandwidth of the input signal, filtered as required. The factor that determines how much aliasing can be tolerated is ultimately the resolution of the system. If the system has low resolution, then the noise floor is already relatively high and aliasing does not have a significant effect. However, with a high resolution system, aliasing can increase the noise floor considerably and therefore needs to be controlled more completely.

One way to prevent aliasing is to increase the sampling rate, as shown. However, the frequency is limited by the type of converter used and also by the maximum clock rate of the digital processor receiving and transmitting the data. Therefore, to reduce the effects of aliasing to within acceptable levels, analog filters must be used to alter the input signal spectrum (see Figure 12).

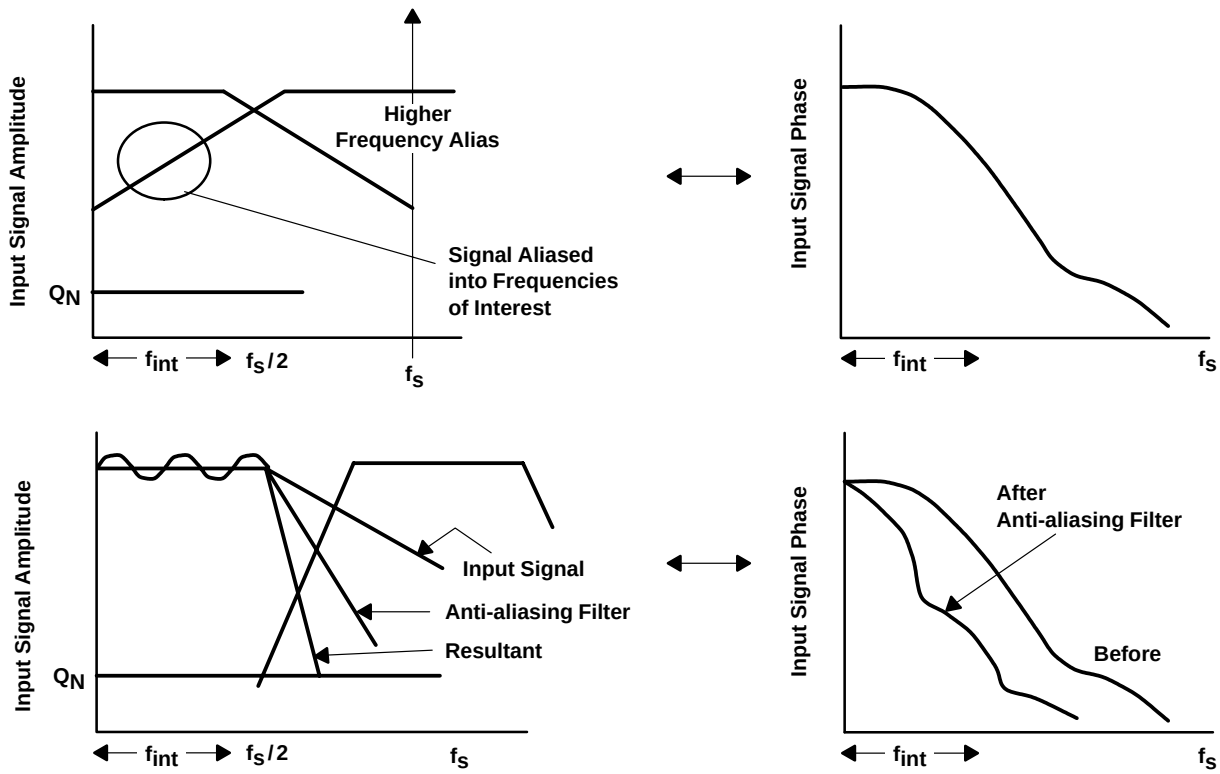


Figure 12. Aliasing Effects and Considerations

8.1 Choice of Filter

As shown with sampling, there is an ideal solution to the choice of a filter and a practical realization that compromises must be made. The ideal filter is a so-called brickwall filter which introduces no attenuation in the passband, and then cuts down instantly to infinite attenuation in the stopband. In practice, this is approximated by a filter that introduces some attenuation in the passband, has a finite rolloff, and passes some frequencies in the stopband. It can also introduce phase distortion as well as amplitude distortion. The choice of the filter order and type must be decided upon so as to best meet the requirements of the system.

8.2 Types of Filter

The basic types of filters available to the designer are briefly presented for comparison purposes. This is not intended to be a full analysis of the subject; therefore, other texts should be referenced for more details.