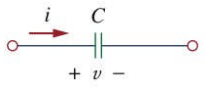
EE303 Lesson 4: Tuned circuits

Tuned circuits

All communications equipment contains tuned circuits, circuits made of capacitors and inductors that resonate at specific frequencies.

Capacitors

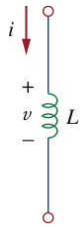
Recall that capacitors oppose changes in voltage.
In ac circuits this opposition is termed reactance (X_C) and given



$$Z_C = \frac{-j}{\omega C} = \frac{-j}{2\pi fC}$$

Inductors

Inductors oppose changes in current and inductive reactance (X_L) is given

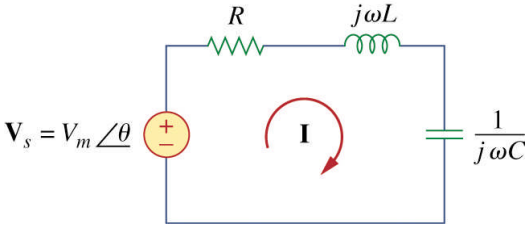


$$Z_L = j\omega L = j2\pi fL$$

Series *RLC* circuits

A series *RLC* circuit is made up of inductance, capacitance and resistance as shown below.

What is the impedance of the circuit as seen by the source?



The total impedance is given **Z** =

The magnitude of the impedance is

$$|\mathbf{Z}| = Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Impedance (Z) is minimized when the inductive and capacitive reactance are equal ($X_L = X_C$).

$$X_L - X_C = \omega L - \frac{1}{\omega C} = 2\pi fL - \frac{1}{2\pi fC} = 0$$

At what frequency f is this true?

Resonance

Impedance (Z) is minimized when the inductive and capacitive reactance are equal ($X_L = X_C$).
The frequency where this occurs is called the _____ (f_r) and is given by

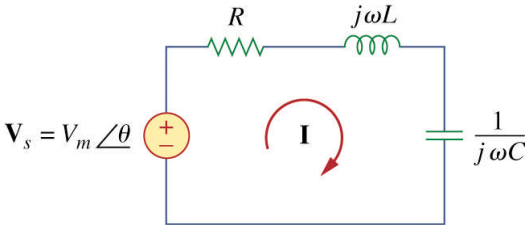
$$f_r =$$

Resonance is a balanced condition between inductive and capacitive reactance.

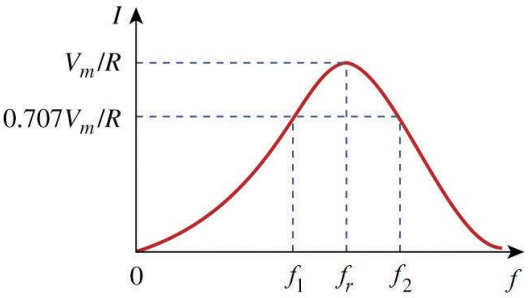
At resonance, the series *RLC* impedance is purely _____.

Let us consider the magnitude of the current **I** as a function of frequency.

$$I = |\mathbf{I}| = \left|\frac{\mathbf{V}_S}{\mathbf{Z}}\right| = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Below is a plot the current amplitude versus frequency.



Half-power frequencies

The average power dissipated by this circuit is $P = \frac{1}{2} I^2 R$

which peaks at $f = f_r$ when $P = \frac{1}{2} I^2 R =$

The frequencies f_1 and f_2 are defined as the _____ or _____ frequencies. It can be shown that the resonant frequency is the geometric mean of the cutoff frequencies

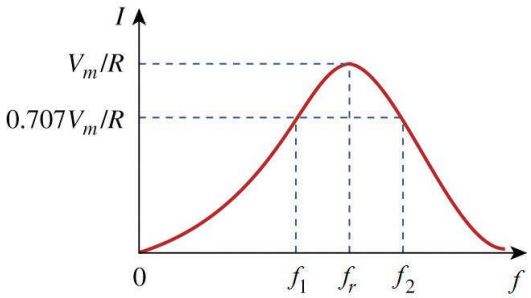
$$f_r =$$

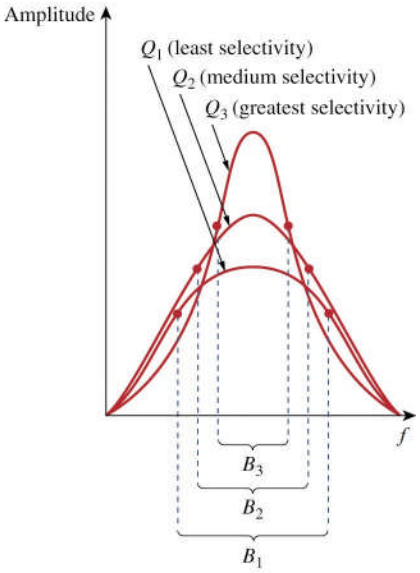
The upper and lower cutoff frequencies f_1 and f_2 are often called the _____ frequencies because at these points to the circuit's power dissipation down by a factor of one-half from its peak.

Bandwidth

The bandwidth (BW) of the tuned circuit is defined as the difference between upper and lower cutoff frequencies.

BW =





Quality factor

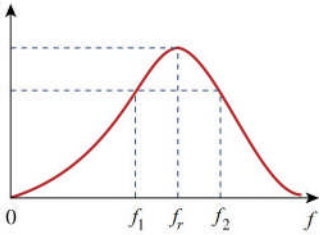
The "sharpness" of the resonance is measured by the quality factor Q. The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth

$Q =$

It can be shown for a series RLC circuit that

$Q =$

For a high-Q circuit ( ), the upper and lower half-power frequencies (f_1, f_2) are approximately symmetric about f_r so

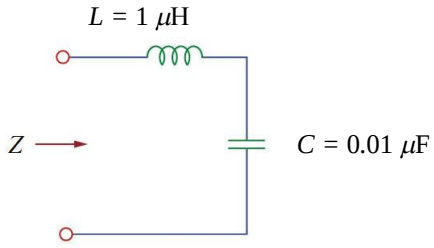


$f_1 \simeq$

$f_2 \simeq$

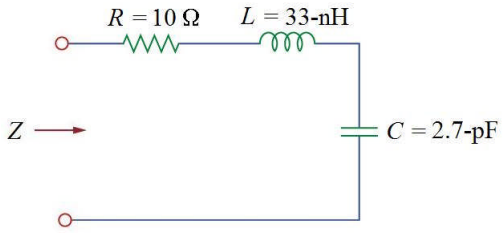
Example Problem 1

Calculate the impedance (Z) of the circuit below at f = 2 MHz with the indicated component values?



Example Problem 2

What is the resonant frequency f_r of the series RLC circuit below? At this frequency, what is the impedance of the circuit?



Example Problem 3

For the circuit below, determine the

a. Resonant frequency and half-power frequencies.

b. Quality factor and bandwidth