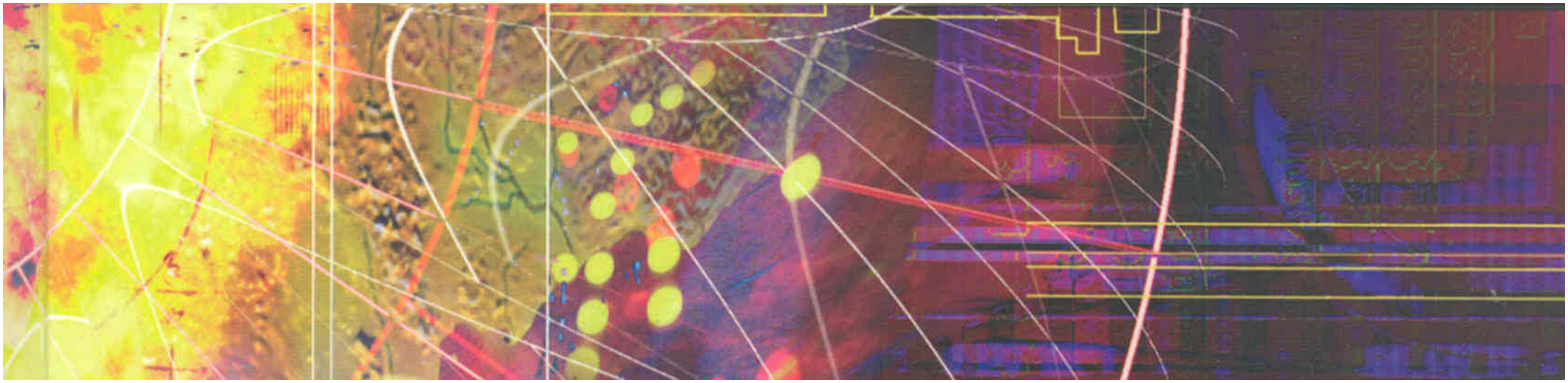


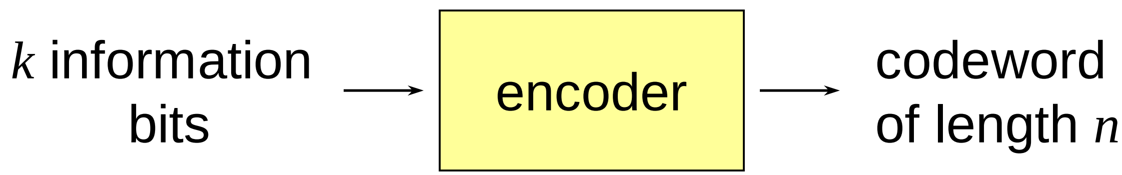
EE303 Lesson 23: Error control coding 2

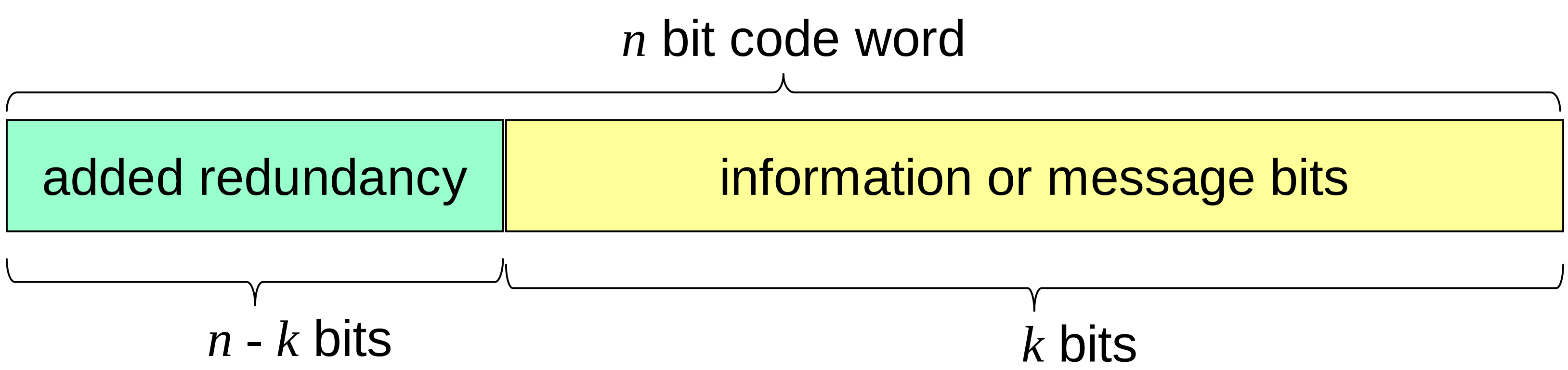
Error detection/correction

- **Error detection codes** only have the ability to confirm that bit error(s) has occurred, however they cannot tell you which bit was in error.
  - Parity codes
  - Cyclical Redundancy Check (CRC)
- **Error correcting codes** have the ability to correct bit errors without requiring a retransmission.
  - Longitudinal Redundancy Check (LRC)
  - Hamming codes
  - Reed-Solomon codes

Block codes

- An (n, k) linear block code contains k information bits and n-k bits of redundancy.



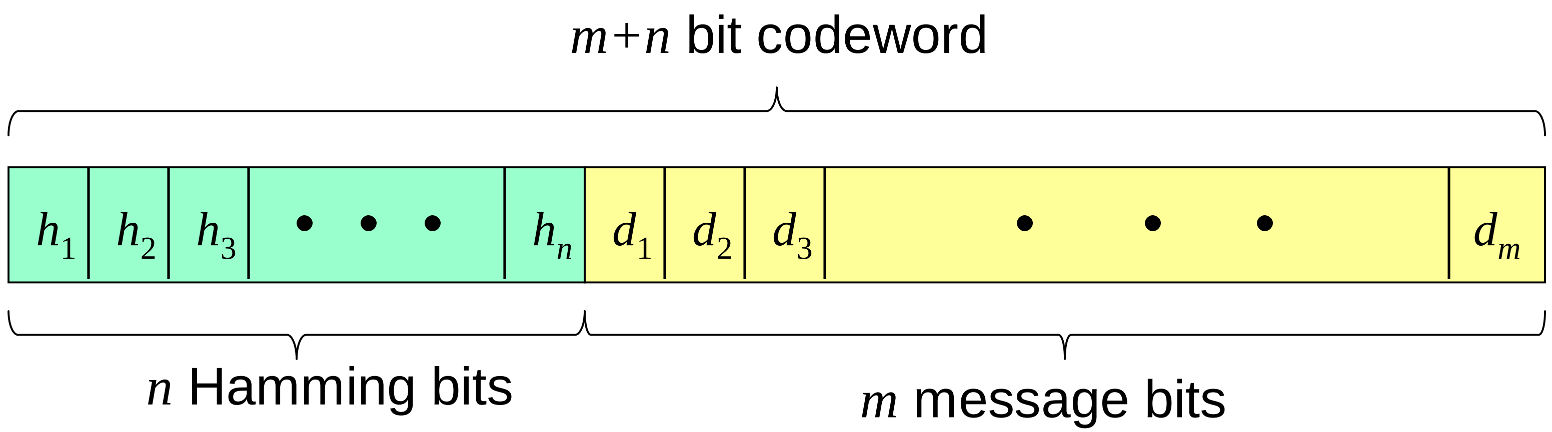


Hamming codes

- Hamming codes are a popular example of an error correcting linear block code.
- In 1950, the mathematician Richard W. Hamming published a general method for constructing error-correcting codes with a minimum distance of 3.
  - The Hamming code is capable of correcting only single bit errors.
- Hamming codes are commonly used in computer memory systems.

Hamming codes

- A Hamming code word consists of m message bits and n Hamming bits.
  - Note: this is different from the n and k used previously.

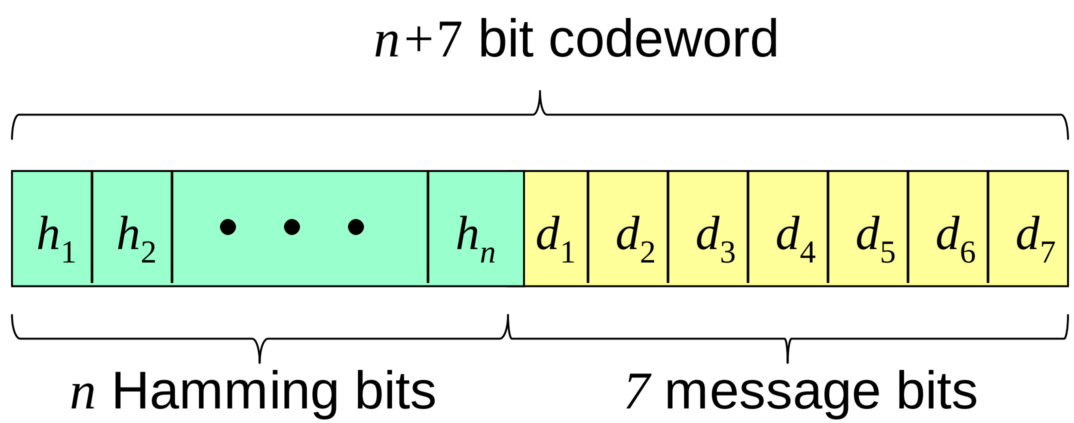


- The required of number Hamming bits (n) is given by the smallest value of $n \geq 3$ that satisfies following expression

$$2^n \geq m + n + 1$$

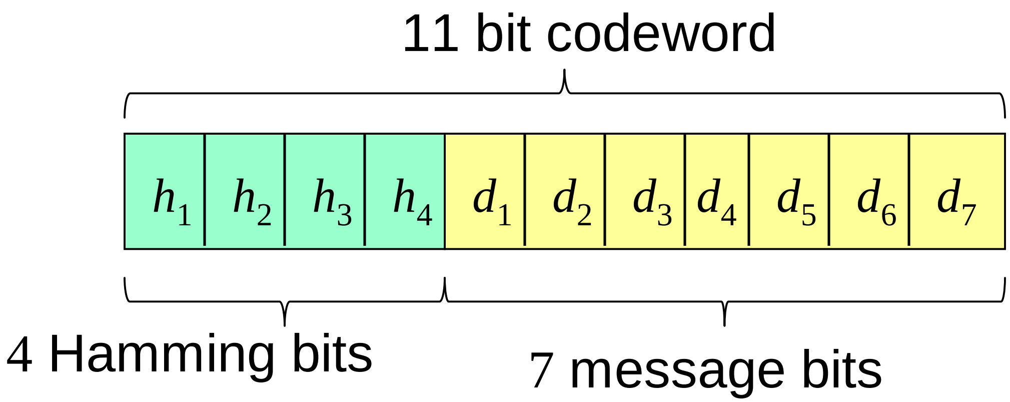
Example Problem 1

It is desired to use a Hamming code to encode a 7 bit ASCII character. How many Hamming bits are required? What is the resulting code rate R_c?

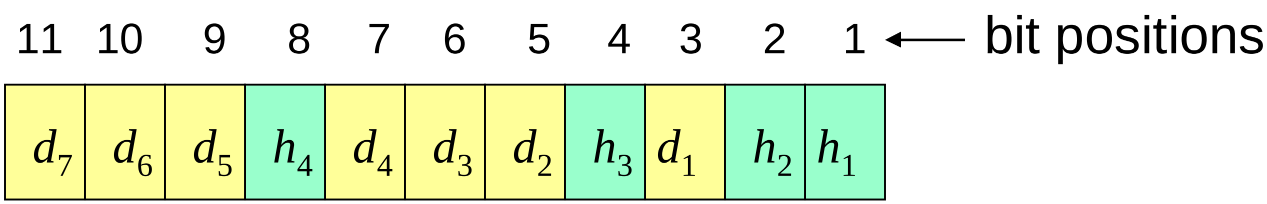


Hamming codes

- The Hamming bits need not be grouped together.



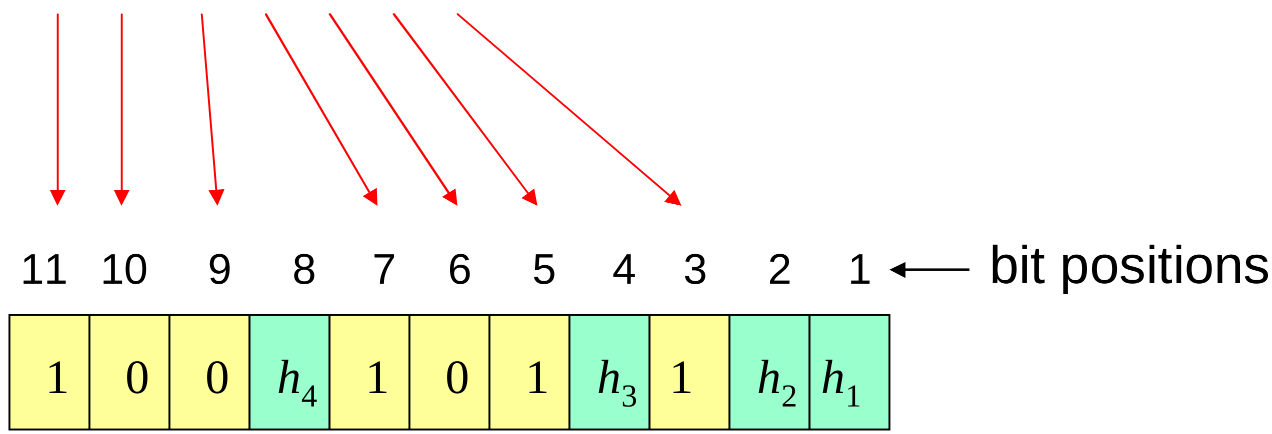
- Consider the following arrangement in which the Hamming bits are located at integer powers of 2.



Hamming encoding

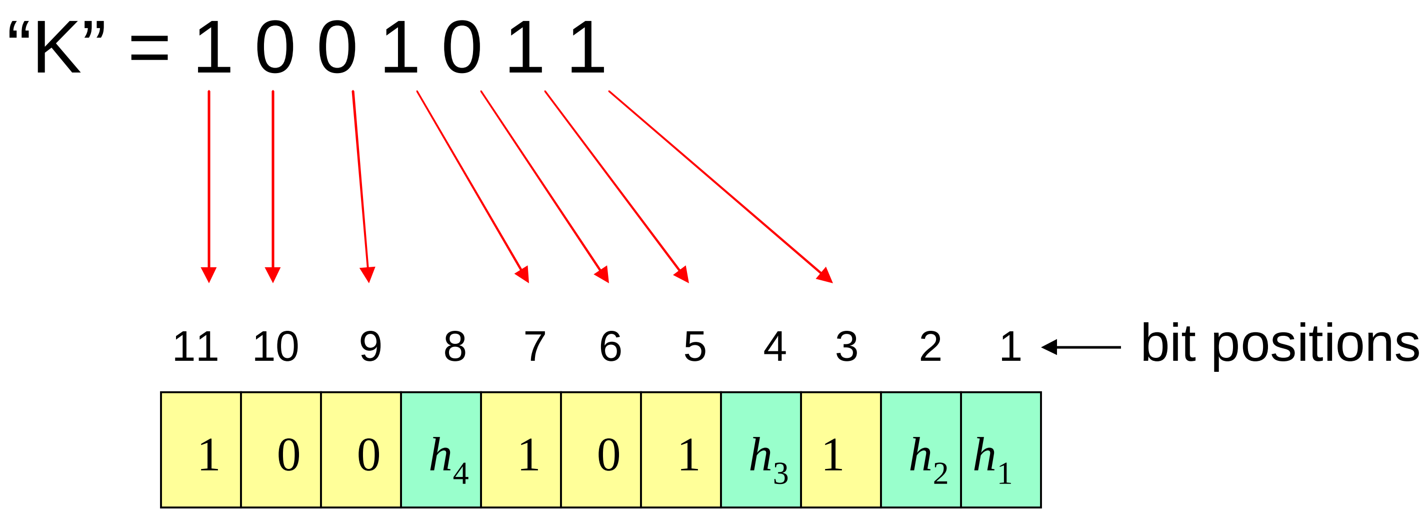
- We now need to determine the value of the Hamming bits for a given 7 bit message.
- Consider transmitting the 7-bit ASCII code corresponding to the letter “K”

“K” = 1 0 0 1 0 1 1



Hamming encoding

- The following process is used to compute the Hamming bits:
  - Express all the bit positions that contain a value of "1" as binary numbers.
    - In the case below, positions 11, 7, 5, and 3.
  - XOR these binary numbers together.
  - The resulting four bit binary number correspond the values of the Hamming bits.

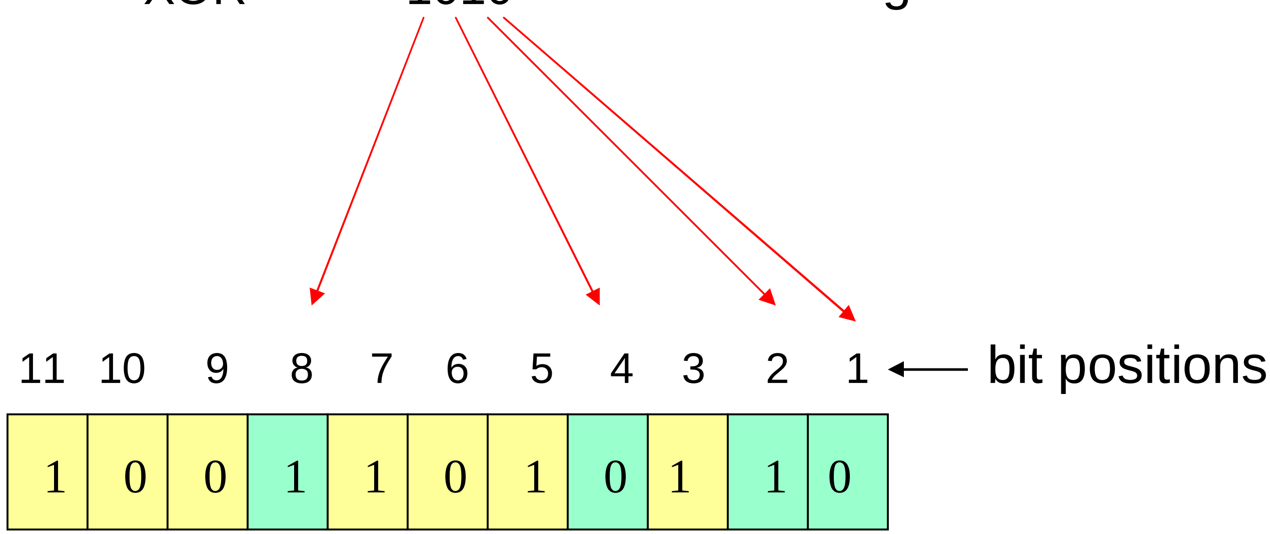


Hamming encoding

- From the previous example (positions 11,7,5,3)

Bit position	Binary
11	1011
7	0111
XOR	1100
5	0101
XOR	1001
3	0011
XOR	1010

← Hamming bits



Example Problem 2

Use the previous encoding process to encode the 7-bit ASCII code corresponding to letter "X"

"X" = 1 0 1 1 0 0 0

11	10	9	8	7	6	5	4	3	2	1 ← bit positions
d_7	d_6	d_5	h_4	d_4	d_3	d_2	h_3	d_1	h_2	h_1

Hamming decoding

- Decoding follows a similar process:
  - Express all the bit positions in the received codeword that contain a value of “1” as binary numbers (not including the Hamming bits)
    - In the case below, positions 11, 7, 5, and 3.
  - XOR these binary numbers together with 4-bit string formed by the Hamming bits (in this case 1010)
  - If no error is present, the resultant will be all zeros.

11	10	9	8	7	6	5	4	3	2	1	← bit positions
1	0	0	1	1	0	1	0	1	1	0	

Hamming decoding (no errors)

- From the previous example (positions 11,7,5,3)

Bit position	**Binary**	
11	1011	
7	0111	
XOR	1100	
5	0101	
XOR	1001	
3	0011	
XOR	1010	
Hamming bits	1010	
XOR	0000	← no errors present

11	10	9	8	7	6	5	4	3	2	1
1	0	0	1	1	0	1	0	1	1	0

Hamming decoding (with errors)

- Let's consider the case in which a single bit error occurs.
- Say an error occurs in bit position 7

1	0	0	1	~~1~~	0	1	0	1	1	0

and the following codeword is received.

11	10	9	8	7	6	5	4	3	2	1
1	0	0	1	0	0	1	0	1	1	0

- Apply the Hamming decoding process.

Hamming decoding (with errors)

- A “1” is present in bit positions 11, 5 and 3 and the Hamming bits are 1010.

Bit position	Binary	
11	1011	
5	0101	
XOR	1110	
3	0011	
XOR	1101	
Hamming bits	1010	
XOR	0111	← position of the bit error

- The resulting 4 bit number reveals the bit position of the error. ($0111_2 \rightarrow 7_{10}$)

11	10	9	8	7	6	5	4	3	2	1
1	0	0	1	0	0	1	0	1	1	0

Example Problem 3

The following codeword is received. Use the Hamming decoding process to determine whether an error exists. If an error exists, determine which bit is in error.

11	10	9	8	7	6	5	4	3	2	1	← bit positions
0	0	1	0	1	0	0	1	0	0	1	

Example Problem 3 answer

The error was in bit position 11.

11	10	9	8	7	6	5	4	3	2	1
0	0	1	0	1	0	0	1	0	0	1

← bit positions

The corrected code word is

1	0	1	0	1	0	0	1	0	0	1

Hamming codes

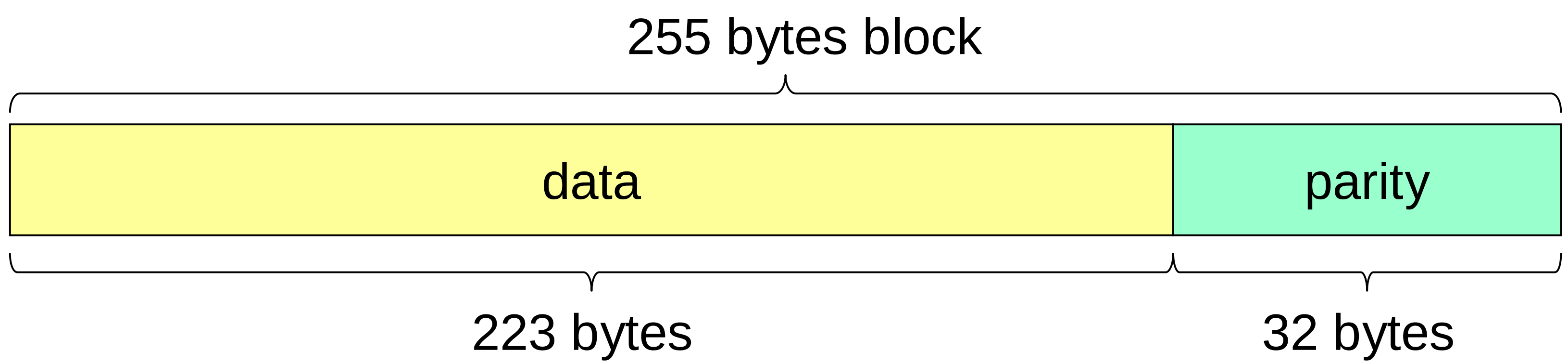
- In the previous examples using an (11,7) Hamming code, the code rate is

$$R_c = \frac{7}{11} = 63.6\%$$

  and the minimum Hamming distance between code words is 3.
  - With a minimum distance of 3, this code can correct all single bit errors.

Reed-Solomon codes

- Reed-Solomon codes were introduced in 1960.
- Like Hamming codes, they are a forward error correcting code, however Reed-Solomon codes are capable of correcting multiple bit errors and bursts of errors.
- A very popular Reed-Solomon codes is RS(255,223)



RS(255,223) can correct up to 16 bit errors

Reed-Solomon codes

- Uses include CDs, DVDs, barcodes, DSL modems, cellular phones, satellite communications, digital television

COMPACT disc DIGITAL AUDIO

Voyager spacecraft

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