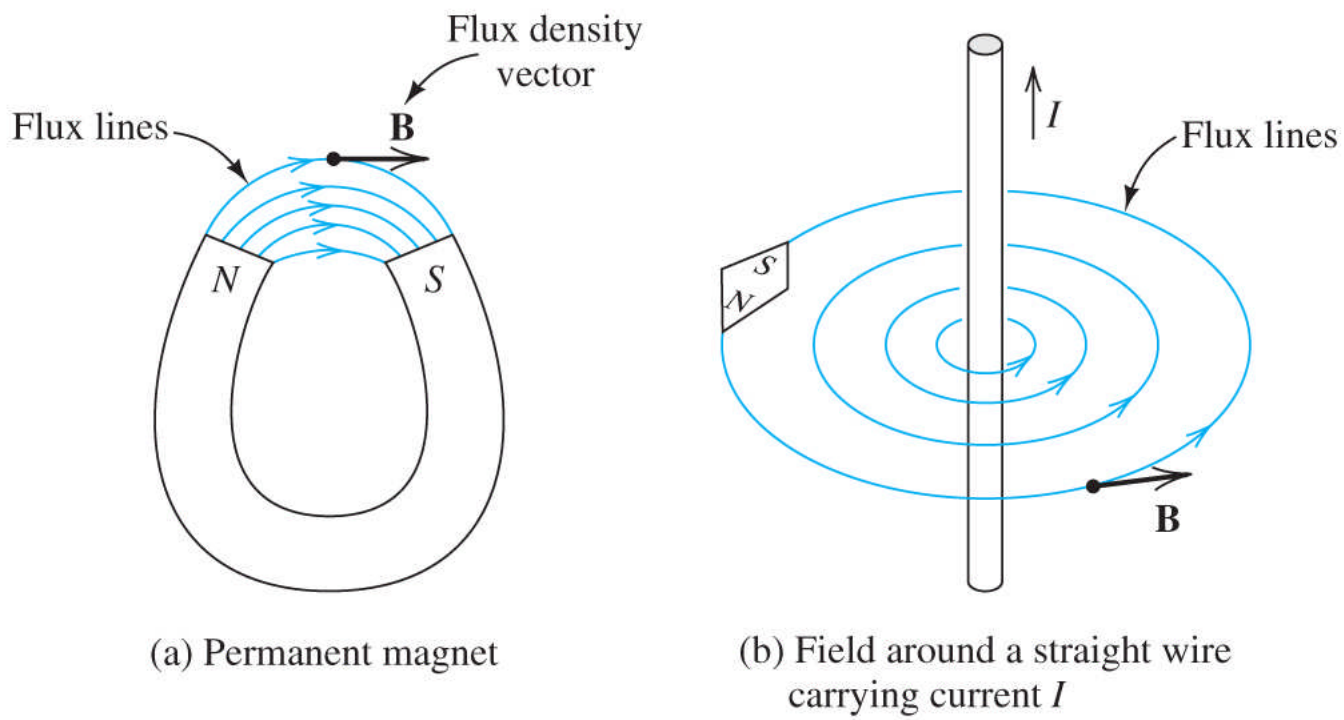
Lesson 12: Magnetism and linear machines

Magnetism

- Electric fields are created by electric charge.
- Magnetic fields arise from electric charge in motion.
  - Charges traveling in magnetic fields experience forces.
  - Time-varying magnetic fields induce voltages in conductors.
- Magnetic fields are the basis of most devices which convert energy between electrical and mechanical forms.

Magnetic fields

- Magnetic fields arise from charge in motion.
  - Permanent magnets – spin of electrons (aligned)
  - Current carrying wire – flowing electrons

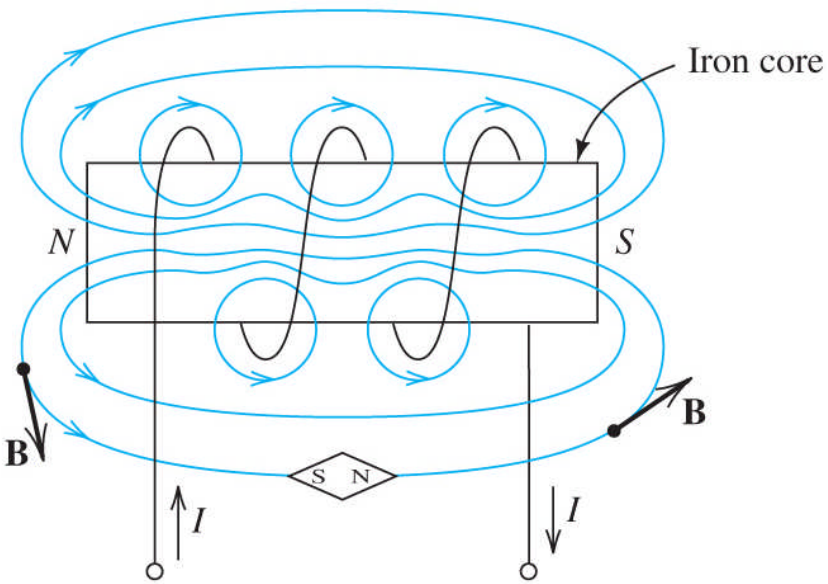


(a) Permanent magnet

(b) Field around a straight wire carrying current I

Magnetic flux

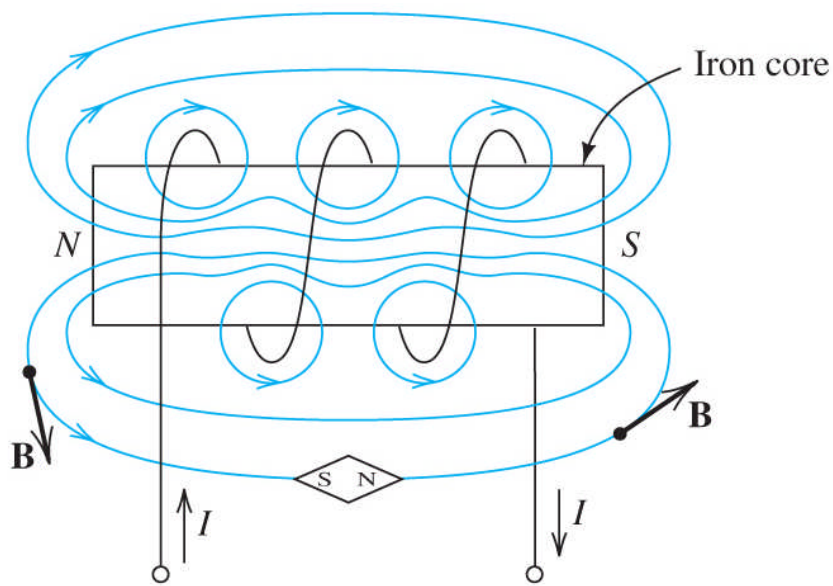
- Visualize magnetic fields using lines of **magnetic flux** that form closed paths.
  - Density of flux lines indicate intensity of the field.
- Units of magnetic flux are **webers** (Wb).



(c) Field for a coil of wire

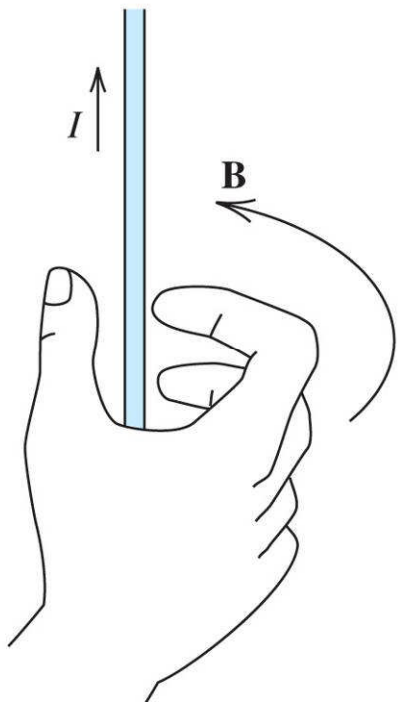
Magnetic flux density

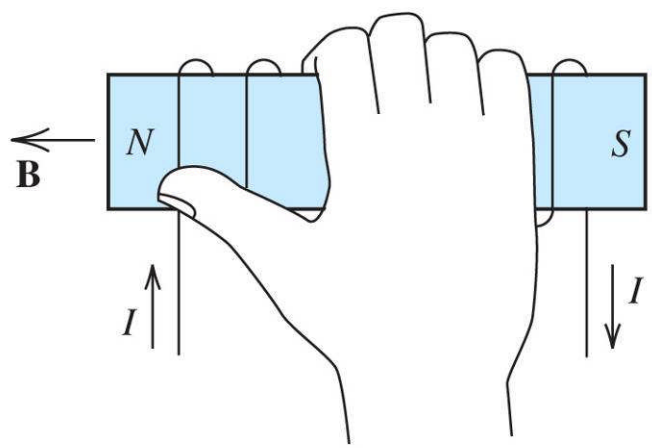
- Mathematically, we represent **magnetic flux density** as a vector quantity **B**.
- Units of **B** are webers/meter^2 (Wb/m^2) or **teslas** (T).
- The direction of **B** is tangent to lines of flux.



Right hand rule

- The right hand rule is useful for determining the direction of the magnetic field produced by a current (and the converse is also true).





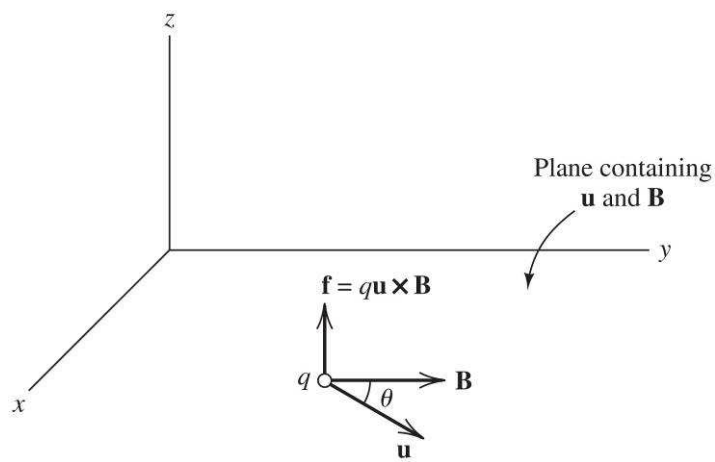
Forces on charges in magnetic fields

- A charge q with velocity vector $\mathbf{u}$ through magnetic field $\mathbf{B}$ experiences a for $\mathbf{f}$ given by

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B}$$

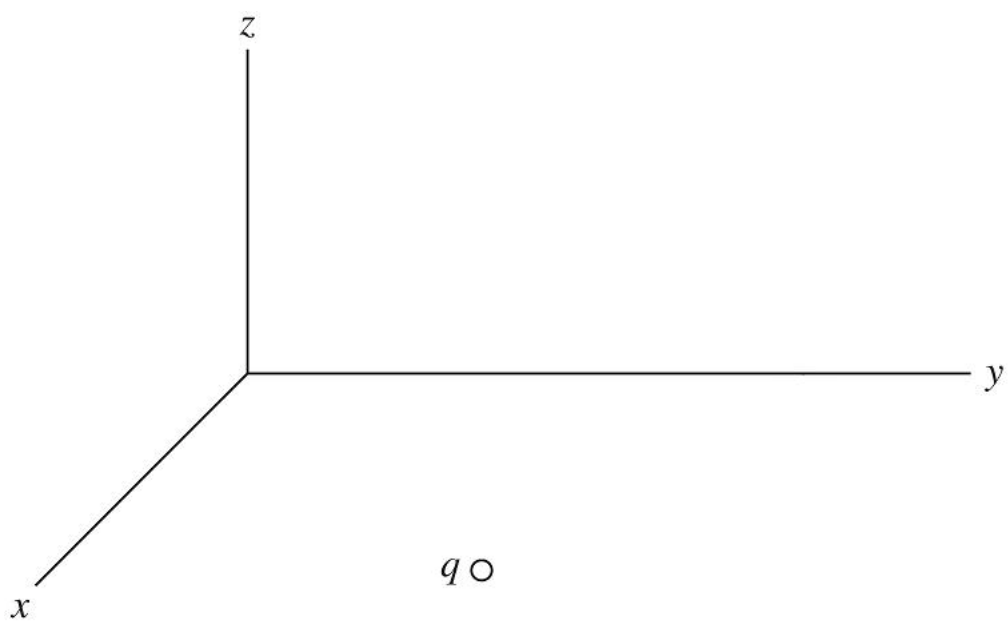
- The magnitude of the force is given by f where θ is the angle between $\mathbf{u}$ and $\mathbf{B}$.

$$f = quB\sin\theta$$



Example Problem 1

An electron ($q = -1.602 \times 10^{-19}$) travels at 10^5 m/s in the positive x direction. The magnetic flux density is 1 T in the positive y direction. Find $\mathbf{f}$.



Forces on current-carrying wire

- A current carrying wire of incremental length $d\mathbf{l}$ immersed magnetic field $\mathbf{B}$ experiences a force $d\mathbf{f}$ given by

$$d\mathbf{f} = i\, d\mathbf{l} \times \mathbf{B}$$

- For a straight wire of length l, force is given by

$$f = ilB\sin\theta$$

where θ is the angle between $\mathbf{l}$ and $\mathbf{B}$.

Example Problem 2

A wire of length $l = 1$ m carries a current of 10 A perpendicular to a field of $B = 0.5$ T. Compute the magnitude of the force on the wire.

Flux linkages

- The magnetic flux (ϕ) passing through a surface area $\mathbf{A}$ is given by the surface integral.

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

- If flux is constant and perpendicular to the surface then

$$\phi = BA$$

- If a coil has N turns, the total flux linkage (λ) is

$$\lambda = N\phi$$

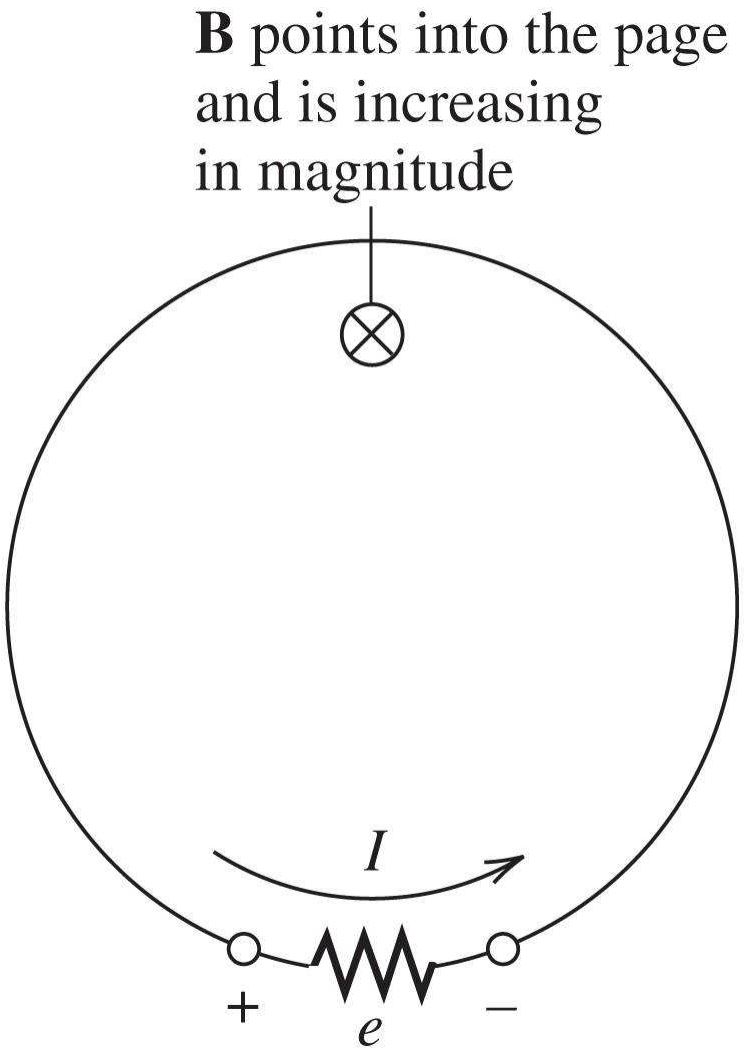
Faraday's law

- Faraday's law states that a voltage (e) is induced in a coil whenever its flux linkages are changing.

$$e = \frac{d\lambda}{dt}$$

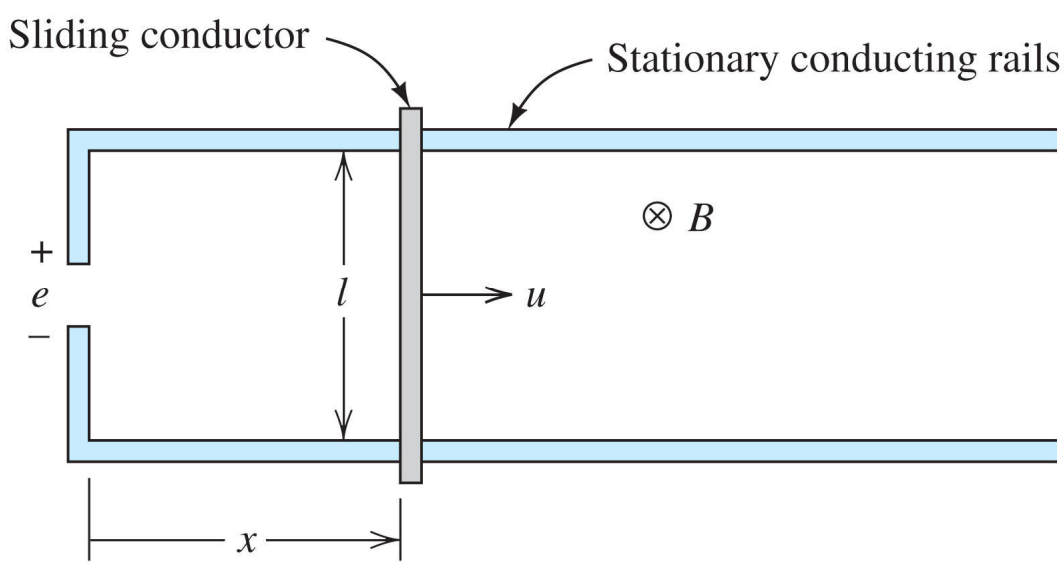
- Lenz's law states that the polarity of the induced voltage is such that the voltage produces a current that opposes the change in flux linkage.

Faraday's law



Voltages induced in a linear motor

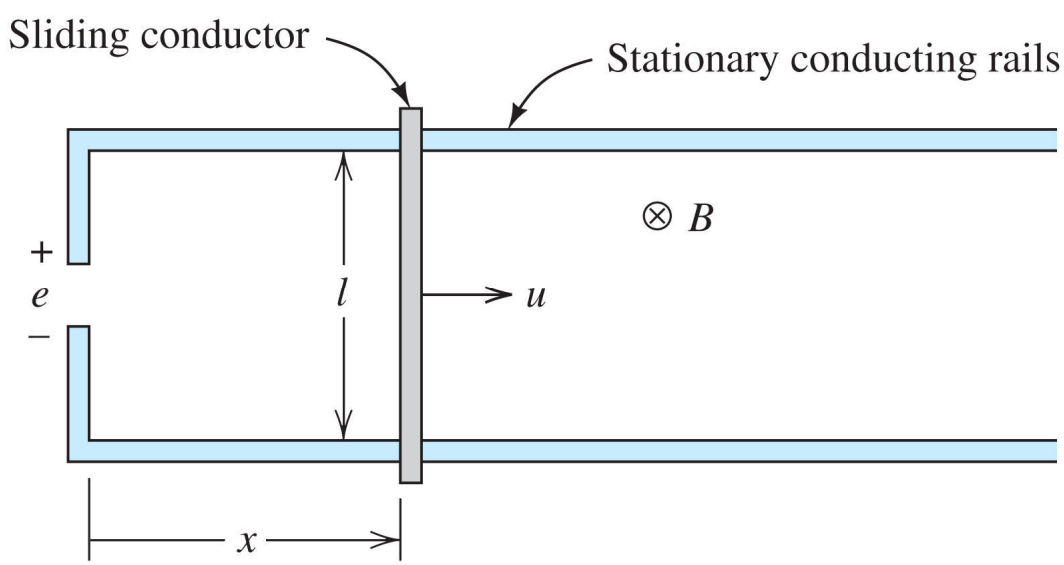
- Consider linear motor depicted below.
  - B is constant, into the page
  - l is the length of the sliding conductor
  - u is the velocity of the sliding conductor
  - The flux linkage is given $\lambda = BA = Blx$



Voltages induced in linear motor

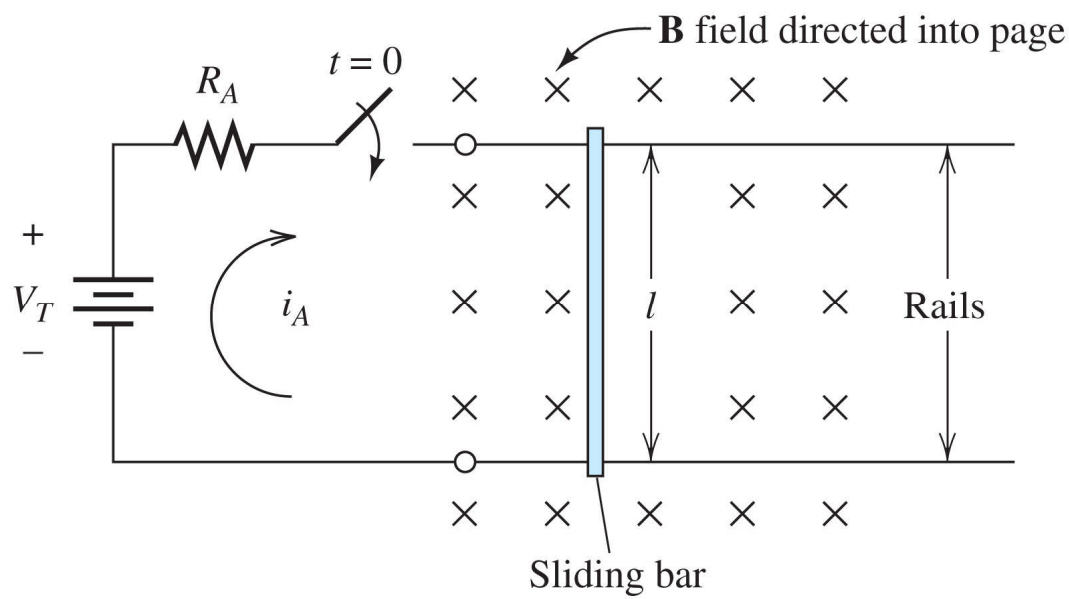
- The induced voltage in the coil is given

$$e = \frac{d\lambda}{dt} = \frac{d}{dt} Blx = Bl\frac{dx}{dt} = Blu$$



Simple dc machine

- Modify the linear motor by adding a voltage source V_T and resistance R_A and a switch that closes at $t = 0$.
- Assume the sliding bar is stationary at $t = 0$.

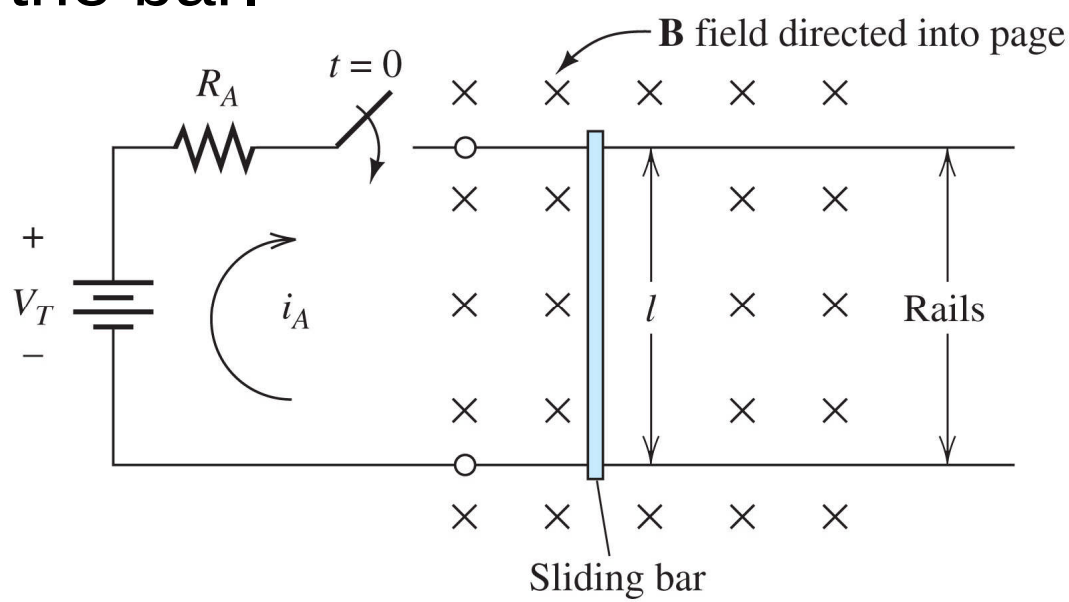


Simple dc machine

- The force exerted on the bar at $t = 0$ is

$$f = i_A l B$$

- As the bar accelerates, the voltage is induced across the bar.

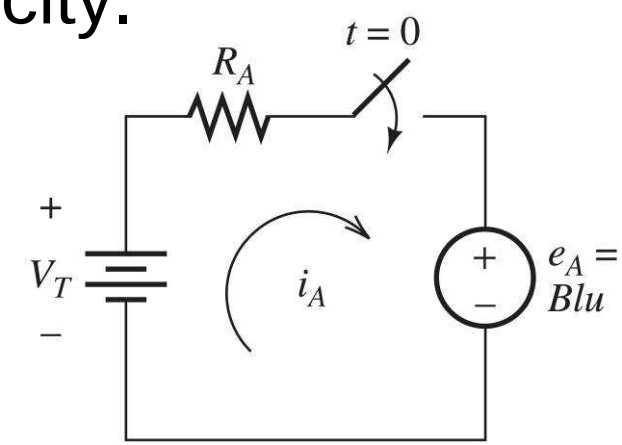


Simple dc machine

- The induced voltage e_A is given $e_A = Blu$
- This induced voltage reduces the current i_A and hence the force on the bar.

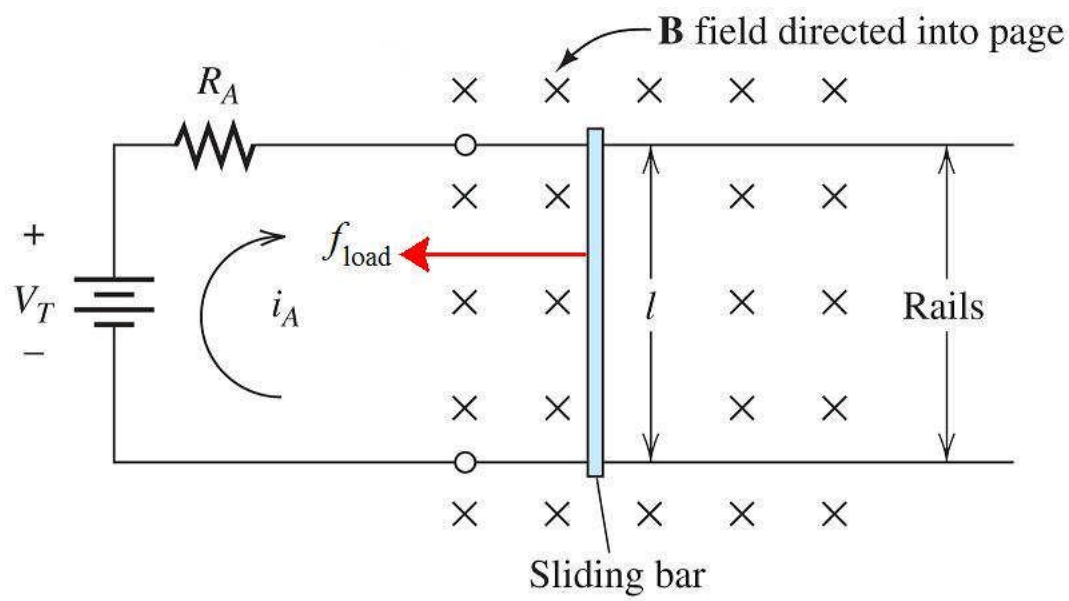
$$i_A = \frac{V_T - e_A}{R_A}$$

- Eventually, $e_A = V_T$ the and the bar travels at a constant velocity.



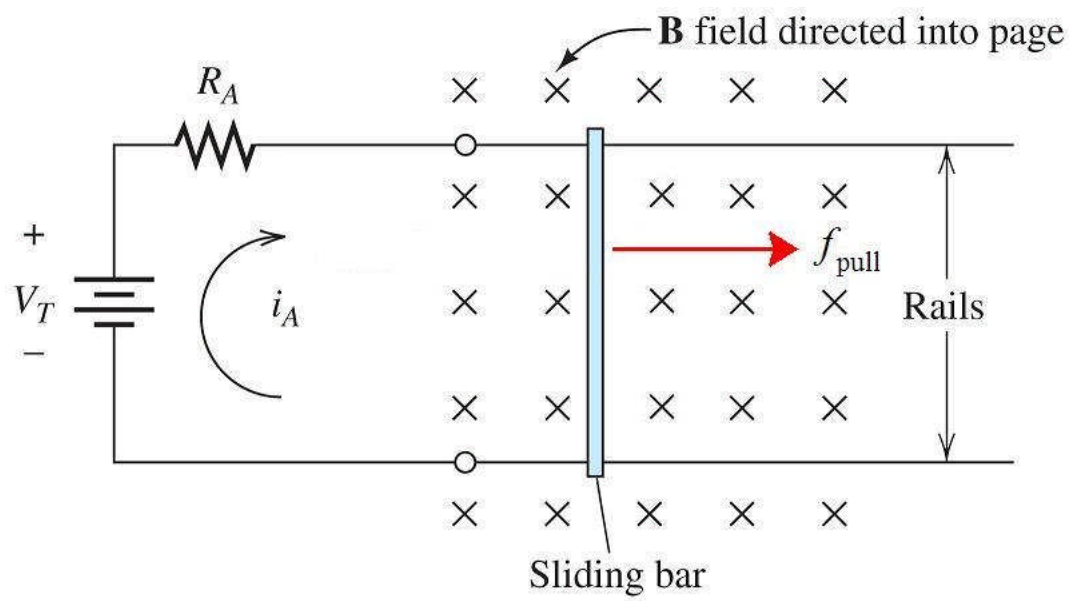
Operation as a motor

- Consider a mechanical load exerting a force f_{load} as depicted.
- What is the effect on the terminal velocity?
- How much power is delivered to the load?



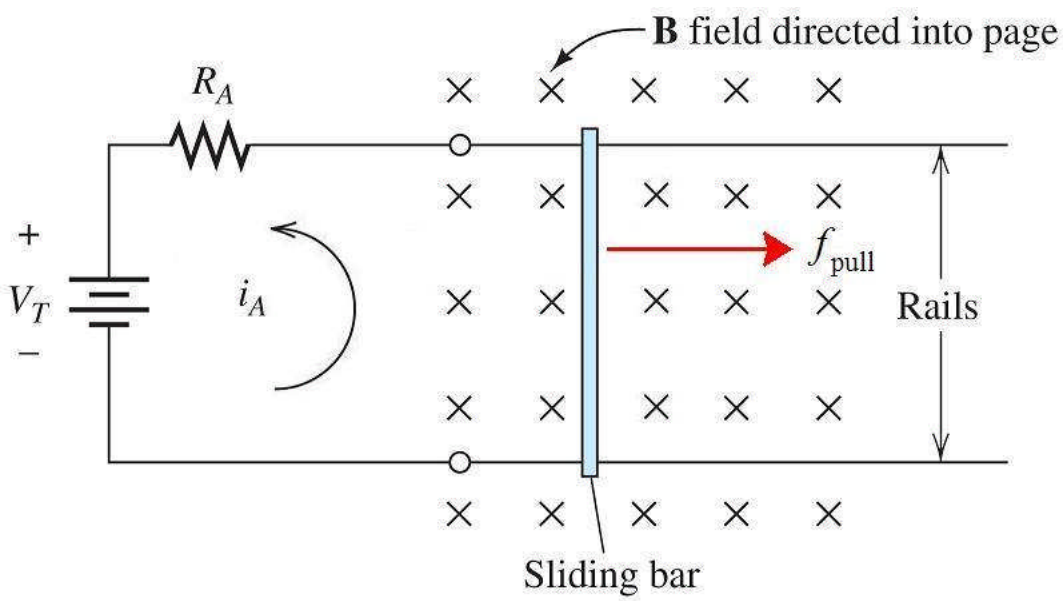
Operation as a generator

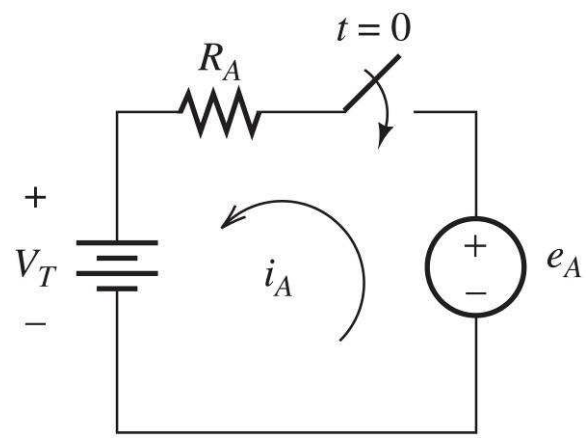
- Instead of a load, consider a mechanical force f_{pull} acting in the direction of motion.
- What is the effect on the current i_A ?



Operation as a generator

- The induced voltage e_A exceeds V_T and current is reversed.
- Mechanical energy is converted into electrical energy.





Example Problem 3

Consider a linear machine with $B = 1$ T, $l = 0.3$ m, $V_T = 2$ V and $R_A = 0.05\ \Omega$.

a. Compute i_A and f at $t = 0$. Also compute steady-state speed (no load).

b. If a load force of 4 N is applied, determine the new speed, the power supplied by V_T, power delivered to the load, power lost to heat and the efficiency.

c. If a pulling force of 2 N is applied, determine the power supplied by the mechanical source, power delivered to the battery, power lost to heat and the efficiency.