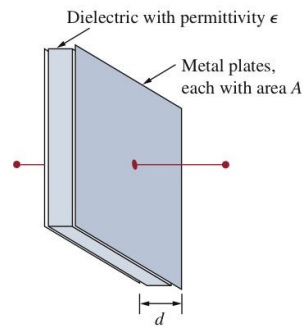
Lesson 16: Capacitors and inductors



Introduction

Thus far we have only considered resistive circuits.

- The voltage-current relations are _____.

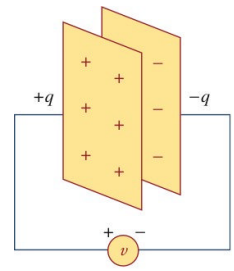
Capacitors and inductors are dynamic elements.

- The voltage-current relations are _____.
- They are dynamic because they store energy.

Capacitor

A capacitor is passive element designed to store energy in its _____ field.
Capacitors consist of parallel plates separated by an insulator (or dielectric).
Capacitors accumulate electric _____.

Capacitance



When voltage is applied, $+q$ charge moves to the positive plate and $-q$ to the negative.
The amount of charge q is directly proportional to the applied voltage v.

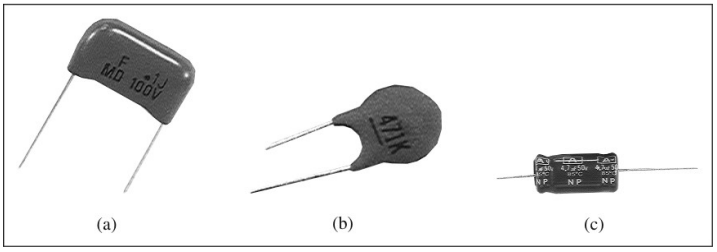
$q =$

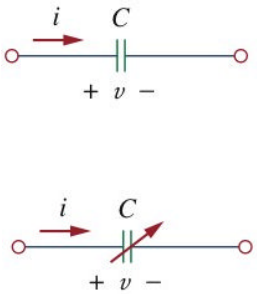
C is the constant of proportionality known as capacitance with units of _____ (F).

Capacitance itself is independent of q or v, but only depends on the physical properties of the capacitor.
For parallel-plate capacitors, capacitance is given $C =$

Practical capacitors

Capacitors have typical values from in the picofarad (pF) to microfarad (μF) range.





Passive sign convention

Capacitors can accumulate or discharge energy.
If $i > 0$ and $v > 0$, or $i < 0$ and $v < 0$ then the capacitor is being _____.
If $v \cdot i < 0$ then the capacitor is being discharged.

Given that $q = Cv$, how to we obtain the current-voltage (i-v) relationship?

Current-voltage relationship
Since current is defined $i =$, the current through a capacitor is given

What is the capacitor current if its voltage is constant?

Voltage-current relationship
The voltage-current (*v-i)* relation is given $v =$

Power and work
The instantaneous power delivered is given $p =$

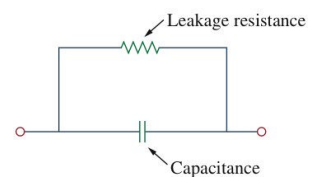
The energy (or work) stored in a capacitor is given by

$w =$

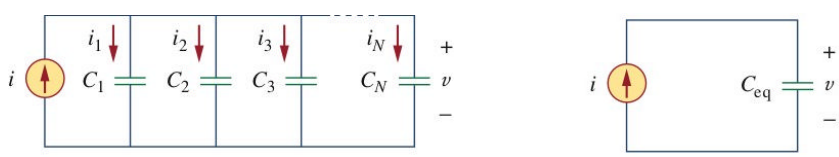
Assuming $v(-\infty) = 0$, we may write $w =$

Capacitor properties

1. When capacitor voltage is not changing in time (dc voltage), capacitor current is _____ .
   A capacitor is an _____ to dc.
2. Capacitor voltage must be _____ (i.e., no instantaneous changes in voltage)
3. Ideal capacitors do not _____.
4. A real (non-ideal) capacitor has a parallel model leakage resistance.



Capacitors in parallel
Consider N capacitors in parallel connected to a current source i.



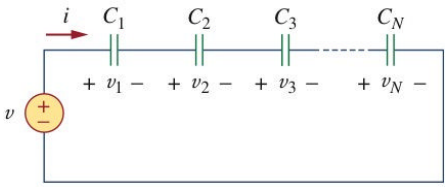
How do we obtain a equivalent capacitance C_{eq}?

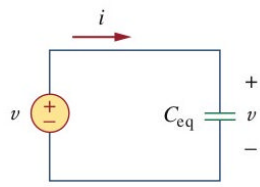
The equivalent capacitance of N parallel- connected capacitors is the sum of the individual capacitances.

$C_{eq} =$

Capacitors in series

Consider N capacitors in series connected to a voltage source v.





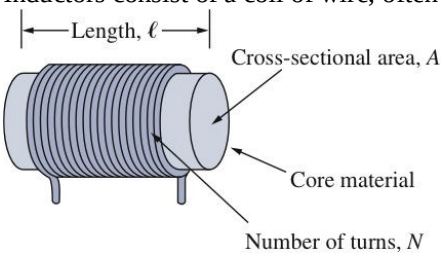
How do we obtain a equivalent capacitance C_{eq}?

The equivalent capacitance of N capacitors connected in series is the reciprocal of the sum of the reciprocals of the individual capacitances.

$$\frac{1}{C_{eq}} =$$

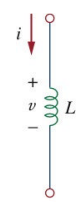
Inductor

An inductor is a passive element designed to store energy in its _____ field.
Inductors consist of a coil of wire, often wound around a core of high magnetic permeability.



Inductance

When current passes through an inductor, the voltage is directly proportional to the time rate of change of the current i.



$v =$

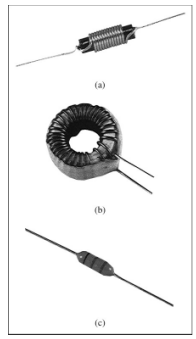
L is the constant of proportionality known as inductance and its unit is the _____ (H).

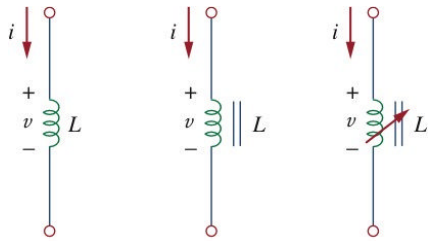
Inductance is dependent on the physical dimensions and construction of an inductor.
For the inductor above, inductance L is given

$L =$

Practical inductors

Inductors have typical values from microhenries (μH) to tens of henries (H).





Current-voltage relationship

Since voltage is defined $v =$, the current through an inductor is given

Power and work

The instantaneous power delivered is given $p =$

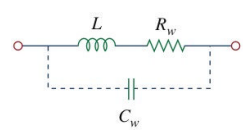
The energy (or work) stored in an inductor is given by

$w =$

Assuming $i(-\infty) = 0$, we may write

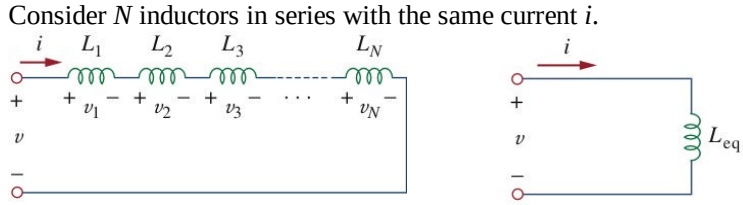
Inductor properties

1. When inductor current is not changing in time (dc) inductor voltage is zero.
   An inductor is a _____ to dc.
2. Inductor _____ must be continuous (i.e., no instantaneous changes in current)
3. Ideal inductors do not dissipate energy.
4. A real (non-ideal) inductor has a model winding resistance R_w and winding capacitance C_w.



Inductors in series

Consider N inductors in series with the same current i.



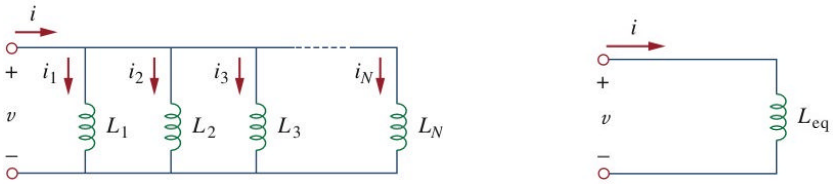
How do we obtain a equivalent inductance L_{eq}?

The equivalent inductance of N inductors connected in series is the sum of the individual inductances.

L_{eq} =

Inductors in parallel

Consider N parallel-connected inductors with the same voltage v.



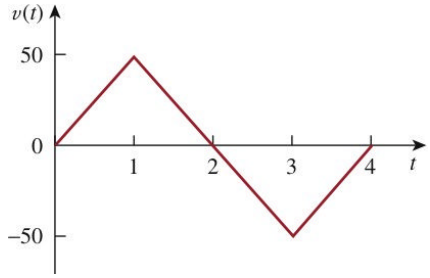
How do we obtain a equivalent inductance L_{eq}?

The equivalent inductance of N inductors connected in parallel is the reciprocal of the sum of the reciprocals of the individual inductances.

$$\frac{1}{L_{eq}} =$$

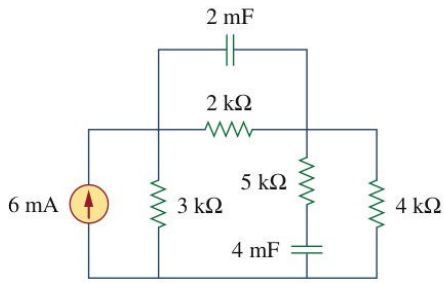
Example Problem 1

Determine the current through a 200-μF capacitor whose voltage is depicted below.



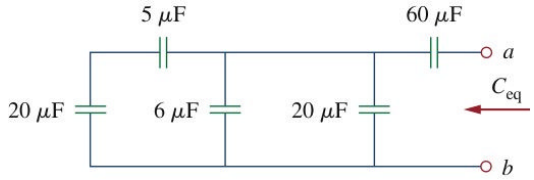
Example Problem 2

Obtain the energy stored in each capacitor under dc conditions.



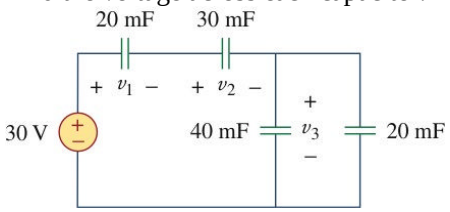
Example Problem 3

Find the equivalent capacitance seen between terminals *a* and *b* of the circuit below.



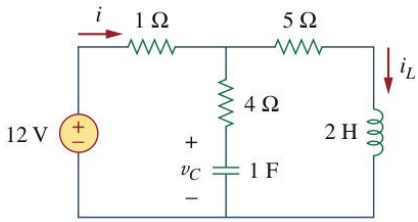
Example Problem 4

Find the voltage across each capacitor.



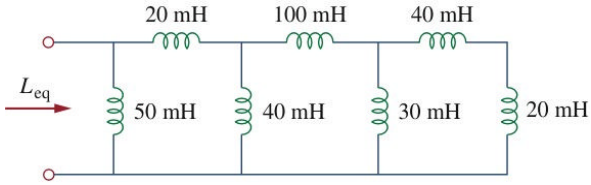
Example Problem 5

Under dc conditions find i, v_C, and i_L. Also, determine the energy stored in the capacitor and inductor.



Example Problem 6

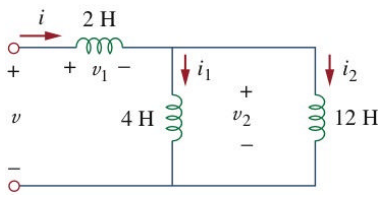
Calculate the equivalent inductance for the inductive ladder network below.



Example Problem 7

For the circuit below, $i(t) = 4\,(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find

$i_2(0)$

$v(t)$, $v_1(t)$, and $v_2(t)$,

$i_1(t)$ and $i_2(t)$