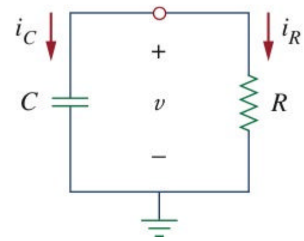
Lesson 17: Transient analysis

Introduction

- Previously we had only considered resistive circuits.
  - The voltages and currents resulting were constant in time (dc).
- Today we consider simple circuits comprised of either a resistor and capactior (RC) or resistor and inductor (RL).
  - These will result in _______________ differential equations.

Source-free RC circuit



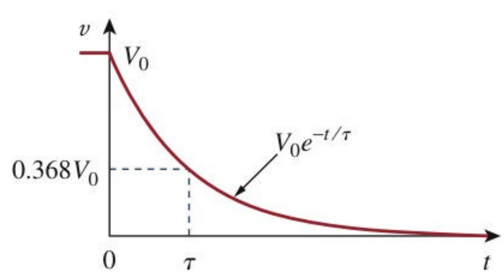
Source-free refers a circuit in which the only energy source the energy already stored in the capacitor.

- Consider a dc source previously connected then removed at $t = 0$.

For the circuit below, assume $v(0) = V_0$.

- What is $v(t)$ for $t > 0$?

Using KCL and evaluating the first-order differential equation with the initial condition $v(0) = V_0$, yields



$v(t) =$

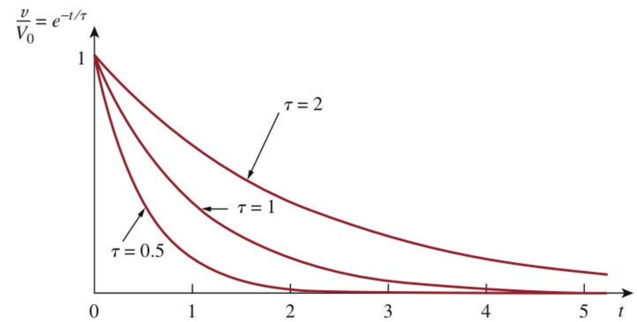
Natural response

This is known as the _________________ response of the circuit because it does not depend on any _____________________ of excitation.

Time constant
The rate of voltage decay is dependent on the time constant τ which is given τ =
The time constant of a circuit is the time required for the response to decay by a factor of 1/e or 36.8 percent of its initial value.

Using the time constant, voltage is given $v(t)$ =

For engineering purposes, it is assumed that the capacitor is fully discharged after 5τ (five time constants).



Power dissipation
The power dissipated by the resistor is

$p(t)$ =

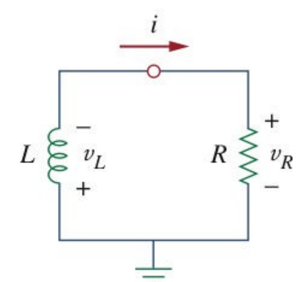
The energy absorbed up to time t is

$w_R(t)$ =

Evaluating source-free RC circuits
1. Determine the initial voltage $v(0) = V_0$ across the capacitor.
2. Find the time constant τ. (note: R is often the Thévenin equivalent resistance as seen from the capacitor).

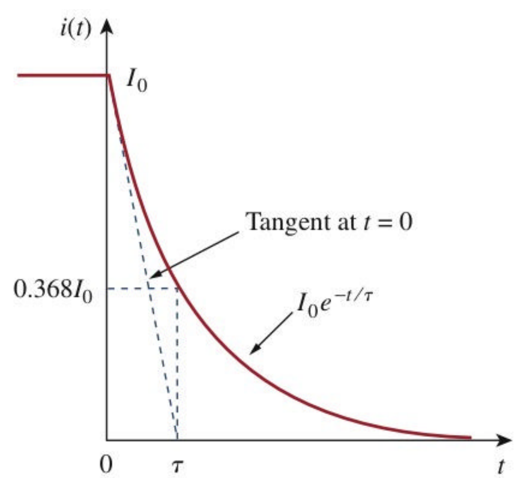
Source-free RL circuit
Consider source free RL circuit below. Assume that at t = 0, the inductor has an initial current I_0.

- What is $i(t)$ for $t > 0$?



Using KVL and evaluating the first-order differential equation with the initial condition $i(0) = I_0$ yields

$i(t)$ =



Power dissipation
The power dissipated by the resistor is $p(t)$ =

The energy absorbed up to time t is $w_R(t)$ =

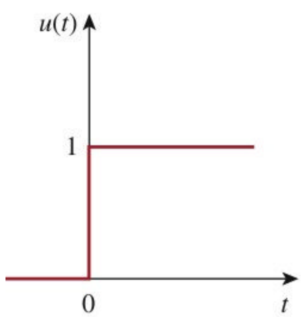
Evaluating source-free RL circuits

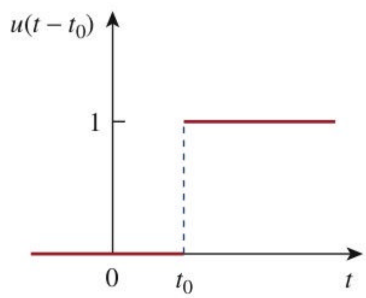
1. Determine the initial current $i(0) = I_0$ through the inductor.
2. Find the time constant τ. (note that R is often the Thévenin equivalent resistance as seen from the inductor).

Step function
The unit step function is a convenient way to mathematically represent the operation of a ____________.

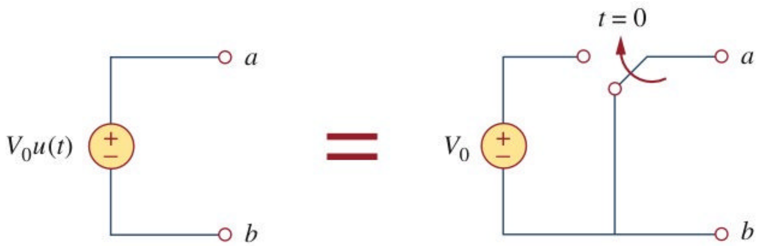
The unit step function $u(t)$ is 0 for $t < 0$ and 1 for $t > 0$. $u(t)$ =

The unit step function can be translated horizontally by adjusting its argument. $u(t- t_0)$ =



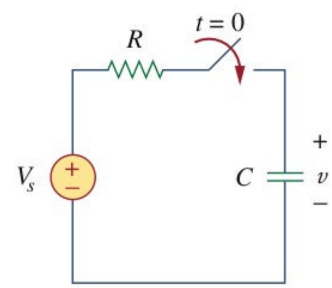


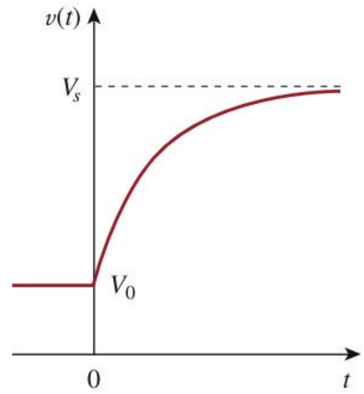
Using the step function
The unit step function can be used to replace a switch as shown below.



Step response of an RC circuit
Consider the circuit below in which a switch is thrown at t = 0.
Assume initial voltage across the capacitor for $t < 0$ is V_0.

- What is $v(t)$?





$v(t) =$

Complete response

This is known as the complete response (or total response) of the circuit because it included the natural response and the response caused by the independent source.

complete response = ____________ response (stored energy) + ____________ response (independent source)

Shortcut

Decompose the problem.

$v =$

Evaluating RC circuits

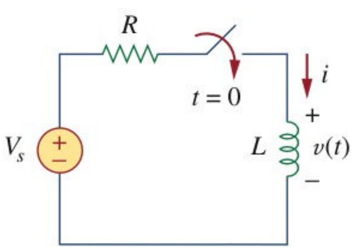
1. Determine initial capacitor voltage v(0).
2. Determine final capacitor voltage v(∞) = V_0.
3. Find the time constant τ.

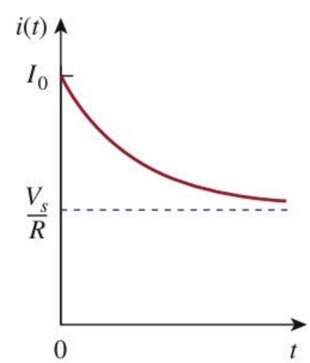
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Step response of an RL circuit

Consider the circuit below in which a switch is thrown at $t = 0$. Assume initial current through the inductor for $t < 0$ is I_0.

- What is $i(t)$?





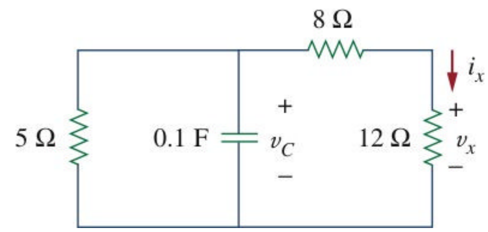
Evaluating RL circuits

1. Determine initial inductor current i(0) at t= 0.
2. Determine final inductor current i(∞).
3. Find the time constant τ.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

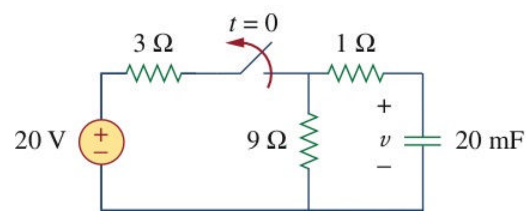
Example Problem 1

For the circuit below, let $v_c(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.



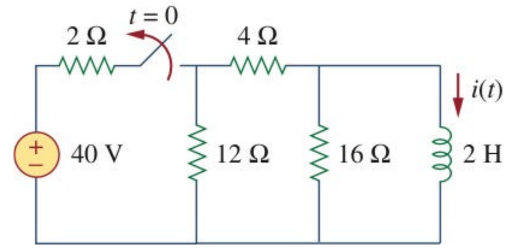
Example Problem 2

The switch in the circuit below has been closed for a long time and is open at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.



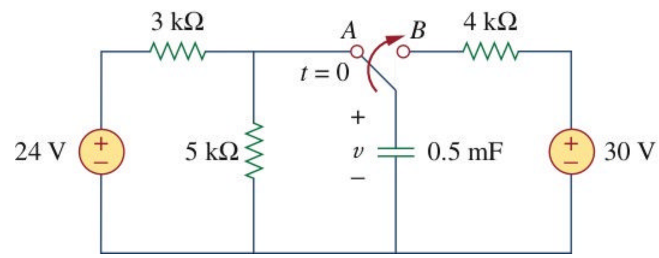
Example Problem 3

The switch in the circuit below has been closed for a long time and is open at $t = 0$. Calculate $i(t)$ for $t \geq 0$.



Example Problem 4

The switch in the circuit below has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t \geq 0$ and calculate its value at $t = 2$ and 4 sec.



Example Problem 5

Find $i(t)$ for $t > 0$. Assume the switch has been closed for a long time.