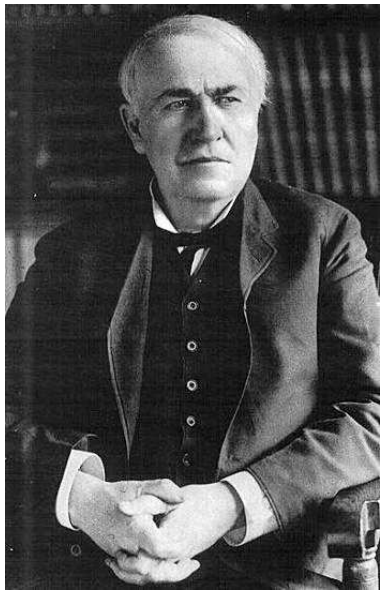
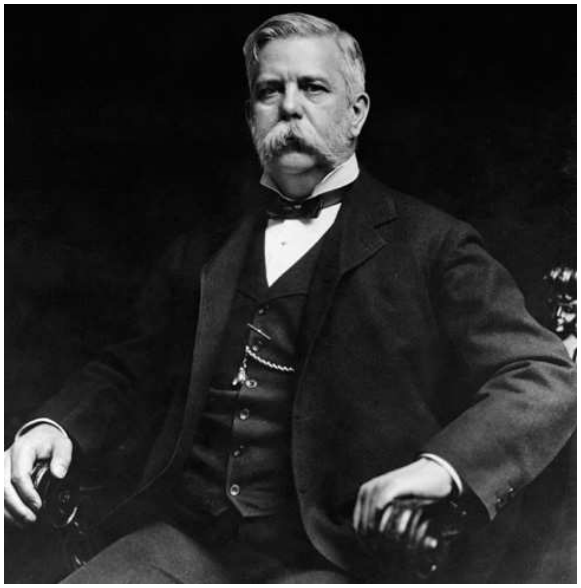
Lesson 18:
Sinusoids and RMS

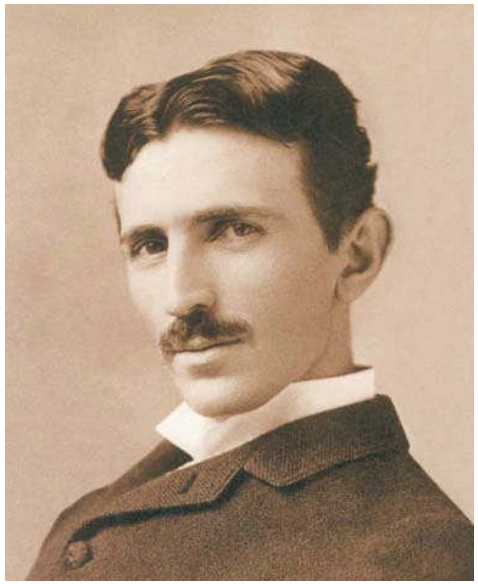
AC vs. DC wars



Thomas Edison



George Westinghouse



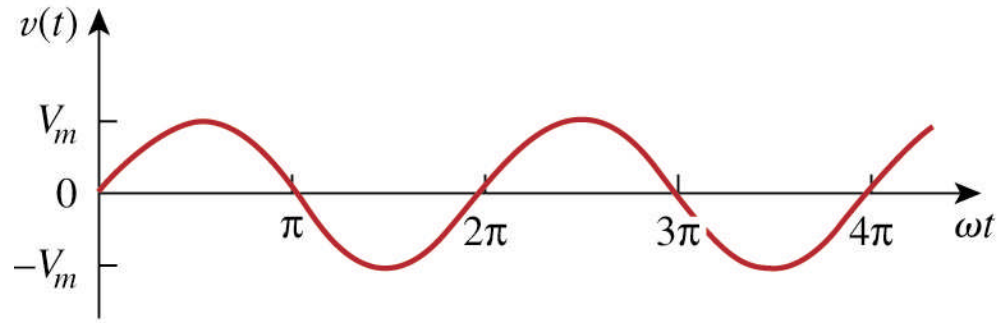
Nikola Tesla

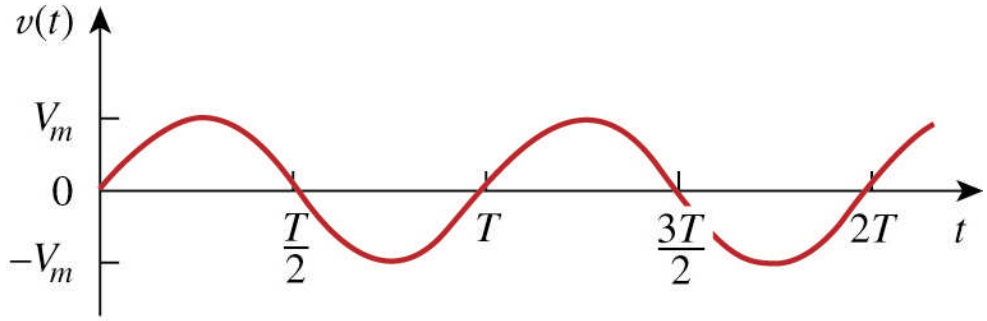
DC

AC

Alternating current (ac)

- An alternating current waveform is a sinusoidal waveform.





Advantages of sinusoids

1. Sinusoids are commonly found in nature.
2. Sinusoidal waveforms are easy to generate and transmit.
3. Through Fourier analysis, any practical periodic waveform can be represented by the sum of weighted sinusoids.
4. Sinusoids are easy to handle mathematically.

Sinusoids

- The general form of a sinusoid is .

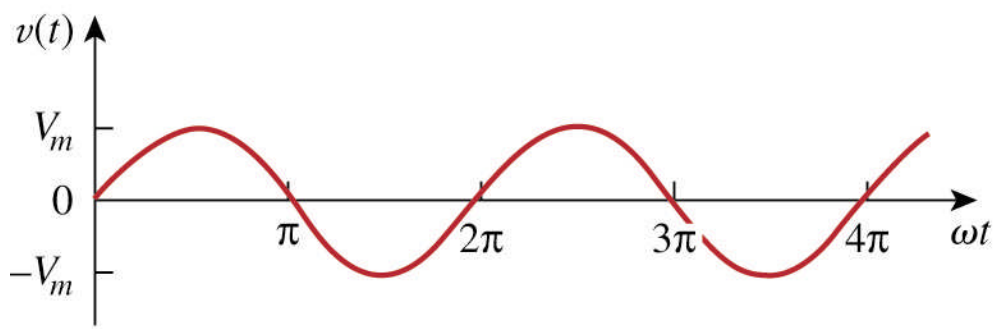
$$v(t) = V_m \sin \omega t$$

where

$V_m =$ the *amplitude* of the sinusoid

$\omega =$ the *angular frequency* in radians/s

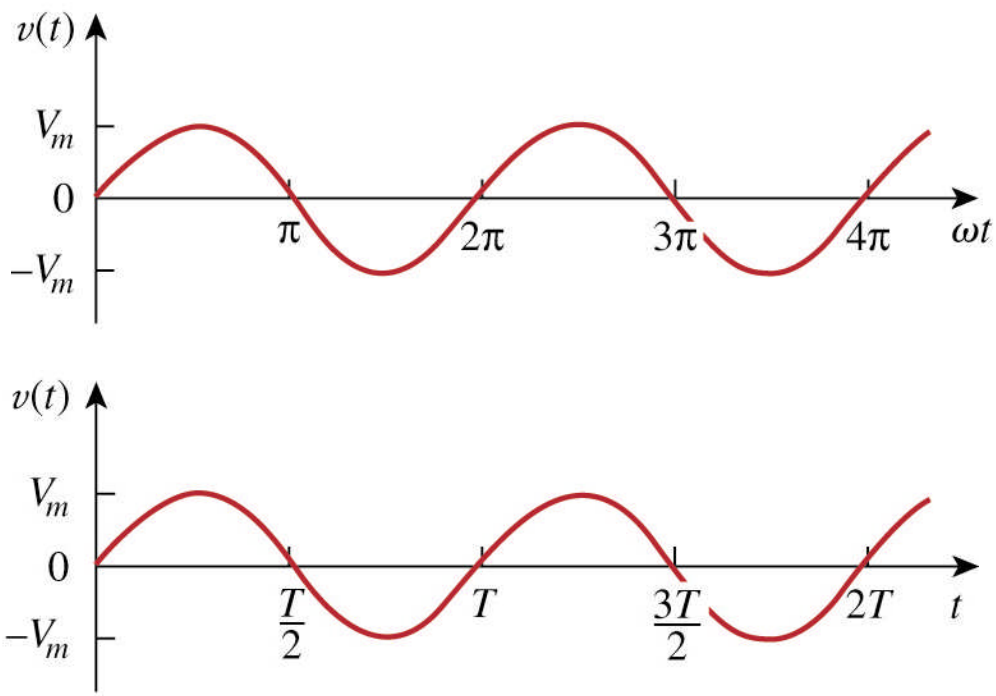
$\omega t =$ the *argument* of the sinusoid



Period

- The sinusoid repeats every T seconds, thus T is the **period** of the sinusoid and

$$T = \frac{2\pi}{\omega}$$



Periodic functions

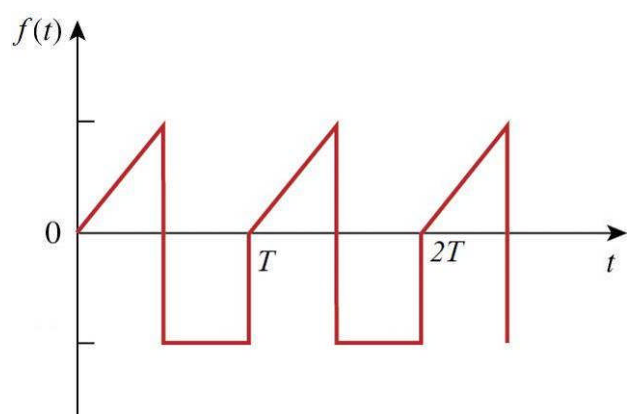
- Since $v(t)$ repeats T seconds, we can write that

$$v(t) = v(t+T)$$

- More generally, any **periodic function** $f(t)$ satisfies

$$f(t) = f(t+nT)$$

for all t and for all integers n.



Frequency

- Frequency f (cycles per second) is the reciprocal of the period T.
  - The unit of frequency is hertz (Hz).

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

  - Alternatively, we could write $\omega = 2\pi f$

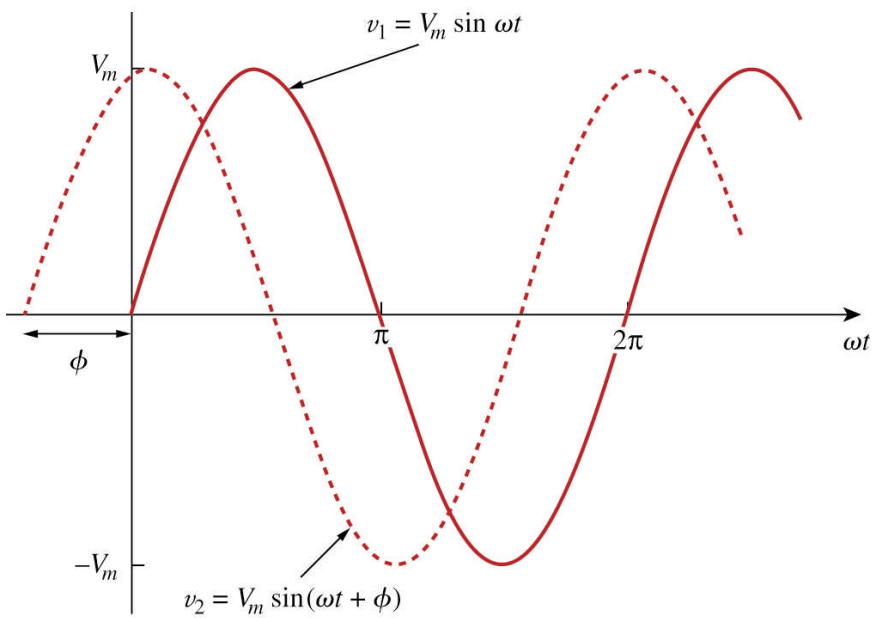
Phase

- A more general form of a sinusoid is

$$v(t) = V_m \sin(\omega t + \phi)$$

where $(\omega t + \phi)$ is the argument and ϕ is the phase.

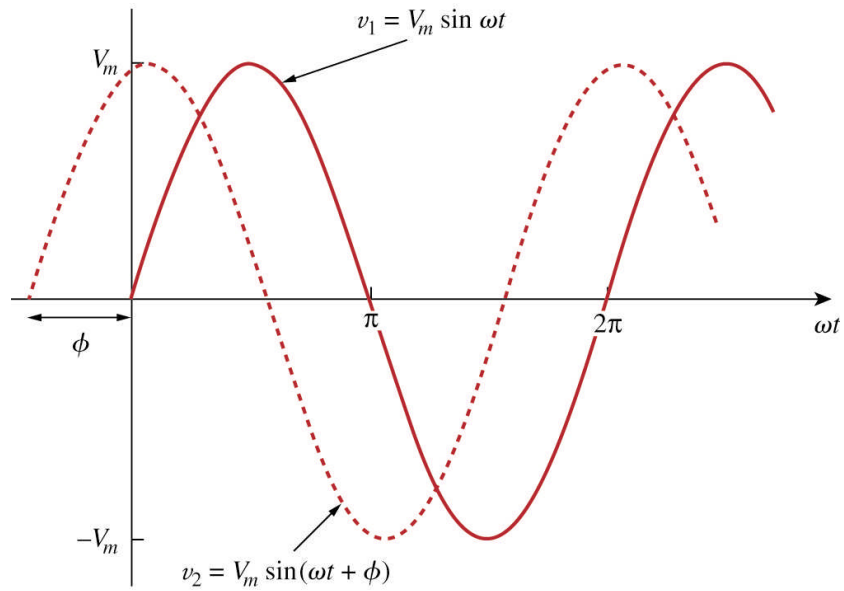
- Phase can be expressed in radians or degrees.
  - Phase is most commonly given in degrees.



Leading and lagging

- Consider two waveforms $v_1(t)$ and $v_2(t)$

$$v_1(t) = V_m \sin \omega t \quad \text{and} \quad v_2(t) = V_m \sin(\omega t + \phi)$$



- We say that v_2 **leads** v_1 by ϕ, or v_1 **lags** v_2 by ϕ.

Trigonometric identities

- When comparing two sinusoids, it is most convenient to represent them as both sine or cosine with positive amplitude.
- The following identities are useful for this conversion

$$\sin\left(\omega t \pm 180^\circ\right) = -\sin \omega t$$

$$\cos\left(\omega t \pm 180^\circ\right) = -\cos \omega t$$

$$\sin\left(\omega t \pm 90^\circ\right) = \pm\cos \omega t$$

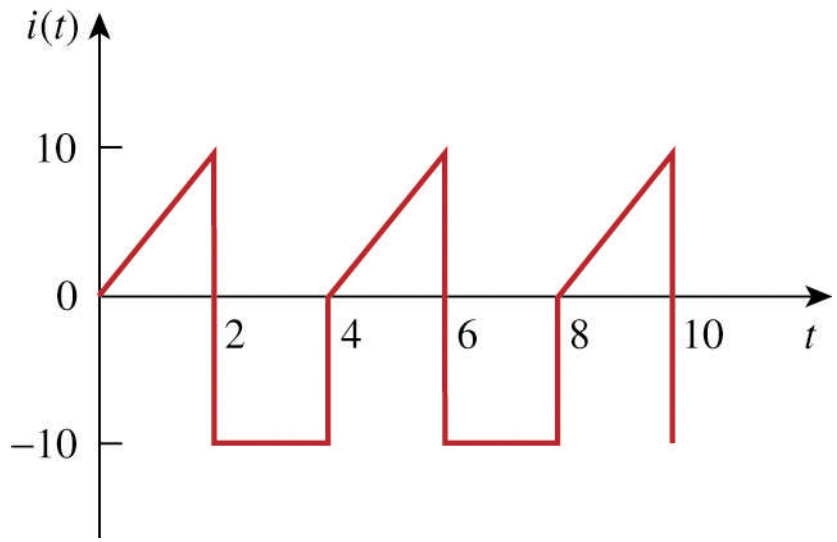
$$\cos\left(\omega t \pm 90^\circ\right) = \mp\sin \omega t$$

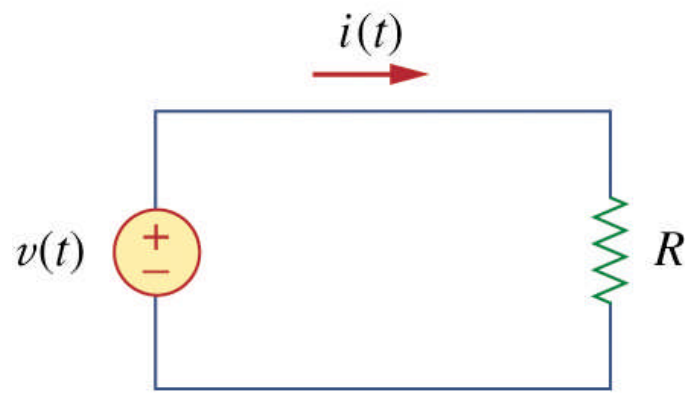
Example Problem 1

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

Effective value

- Consider the following periodic current waveform.

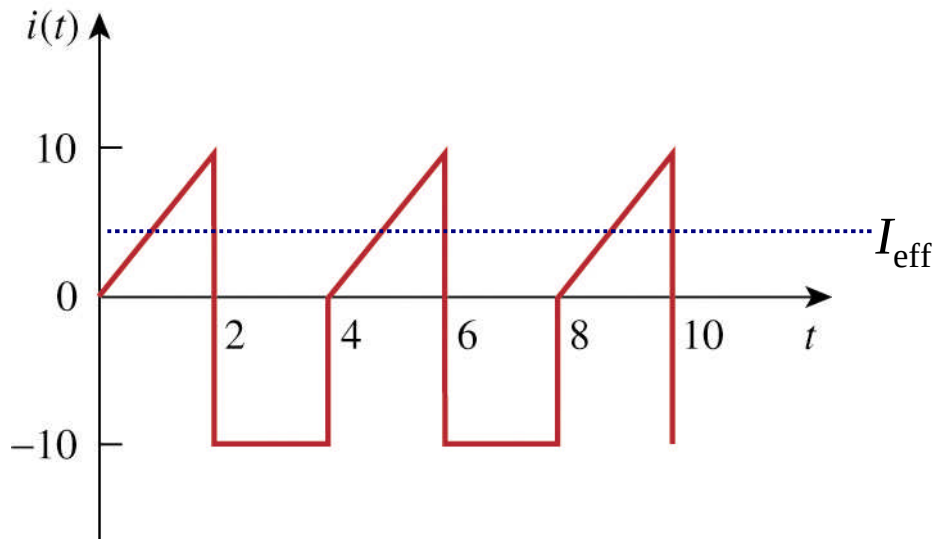


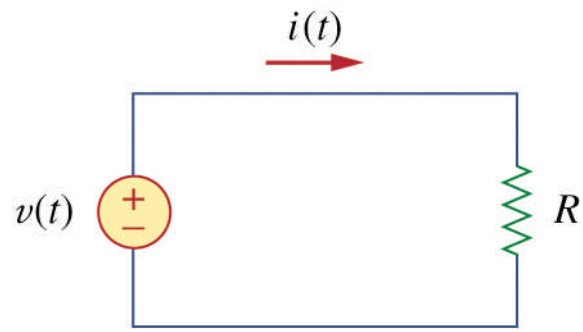


- How would we quantify its effectiveness in delivering power to a resistive load?

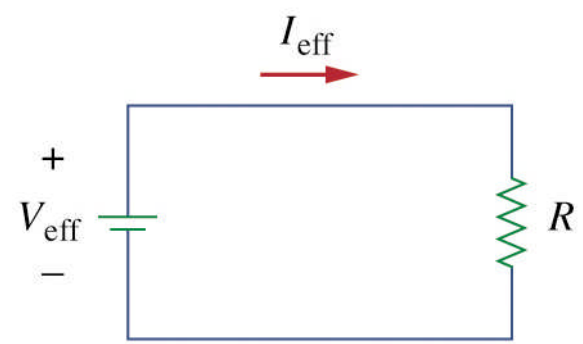
Effective value

- One way would be to determine a dc equivalent that provides the same average power.





original circuit

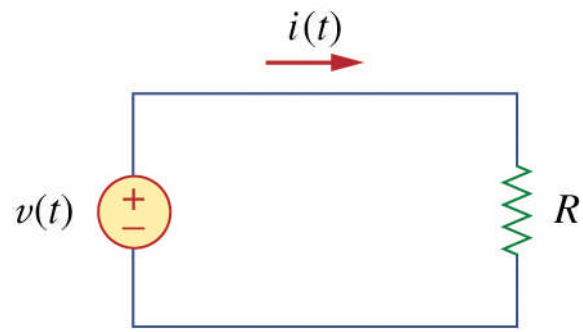


dc equivalent

Effective value

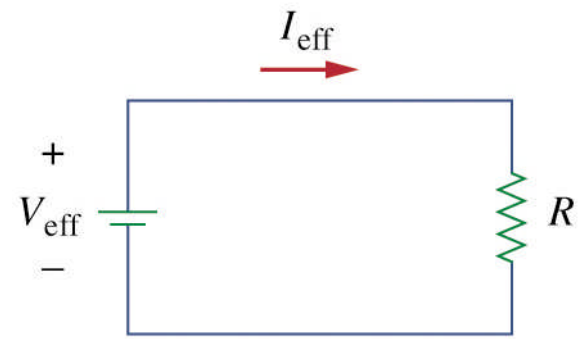
- One way would be to determine a dc equivalent that provides the same average power.

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt$$



original circuit

$$P = I_{\text{eff}}^2 R$$



dc equivalent

Root mean square (rms) value

- Equating the expressions and solving for I_{eff}, we obtain

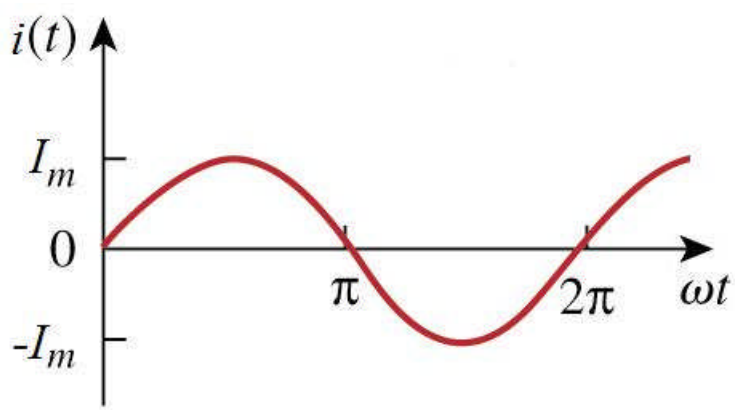
$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

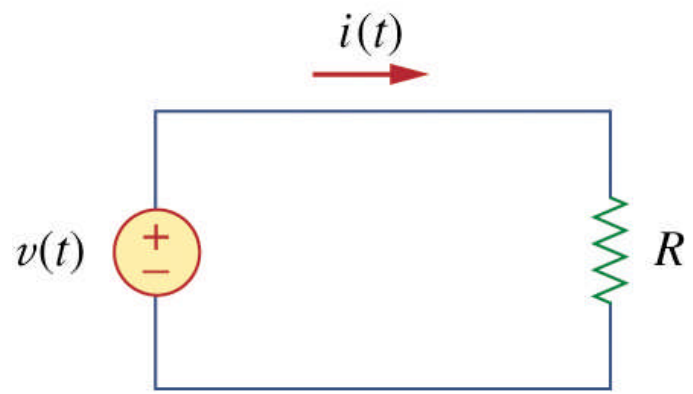
root **mean** **square**

The **effective value** or **root mean square (rms) value** of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

RMS value of a sinusoid

- What is the rms value of the sinusoidal current $i(t) = I_m \sin \omega t$?





RMS value of a sinusoid

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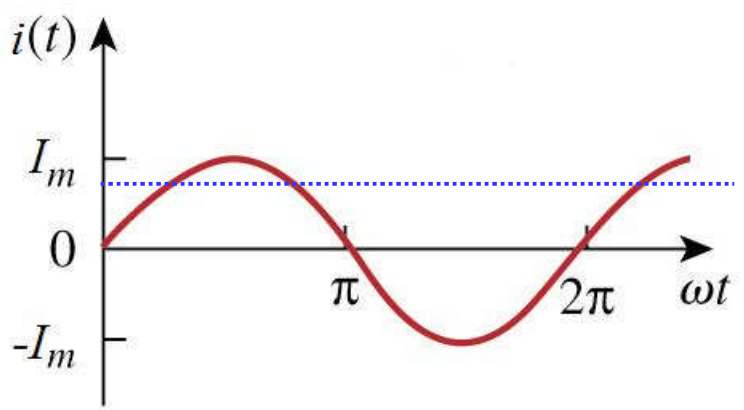


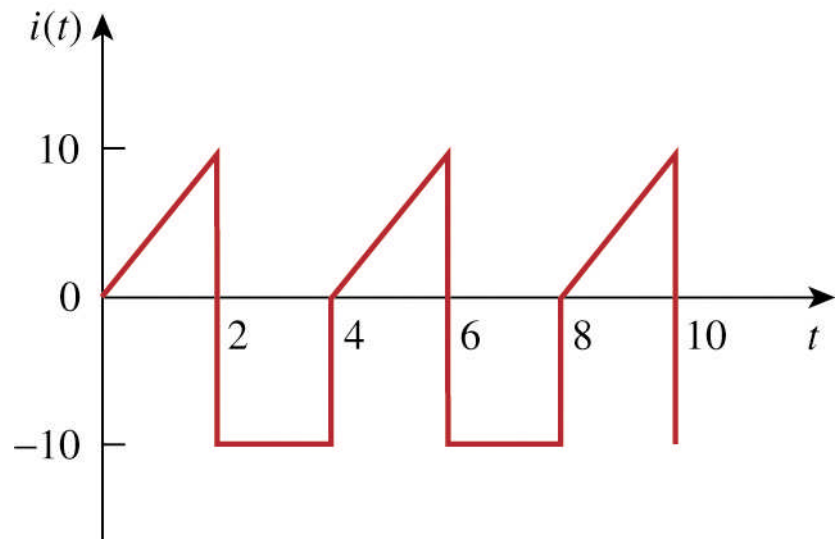
$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

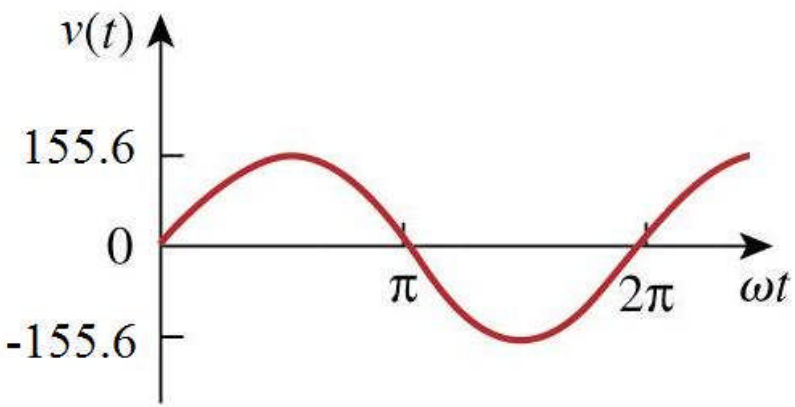
$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1-\cos 2\omega t)\,dt} = \frac{I_m}{\sqrt{2}}$$

Example Problem 2

The waveform below is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 2-Ω resistor.



Example Problem 3

A standard AC voltage wall outlet has a voltage given by $v(t) = 155.6 \cos 2\pi 60t$ V. What is the rms value?