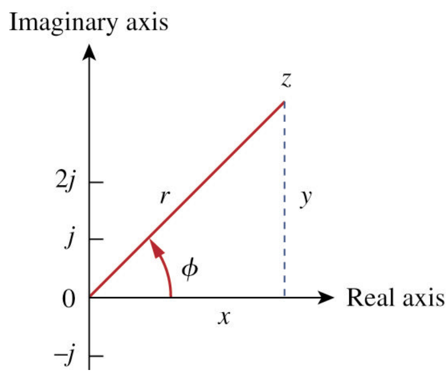
Lesson 19: Phasors and impedance

Complex numbers
A complex number z is a number of the form $z =$

where x, y are real numbers and $j =$

- x is the ___________ part of z, Re (z) =
- y is the ________________ part of z. Im (z) =

The complex number z can be represented geometrically by plotting it in the complex plane.
- The x axis is the real axis.
- The y axis is the imaginary axis.



The complex number z can be represented three ways:

rectangular form

polar form

exponential form

$r =$ $\phi =$

$x =$ $y =$

Complex numbers operations
Given two complex numbers z_1, z_2 defined $z_1 = x_1 + jy_1 = r_1\angle\phi_1$ and $z_2 = x_2 + jy_2 = r_2\angle\phi_2$

Addition: $z_1 + z_2 =$

Subtraction: $z_1 - z_2 =$

Multiplication: $z_1 \cdot z_2 =$

Division: $z_1 / z_2 =$

Reciprocal: $\frac{z_1}{z_2} =$

Square root: $\sqrt{z} =$

Complex conjugate: $z^* =$

Powers of j

Euler's identity

Euler's identity $\qquad e^{\pm j\phi} =$

shows that we may regard cos ϕ and sin ϕ as the real and imaginary parts of $e^{j\phi}$

Representation of sinusoids

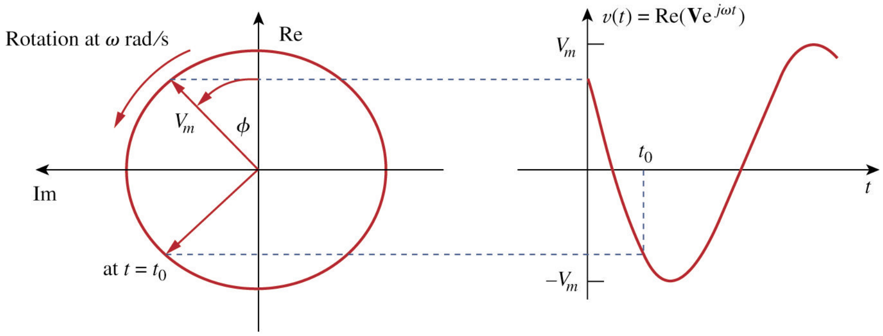
Using Euler's identity, we can represent the sinusoid $v(t) =$

Thus, $v(t) = \mathrm{Re}\left(\mathbf{V}e^{j\omega t}\right)$ where $\mathbf{V} =$

$\mathbf{V}$ is the phasor representation of the sinusoid $v(t)$.

- A phasor is a complex representation of the ____________ and ____________ of a sinusoid.

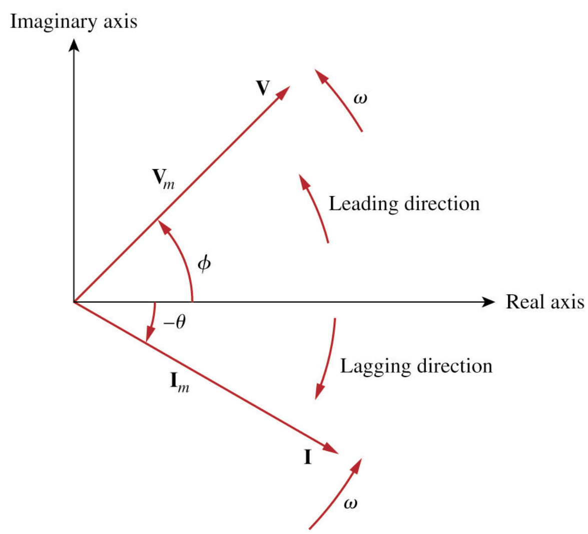
Representation of $\mathrm{V}e^{j\omega t}$



(a) (b)

Phasors

Phasors are complex quantities and may be expressed in rectangular, polar or exponential form.
Phasors have ______________ and ______________and behave as vectors (printed in boldface).



Consider V = $V_m\angle\phi$ and I = $I_m\angle-\theta$.

Time-phasor points to remember

1. $v(t)$ is the instantaneous or time domain representation, while $\mathbf{V}$ is the frequency or phasor domain representation.
2. $v(t)$ is the time dependent, $\mathbf{V}$ is not.
3. $v(t)$ is always real with no complex term, while $\mathbf{V}$ is generally complex.
4. Phasor analysis only applies when frequency is constant.

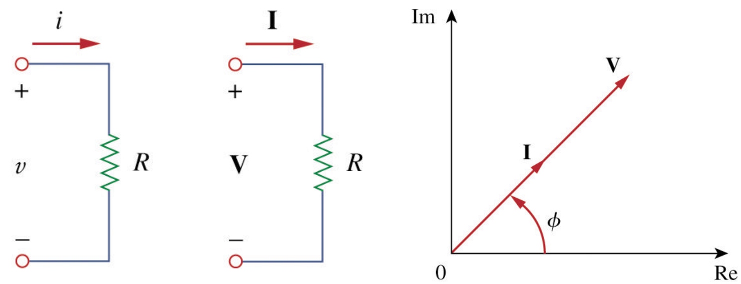
Phasor relationship for resistors

We want to develop voltage-current relationships for circuit elements in the phasor domain.

The voltage-current relation in phasor domain is unchanged from the time domain.

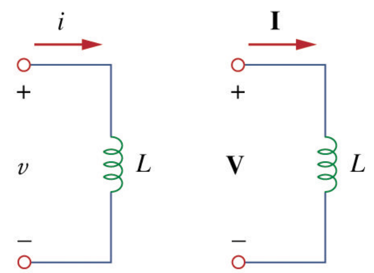
V =

Voltage and current are ____________________.



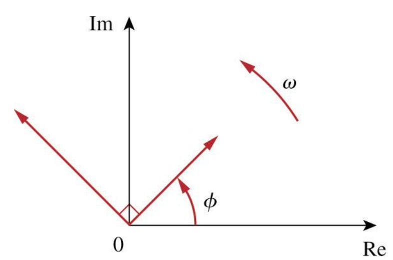
Phasor relationship for inductors

Consider an inductor in which $i = I_m \cos(\omega t + \phi)$



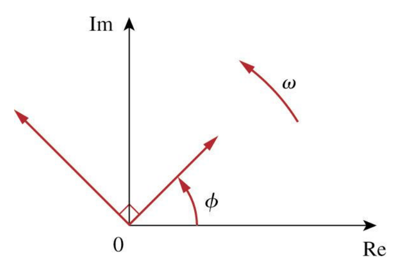
The voltage-current relation in phasor domain is **V** =

Current __________ voltage by 90°.



Phasor relationship for capacitors

Consider a capacitor in which $v = V_m \cos(\omega t + \phi)$



The voltage-current relation in phasor domain is

I = $\Rightarrow$ **V** =

Current ___________ voltage by 90°.

Summary of phasor relations

TABLE 9.2 Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Impedance

Previously we obtained the voltage-current relations $\mathbf{V} = R\mathbf{I}, \quad \mathbf{V} = j\omega L\mathbf{I}, \quad \mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$

These could be written in terms of the ratio of phasor voltage to phasor current as

$$\frac{\mathbf{V}}{\mathbf{I}} = \qquad , \quad \frac{\mathbf{V}}{\mathbf{I}} = \qquad , \quad \frac{\mathbf{V}}{\mathbf{I}} =$$

From these equations we obtain Ohm's law in phasor form for any element as

$$\mathbf{Z} = \qquad \text{or} \quad \mathbf{V} =$$

where **Z** is a frequency-dependent quantity known as impedance, measure in __________.

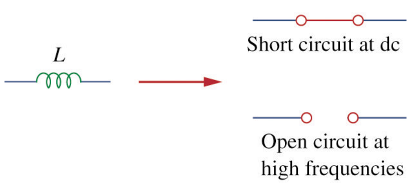
Frequency dependency

The impedance of capacitors and inductors is dependent on frequency.

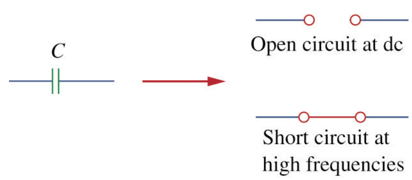
Consider $\omega = 0$ (dc sources) $\qquad Z_L = j\omega L = \qquad \Omega$ and $Z_C = \frac{1}{j\omega C} \rightarrow \qquad \Omega$

Consider $\omega \rightarrow \infty$ (high frequency) $\qquad Z_L = j\omega L \rightarrow \qquad \Omega$ and $Z_C = \frac{1}{j\omega C} = \qquad \Omega$

Inductors



Capacitors



Impedance (Z)

Impedance is a complex quantity and may be expressed in rectangular form $\quad \mathbf{Z} =$

where R is the resistance and X is called ______________________.

Thus,

- $Z = R + jX$ is said to be ______________________.
- $Z = R - jX$ is said to be ______________________.

Impedance may also be expressed in polar form $\quad \mathbf{Z} =$

Since $\qquad \mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta$

Admittance (Y)

Admittance is the reciprocal of impedance $\mathbf{Y} =$

which can also be expressed $\mathbf{Y} =$

where G is the conductance and B is called the susceptance.

Example Problem 1

Evaluate these complex numbers.

$$\left(40\angle 50° + 20\angle -30°\right)^{1/2}$$

$$\frac{\left(10\angle -30° + \left(3 - j4\right)\right)}{(2 + j4)(3 - j5)^*}$$

Example Problem 2

Transform these sinusoids to phasors.

$$i = 6\cos\left(50t - 40°\right) \text{ A}$$

$$v = -4\sin\left(30t + 50°\right) \text{ V}$$

Example Problem 3

Find the sinusoids represented by these phasors.

$$\mathbf{I} = -3 + j4 \text{ A}$$

$$\mathbf{V} = j8e^{-j20°} \text{ V}$$

Example Problem 4

The voltage $v = 12 \cos(60t + 45°)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Example Problem 5

Find $v(t)$ and $i(t)$ in the circuit below.