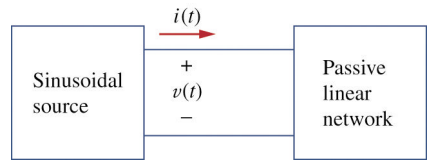
Lesson 22: Single-Phase AC Power

Instantaneous power

Consider the element below with voltage $v(t)$ and current $i(t)$.



$$v(t) = V_m \cos(\omega t + \theta_v), \quad i(t) = I_m \cos(\omega t + \theta_i)$$

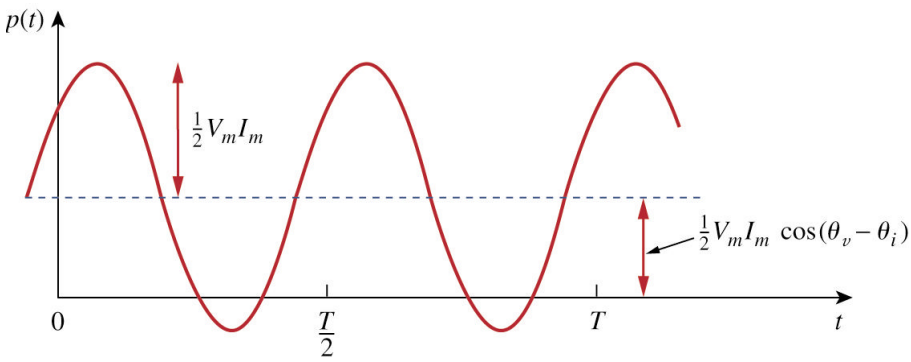
What is the power delivered to this element at time t? $p(t) =$

Instantaneous power

The instantaneous power is the power delivered at any instant in time.

Using the identity, $\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

we have $p(t) =$



Positive/negative power

- Positive power is being _____.
- Negative power is being _____.

Average power

Average power is the average of the instantaneous power over _____.

Average power is given $P =$

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, only the _____ matters.

Average power using phasors

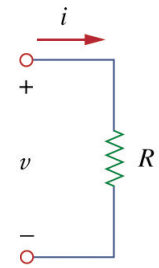
$v(t)$ and $i(t)$ can be expressed as phasors $\mathbf{V} = V_m \angle \theta_v$ and $\mathbf{I} = I_m \angle \theta_i$

notice that $\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i) = \frac{1}{2} V_m I_m \left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$

Thus, average power P is given by $P =$

Purely resistive circuits

When a load is purely resistive, the voltage and current are _____.

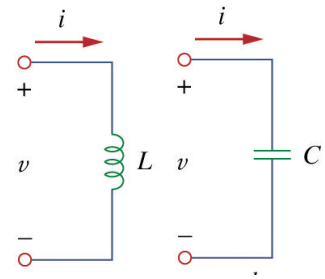


Since $\theta_v = \theta_i$, average power P is given by $P =$

A resistive load _____.

Purely reactive circuits

When a load is purely reactive, the voltage and current are _____.



Since $\theta_v - \theta_i = \pm 90^o$, average power P is given by $P =$

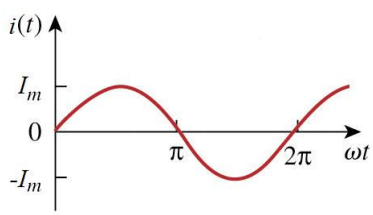
A reactive load absorbs _____.

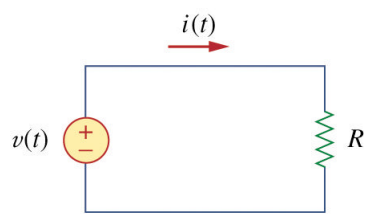
Root mean square (rms) value

The effective value or root mean square (rms) value of a periodic current is the dc current that delivers the _____ to a resistor as the periodic current.

RMS value of a sinusoid

What is the rms value of the sinusoidal current $i(t) = I_m \cos \omega t$?



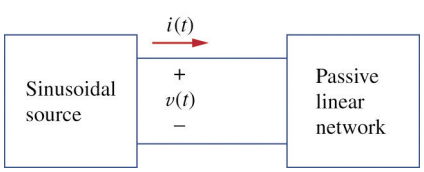


$I_{\text{rms}} =$

Average power with RMS values

Consider the element below with $v(t)$ and $i(t)$ given $\quad v(t) = V_m \cos(\omega t + \theta_v), \quad i(t) = I_m \cos(\omega t + \theta_i)$

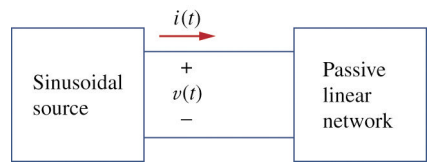
Average power delivered to the load is $P =$



Apparent power

Previously, we determined that the average power delivered to a load with voltage $v(t)$ and current $i(t)$ is given

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$



The apparent power S is the power that "_____" to flow to the load.
It is measured in _____(____), not watts

Power factor

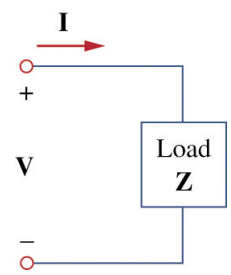
The average power P is the product of two terms

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

- The apparent power S
- A term called the _____ given by $\cos(\theta_v - \theta_i)$

Power factor is the ratio of _____ power to the _____ power. pf =

Power factor angle



The angle $\theta_v - \theta_i$ is called the power factor angle.
How does the power factor angle relate to the load impedance **Z**?

$$\mathbf{Z} = \frac{\mathbf{V}_m}{\mathbf{I}_m} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle(\theta_v - \theta_i)$$

The power factor angle is equal to the angle of the _____.

For purely resistive loads $\theta_v - \theta_i = 0^\circ$ and pf = 1.

- This implies that average power and apparent power are _____.

For purely reactive loads $\theta_v - \theta_i = \pm 90^\circ$ and pf = 0.

- This implies that average power is _____.

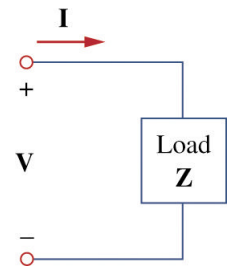
If $\theta_v - \theta_i > 0^\circ$ power factor is said to be lagging because current lags voltage.

- Lagging power factor implies an _____ load.

If $\theta_v - \theta_i < 0^\circ$ power factor is said to be leading because current leads voltage.

- Leading power factor implies a _____ load.

ELI	**ICE**
voltage (**E**) leads current (**I**) in an inductor (**L**)	current (**I**) leads voltage(**E**) in a capacitor (**C**)



Average power using phasors

If $v(t)$ and $i(t)$ are expressed as phasors $\mathbf{V} = V_m \angle \theta_v$ and $\mathbf{I} = I_m \angle \theta_i$

notice that average power P is given by

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \mathrm{Re}\left[V_m \angle \theta_v \cdot I_m \angle -\theta_v\right] =$$

If we convert to rms values,

$$P = \frac{1}{2} \mathrm{Re}\left[\mathbf{V}\mathbf{I}^*\right] = \frac{1}{2} \mathrm{Re}\left[V_m \angle \theta_v \cdot I_m \angle -\theta_v\right] = \frac{1}{2} \mathrm{Re}\left[\sqrt{2} V_{\mathrm{rms}} \angle \theta_v \cdot \sqrt{2} I_{\mathrm{rms}} \angle -\theta_v\right] =$$

Complex power

Notice that the average power is just the _____ portion of a quantity called _____ power.

$$P = \mathrm{Re}[\mathbf{V}_{\mathrm{rms}} \mathbf{I}^*_{\mathrm{rms}}]$$

$$\mathbf{S} =$$

Complex power can also be expressed in terms of the load impedance **Z**.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\mathrm{rms}}}{\mathbf{I}_{\mathrm{rms}}} = \frac{V_{\mathrm{rms}}}{I_{\mathrm{rms}}} \angle (\theta_v - \theta_i)$$

Since $\mathbf{V}_{\mathrm{rms}} = \mathbf{Z}\mathbf{I}_{\mathrm{rms}}$, $\mathbf{S}$ can be expressed $\mathbf{S} =$

Real and reactive power

We can divide the load impedance **Z** into its real and imaginary components.

$$\mathbf{S} = I^2_{\mathrm{rms}} \mathbf{Z} =$$

where P and Q are the real and imaginary components of the complex power.

$P =$ $Q =$

Reactive power

Real power P in watts is the average power delivered to a load.

- Is the only _____ power.

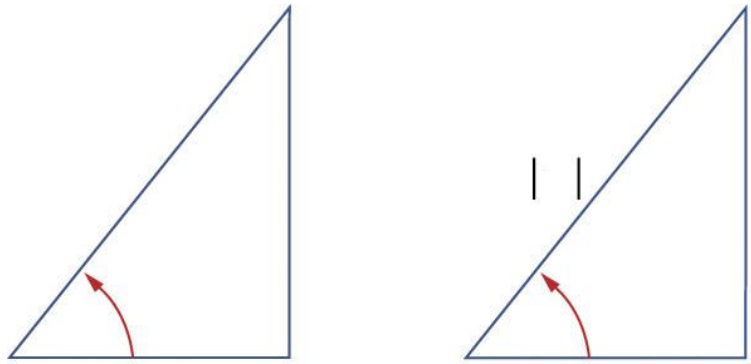
Reactive power Q is a measure of energy exchanged between the source and the reactive part of the load.

- The unit of Q is the volt_____ ( ).

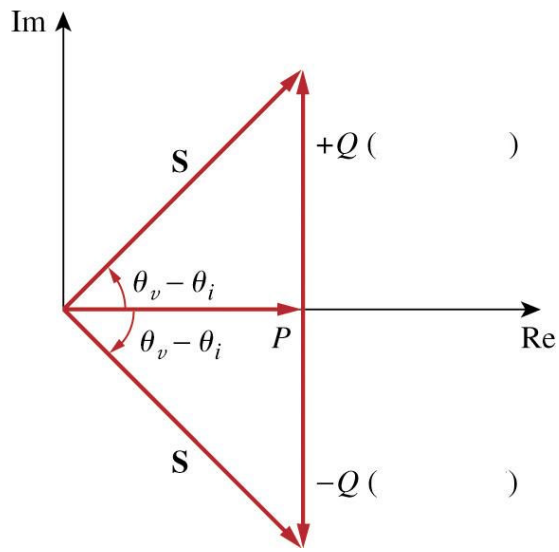
Notice

- Q = 0 for resistive loads (_____ pf).
- Q < 0 for capacitive loads (_____ pf).
- Q > 0 for inductive loads (_____ pf).

Power triangle
All of the relations are conveniently expressed by the power triangle.

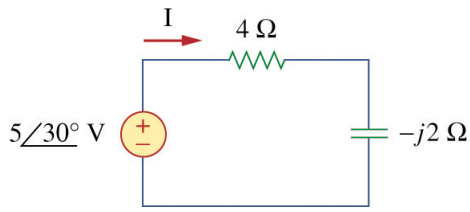


Leading/lagging in power triangle
The power triangle's orientation depends on the sign of Q.



Example Problem 1

Find the average power supplied by the source and the average power absorbed by the resistor.

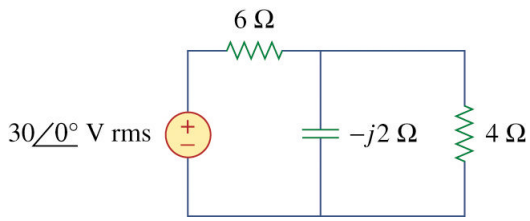


Example Problem 2

A series connected load draws a current $i = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v = 120\cos(100\pi t - 20°)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Example Problem 3

Determine the power factor of the entire circuit as seen by the source. Calculate the average power delivered by the source.



Example Problem 4

The voltage across a load is $v = 60\cos(\omega t - 10°)$ V and the current through the element is $i = 1.5\cos(\omega t + 50°)$ A.

Find:

a. The complex and apparent powers
b. The real and reactive powers
c. Power factor and load impedance.