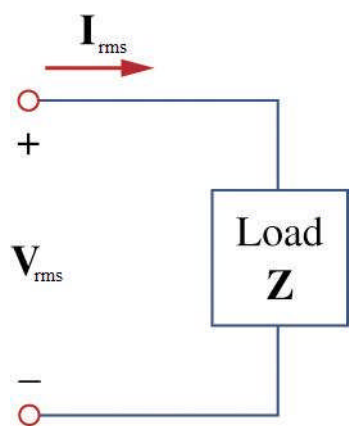
Lesson 23:
AC Power

AC power

- Previously we addressed the question of power delivered to a load impedance **Z**.



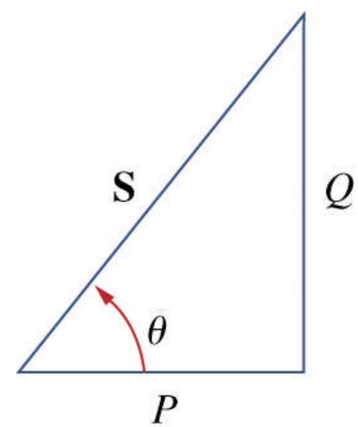
- Given the phasors $\mathbf{V}_{\text{rms}}$ and $\mathbf{I}_{\text{rms}}$ we found the complex power **S** is given

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^{*}_{\text{rms}} = S\angle\theta$$

Complex power (rectangular form)

- As its name implies, $\mathbf{S}$ is complex quantity and its units are volt-amperes (VA).
- Complex power in rectangular form reveals its real (P) and imaginary (Q) components.
  - The real part is the average power (P) in watts.
  - The imaginary part is the reactive power (Q) in VAR.

$$\mathbf{S} = P + jQ$$

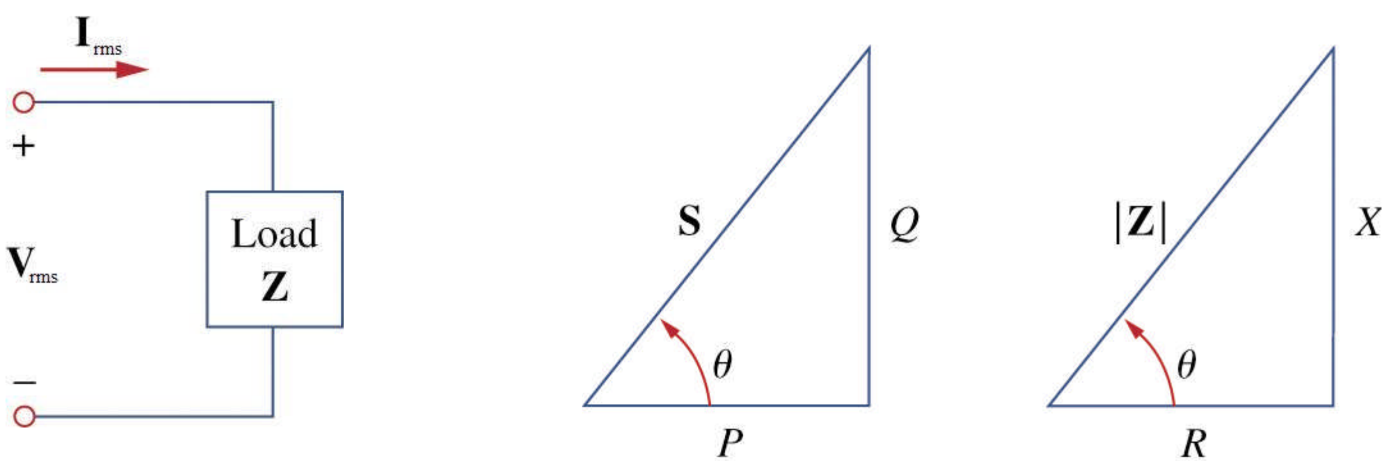


Complex power (polar form)

- Complex power in polar form is comprised of

$$\mathbf{S} = S\angle\theta$$

  - Apparent power (S) in volt-amperes. $S = |\mathbf{S}|$
  - Power factor angle (θ) which is the same angle as the load impedance **Z**.



Power factor angle

- If $\theta_v - \theta_i > 0^o$ power factor is said to be **lagging** because current lags voltage.
  - Lagging power factor implies an inductive load.
- If $\theta_v - \theta_i < 0^o$ power factor is said to be **leading** because current leads voltage.
  - Leading power factor implies a capacitive load.

ICE

Current (**I**) leads voltage(**E**) in a capacitor (**C**)

ELI

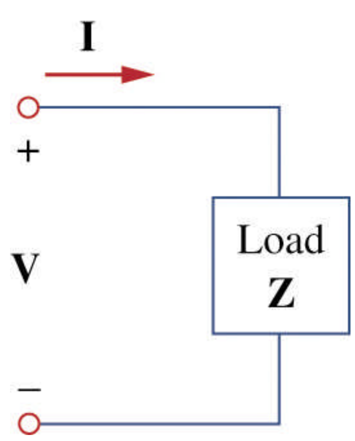
Voltage (**E**) leads current (**I**) in an inductor (**L**)

Complex power

- Complex power can also be expressed in terms of the load impedance **Z**.

$$\mathbf{Z} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}\angle\theta_v}{I_{\text{rms}}\angle\theta_i}$$

Since $\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$, $\mathbf{S}$ can be expressed



$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = (\mathbf{Z}\mathbf{I}_{\text{rms}})\mathbf{I}^*_{\text{rms}} = I^2_{\text{rms}}\mathbf{Z}$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\left(\frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}}\right)^* = \frac{V^2_{\text{rms}}}{\mathbf{Z}^*}$$

Real and reactive power

- Dividing the load impedance $\mathbf{Z}$ into its real and imaginary components

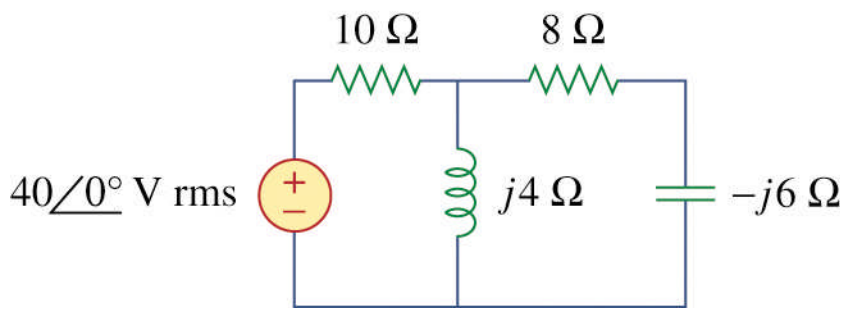
$$
\begin{aligned}
\mathbf{S} &= I_{\text{rms}}^2 \mathbf{Z} \\
&= I_{\text{rms}}^2 \left(R + jX \right) \\
&= I_{\text{rms}}^2 R + j I_{\text{rms}}^2 X \\
&= P + jQ
\end{aligned}
$$

we observe that P and Q can be expressed

$$P = \mathrm{Re}(\mathbf{S}) = I_{\text{rms}}^2 R, \quad Q = \mathrm{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

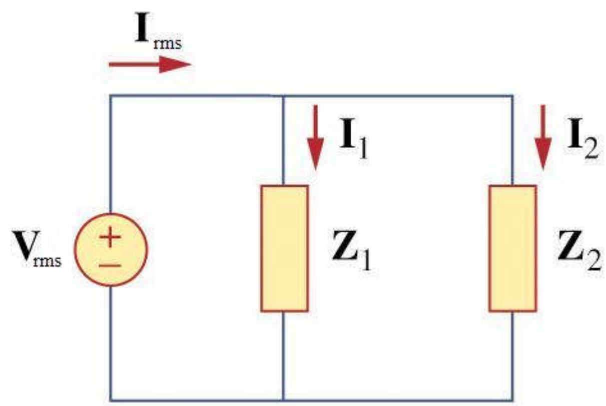
Example Problem 1

Determine the complex power delivered by the source. What is the average power, reactive power and power factor of the load?



Conservation of AC power

- Consider the circuit below (all phasor are in RMS)



- Determine the complex power **S** supplied by the source and delivered to each load.

Conservation of AC power

- The total complex power $\mathbf{S}$ supplied by the source equals the total complex power delivered to the load.

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \mathbf{V}_{\text{rms}}(\mathbf{I}^*_1 + \mathbf{I}^*_2) = \mathbf{V}_{\text{rms}}\mathbf{I}^*_1 + \mathbf{V}_{\text{rms}}\mathbf{I}^*_2 = \mathbf{S}_1 + \mathbf{S}_2$$

- This can be extended to N loads.

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N$$

Conservation of AC power

- We also notice that this is true of average power (P) and reactive power (Q).

$$\begin{aligned}\mathbf{S}_{\text{total}} &= \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N \\ P_{\text{total}} + jQ_{\text{total}} &= \left(P_1 + jQ_1\right) + \left(P_2 + jQ_2\right) + \cdots + \left(P_N + jQ_N\right) \\ &= \left(P_1 + P_2 + \cdots + P_N\right) + j\left(Q_1 + Q_2 + \cdots + Q_N\right)\end{aligned}$$

Thus

$$\begin{aligned}P_{\text{total}} &= P_1 + P_2 + \cdots + P_N \\ Q_{\text{total}} &= Q_1 + Q_2 + \cdots + Q_N\end{aligned}$$

- However, this is **not** true for apparent power (S)

$$S_{\text{total}} \neq S_1 + S_2 + \cdots + S_N$$

Example Problem 2

The figure below depicts a load being fed by a voltage source through a transmission line with impedance $4 + j2\ \Omega$. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.