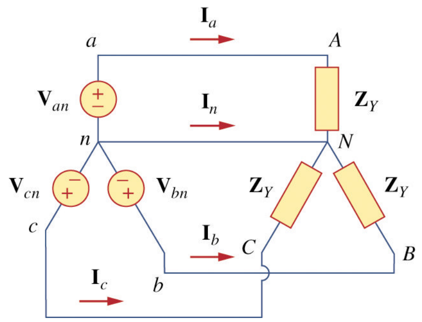
Lesson 28: 3-Phase Circuit Analysis 2

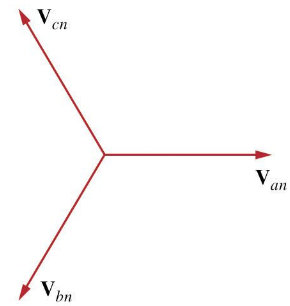
Line voltages (Y-Y)

$$\mathbf{V}_{ab} = \mathbf{V}_{ab} - \mathbf{V}_{bn} =$$

$$\mathbf{V}_{bc} = \sqrt{3}V_P\angle -90°$$

$$\mathbf{V}_{ca} = \sqrt{3}V_P\angle -210°$$

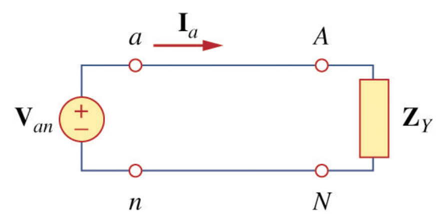




Line voltage are ______ times larger and lead their respective phase voltages by _____.

Line currents (Y-Y) $\mathbf{I}_a =$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle -240°$

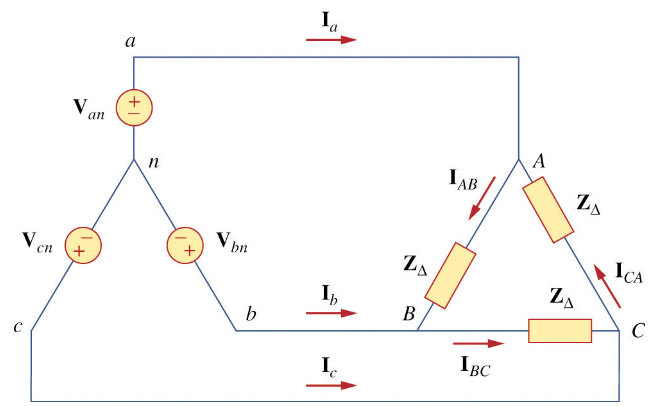
Since $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$, if follows that $\mathbf{I}_n =$ ______ .



Single-phase equivalent (Y-Y)
In analyzing a balanced Y-Y system, it is sufficient to examine just one of the phases and apply the results to the other two phases (with appropriate phase delays).

Balanced Y-Δ connection
A balanced Y-Δ system consists of a Y-connected source feeding a Δ-connected load.

- There is no ________________ connection.



Line voltages (Y-Δ)
For the Y-Δ system, the line voltages are still given

$$\mathbf{V}_{ab} = \qquad\qquad = \sqrt{3}V_P\angle 30° = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = \sqrt{3}V_P\angle -90° = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_P\angle -210° = \mathbf{V}_{CA}$$

Line currents (Y-Δ)
The currents through each phase of the Δ-load ($\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, $\mathbf{I}_{CA}$) are ____________ the same as the line currents ($\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$)..

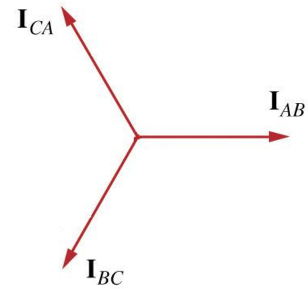
$$\mathbf{I}_{AB} = \frac{\quad}{\quad} \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

How do we obtain the line currents ($\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$). in terms of the phase currents ($\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, $\mathbf{I}_{CA}$) ?
We write a ______ expressions at *A*, *B* and *C*.

$$\mathbf{I}_a =$$

Line currents are _____ times larger and _______ their respective phase currents by 30°.

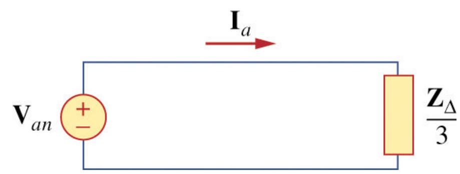
$$\mathbf{I}_a = \sqrt{3}\mathbf{I}_{AB}\angle -30°$$
$$\mathbf{I}_b = \sqrt{3}\mathbf{I}_{AB}\angle -150°$$
$$\mathbf{I}_c = \sqrt{3}\mathbf{I}_{AB}\angle -270°$$



Single-phase equivalent
In analyzing a balanced Y-Δ, we can transform the load to form an equivalent Y-Y system.

$\mathbf{Z}_Y =$

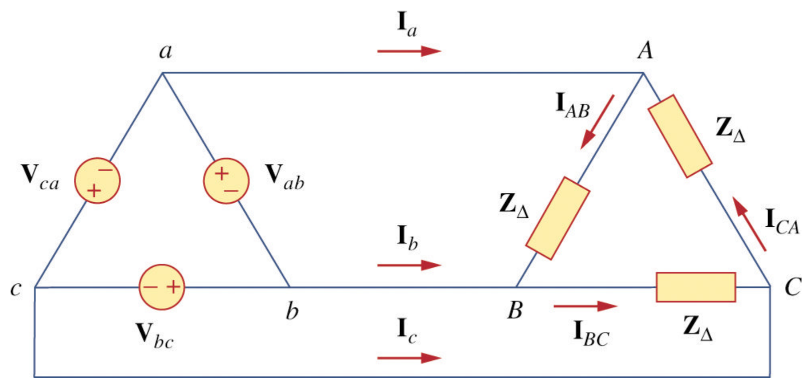
Then a single phase can be examined to determine the phase current.



Balanced Δ-Δ connection
In a balanced Δ-Δ system, both source and load are Δ-connected.

Line voltages (Δ -Δ)
For the Δ-Δ system, the line voltages and phase voltages are the same.

$$\mathbf{V}_{ab} = V_p\angle 0° = \mathbf{V}_{AB}$$
$$\mathbf{V}_{bc} = V_p\angle -120° = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = V_p\angle +120° = \mathbf{V}_{CA}$$



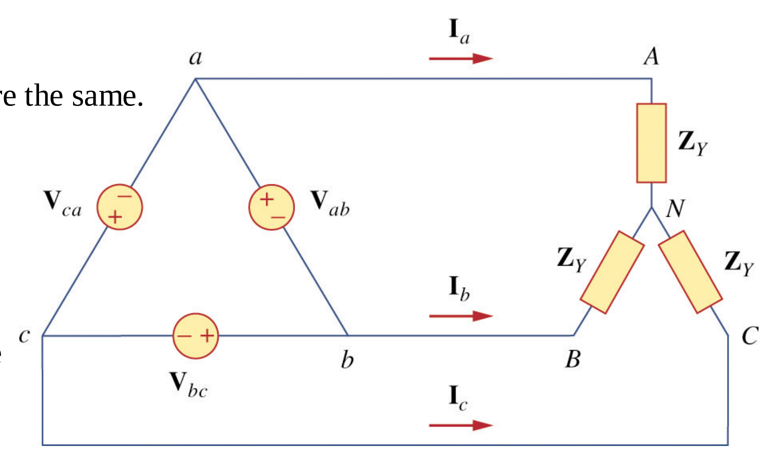
Line currents (Δ -Δ)
The line currents for the Δ-connected load are identical to the previous Y-Δ system.

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} - \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} - \frac{\mathbf{V}_{ab}\angle -240°}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}\left(1 - 1\angle -240°\right) =$$

Balanced Δ-Y connection
A balanced Δ-Y system consists of a Δ-connected source feeding a Y-connected load.

Line voltages (Δ-Y)
The line voltages and source phase voltages are the same.

$$\mathbf{V}_{ab} = V_p\angle 0°$$
$$\mathbf{V}_{bc} = V_p\angle -120°$$
$$\mathbf{V}_{ca} = V_p\angle -240°$$



At the load, the per phase voltage relations are

$$\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{AN}\angle 30° \Rightarrow \mathbf{V}_{AN} =$$

Line currents (Δ-Y)

The phase currents for the Y-connected load are identical to the line currents.

$$I_{AN} = I_a \quad I_{BN} = I_b \quad I_{CN} = I_c$$

We can express these in terms of the source voltages. $\mathbf{I}_a = \mathbf{I}_{AN} = \dfrac{\mathbf{V}_{AN}}{\mathbf{Z}_Y} =$ ________________

Summary of balanced 3-phase systems

TABLE 12.1

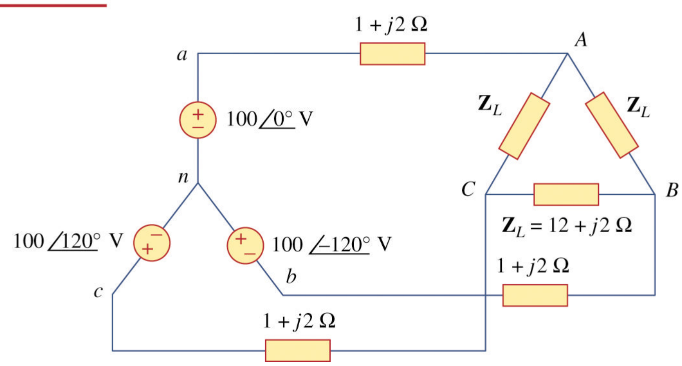
Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30°}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$

Beware!

Consider the following

When _________ impedances (1+ j2 Ω) are present, we must be more judicious in the application of our formulas.



$$\mathbf{I}_{AB} \neq \frac{\mathbf{V}_{ab}}{\mathbf{Z}_L} \qquad \mathbf{V}_{AB} \neq \sqrt{3}\mathbf{V}_{an}\angle 30°$$

Example Problem 1

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 1000\angle\ 10\ ^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

Example Problem 2

A balanced Δ-connected load having an impedance $20+j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $V_{ab} = 330\angle\ 0°$ V. Calculate the phase currents of the load and the line currents.

Example Problem 3

A balanced Y-connected load with a phase impedance of $40+j25\ \Omega$ is supplied by a balanced, positive-sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use V_{ab} as a reference.