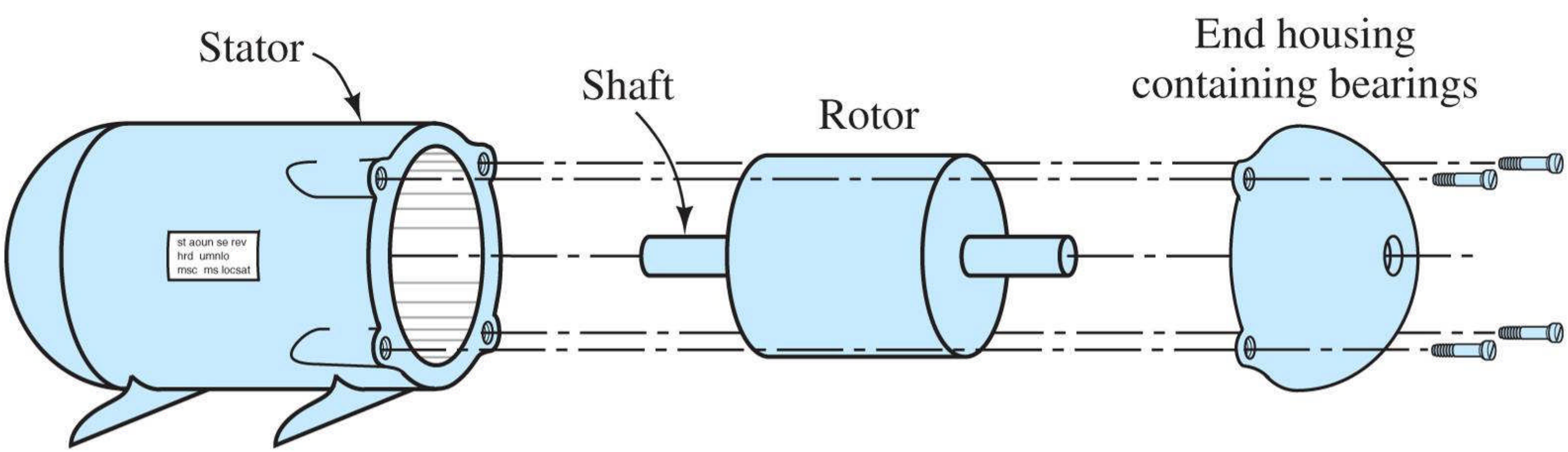
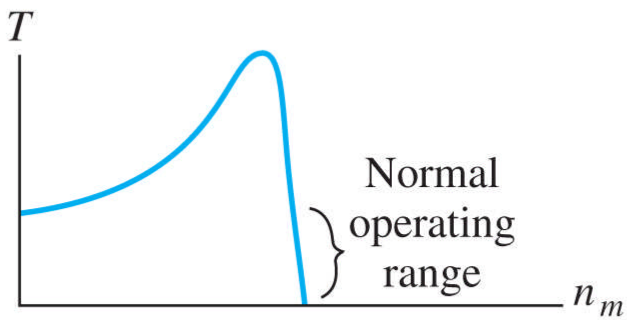
Lessons 30-31: Synchronous Machines

Electric motors

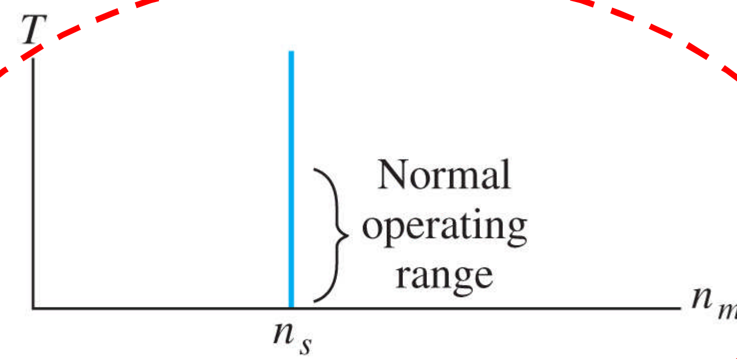
- DC motors
  - High start torque, easy speed control
  - Up to ~200 hp output, commutator wear
- AC motors
  - Most common (> 5 hp), up to ~50,000 hp output
  - Less start torque



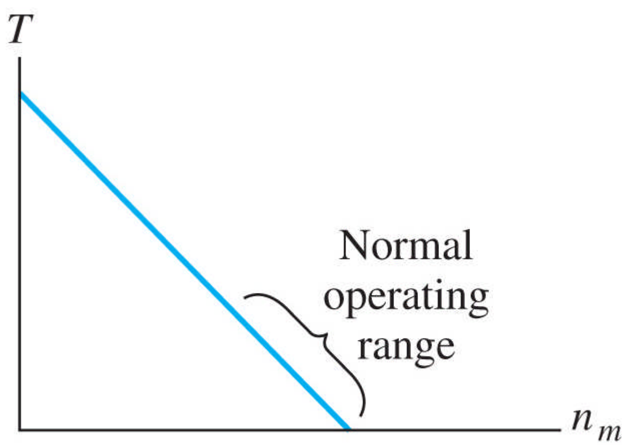
Motor torque-speed characteristics



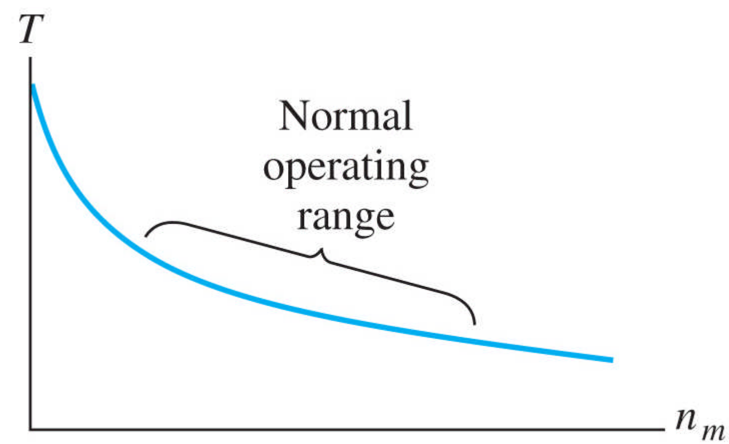
(a) Ac induction motor



(b) Synchronous motor



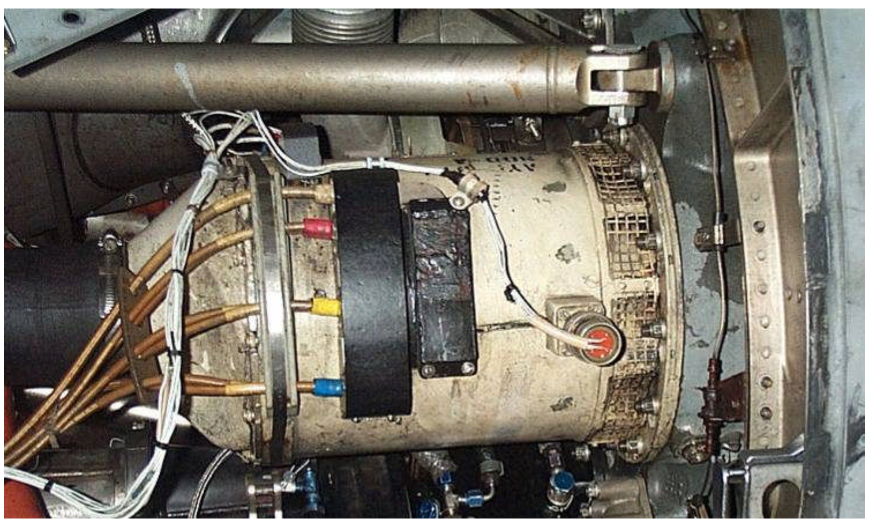
(c) Shunt-connected or permanent-magnet dc motor



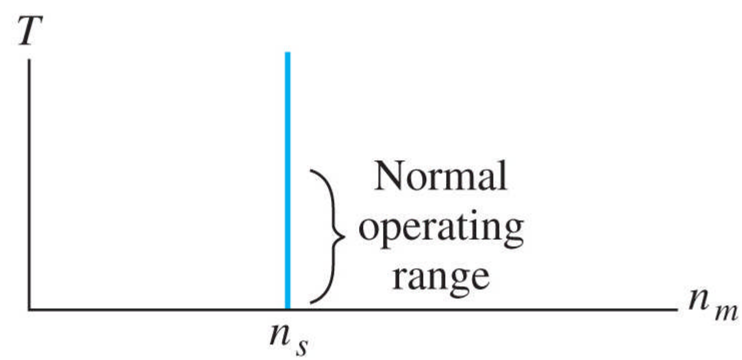
(d) Series-connected dc motor or universal motor

Synchronous AC machines

- Used for nearly all electrical energy generation.
- Used for high-power, low-speed applications as motors.

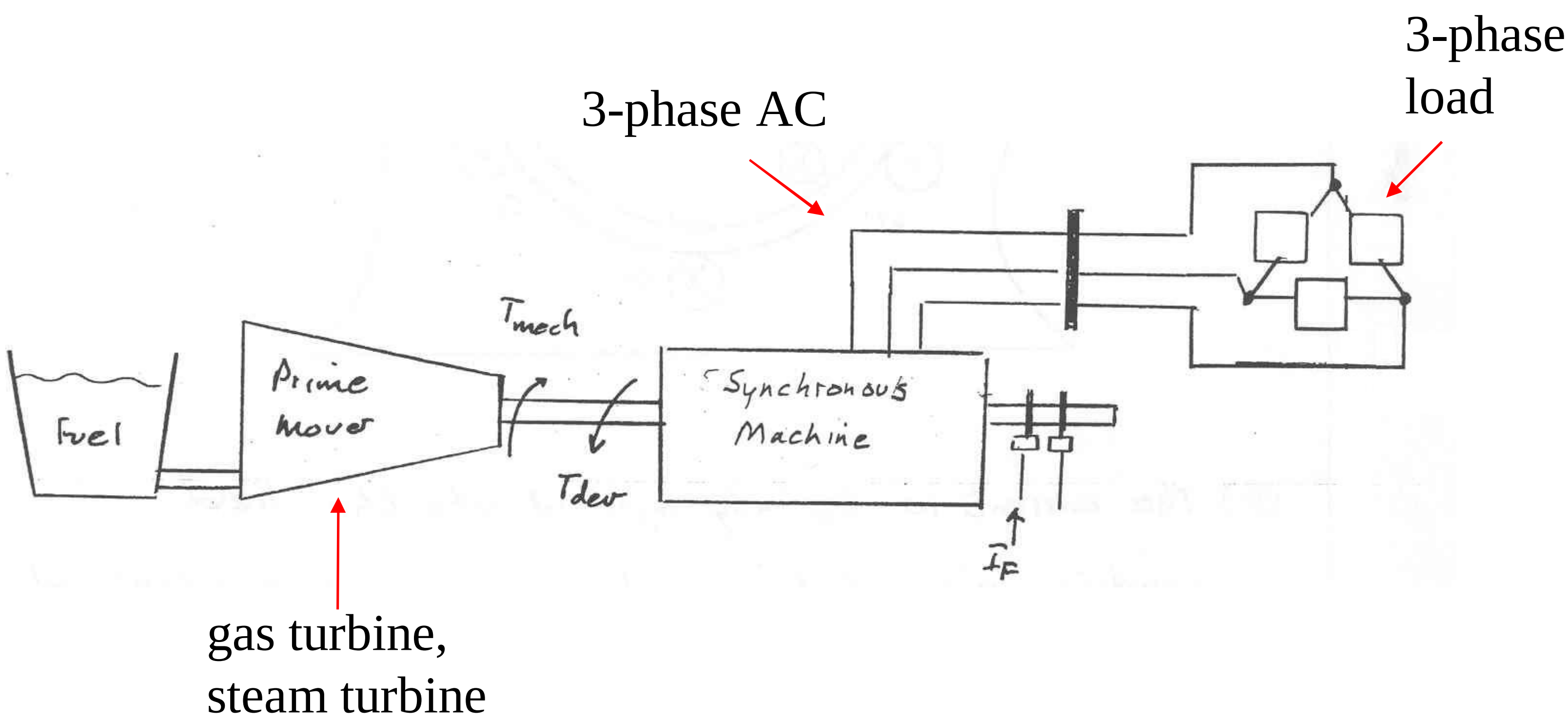


P-3C Orion generator
(120-V, 400 Hz, 90 kVA)



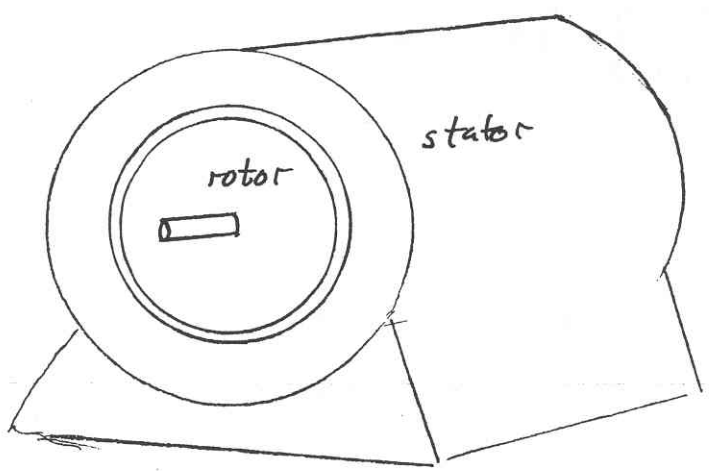
Synchronous AC generator

- The machine can operate as a generator producing AC power if driven by a prime mover (gas, steam, hydroelectric turbine).



Goals

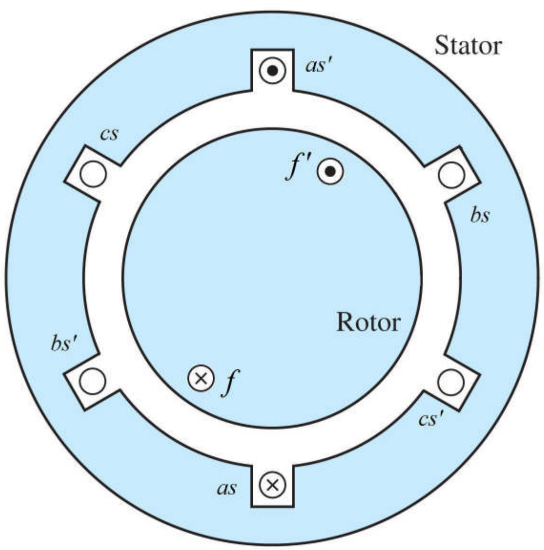
- Understand how a rotating magnetic field is created in the stator
- Understand the power (torque) angle δ
- Use the equivalent circuit model to solve problems.



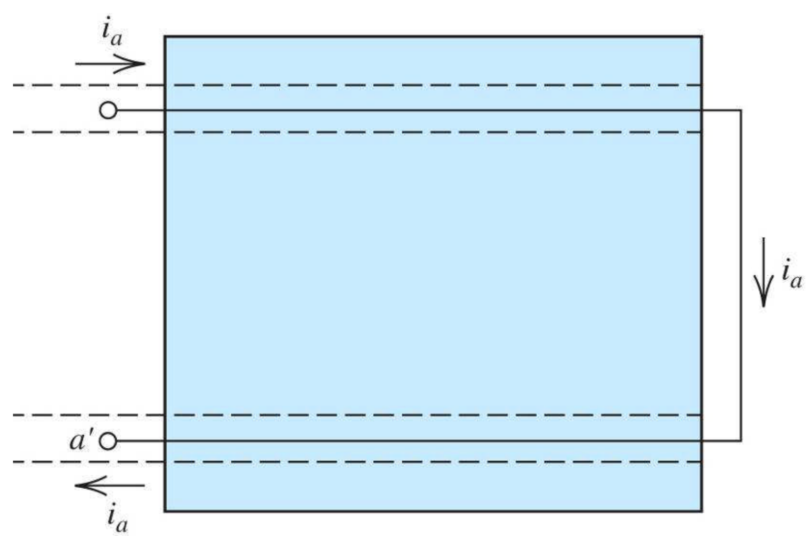
(one of numerous professionally drawn illustrations)

Stator

- The stator (the stationary part) contains set of 3-phase windings to establish the stator field.
  - The number of slots in the stator is an integer multiple of 6. (why?)



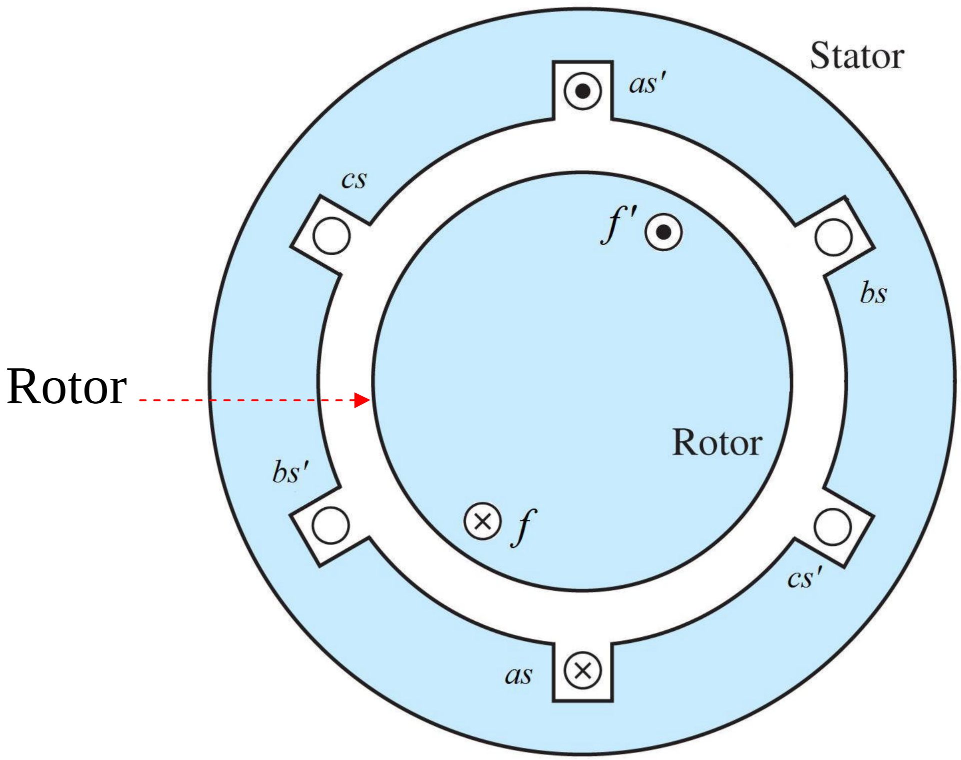
End-view



Lengthwise view of just phase *a*

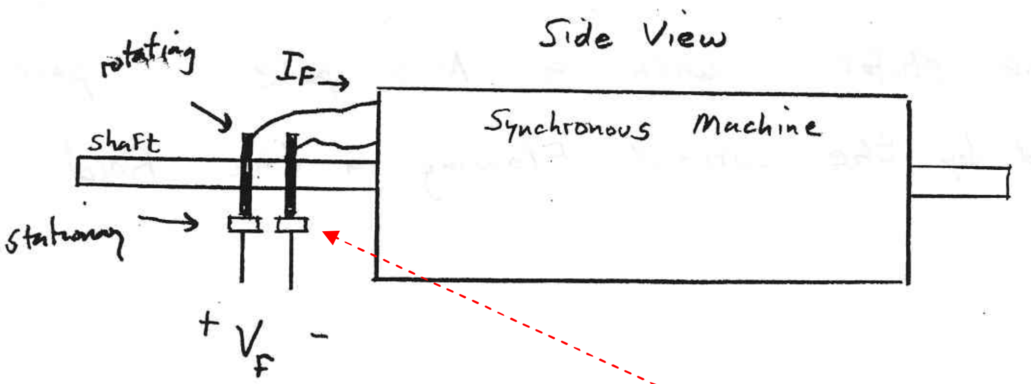
Rotor

- The rotor contains a winding called the field winding.
  - Unlike the stator, it carries dc current.

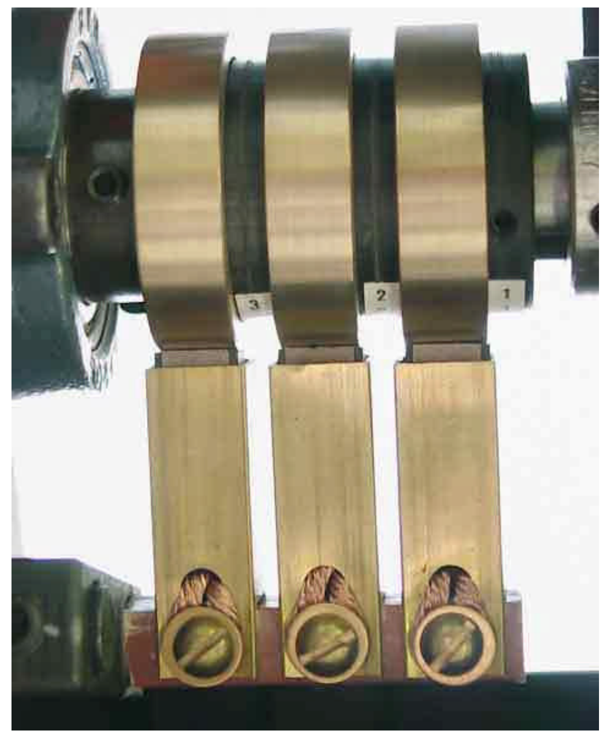


Rotor

- The dc current is often connected to the rotor via slip rings.
  - Because these contacts aren't commutating (changing current direction) wear is much less of an issue than in DC motors.

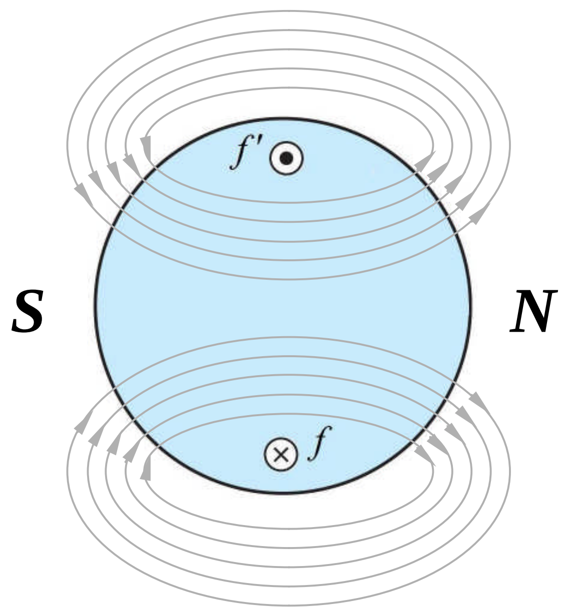


slip rings

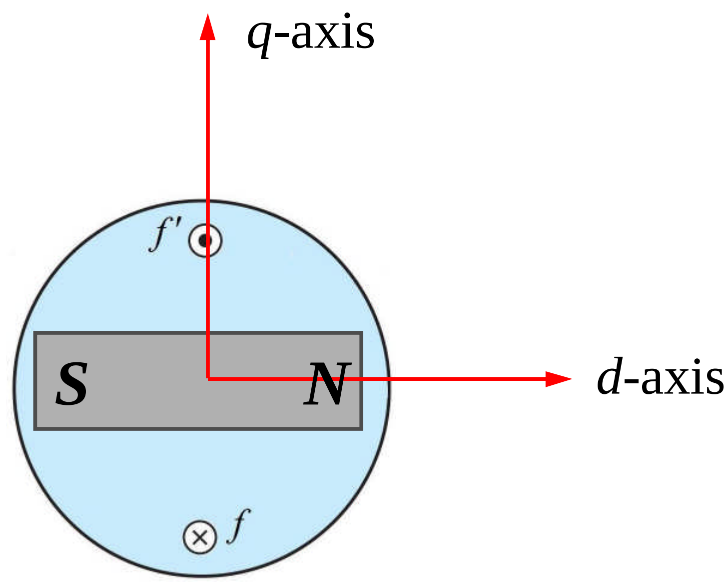


Rotor magnetic field

- The dc current in the field windings produces a magnetic flux.
  - We assign an axis to the direction of the flux labeled *d*-axis (direct axis) and *q*-axis (quadrature axis, 90° ahead).



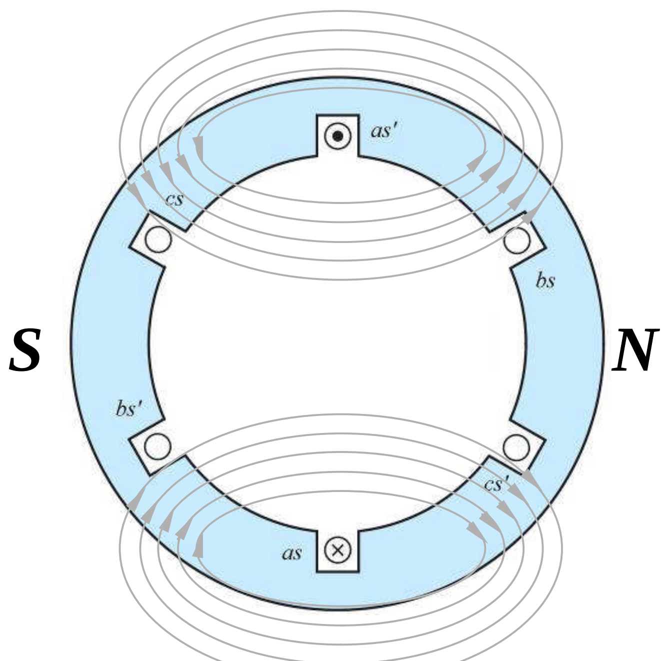
rotor magnetic field



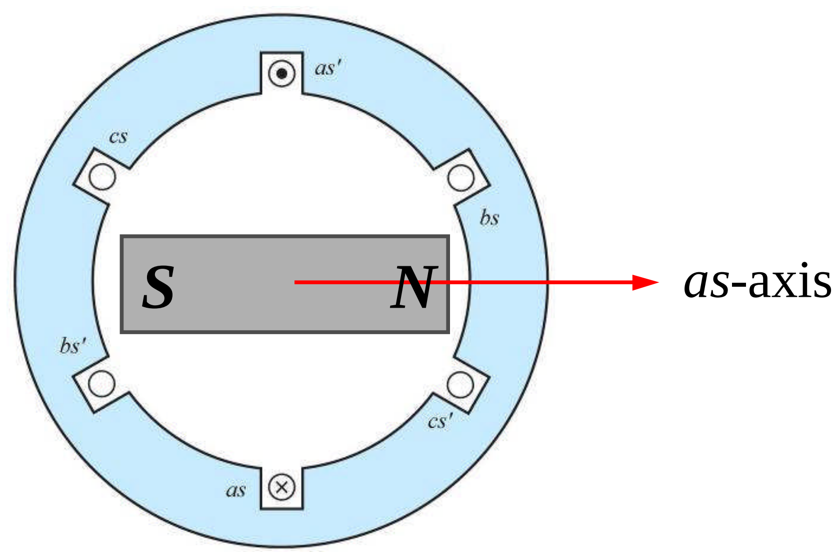
bar magnet equivalent to rotor field

Stator magnetic field

- If current were flowing in the stator windings, it too would create magnetic flux.
  - We will assign the *as*-axis to denote the direction of the magnetic field of the a-phase of the stator.



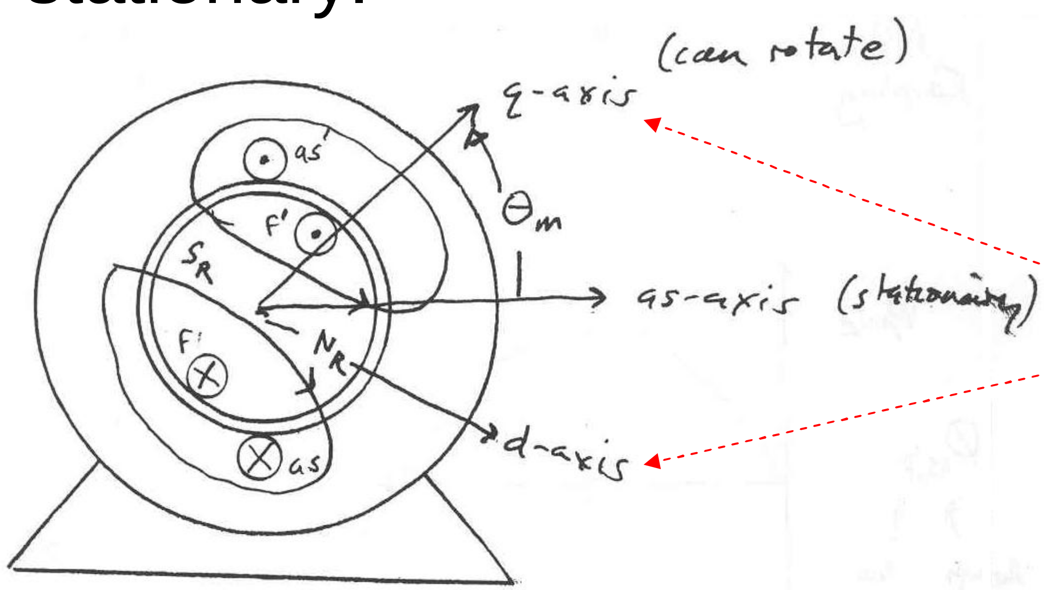
stator magnetic field



bar magnet equivalent to stator field

Magnetic fields

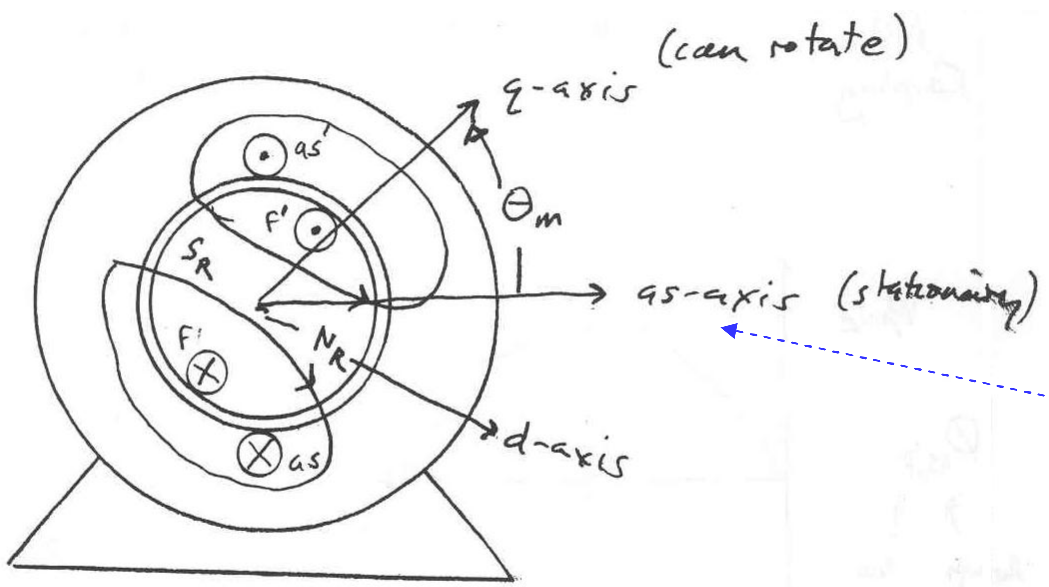
- Recall that the rotor rotates at a speed of ω_m, therefore the rotor's d-axis and q-axis also rotate.
- The stator is fixed, so as-axis the remains stationary.



magnetic field axes associated with the rotor

Stator magnetic flux

- Current flowing in the stator windings would also produce a field along an axis (labeled *as*-axis)
- Because the stator is fixed, the *as*-axis is also fixed.

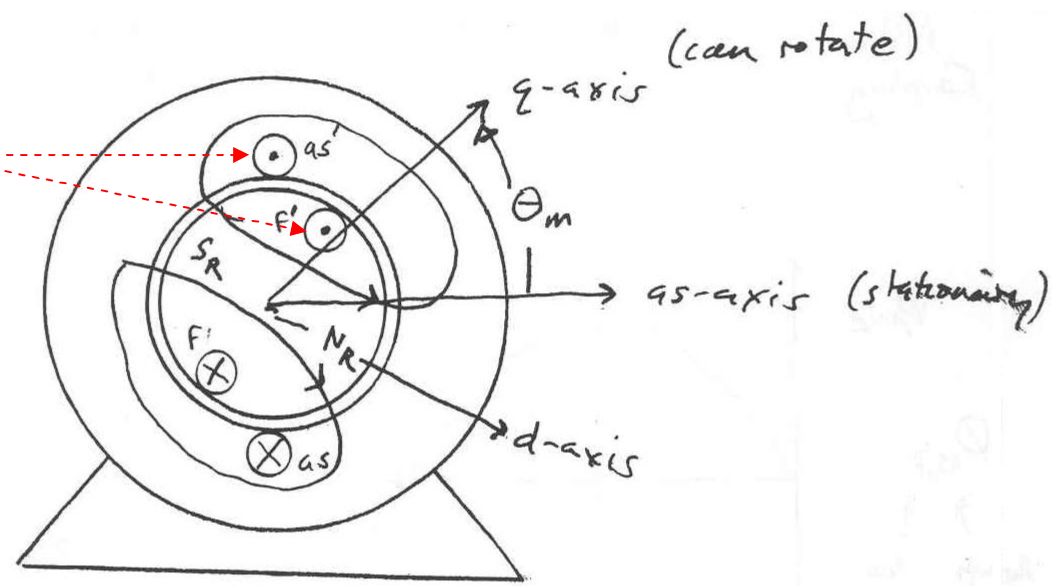


magnetic field axis associated with the stator

Flux coupling

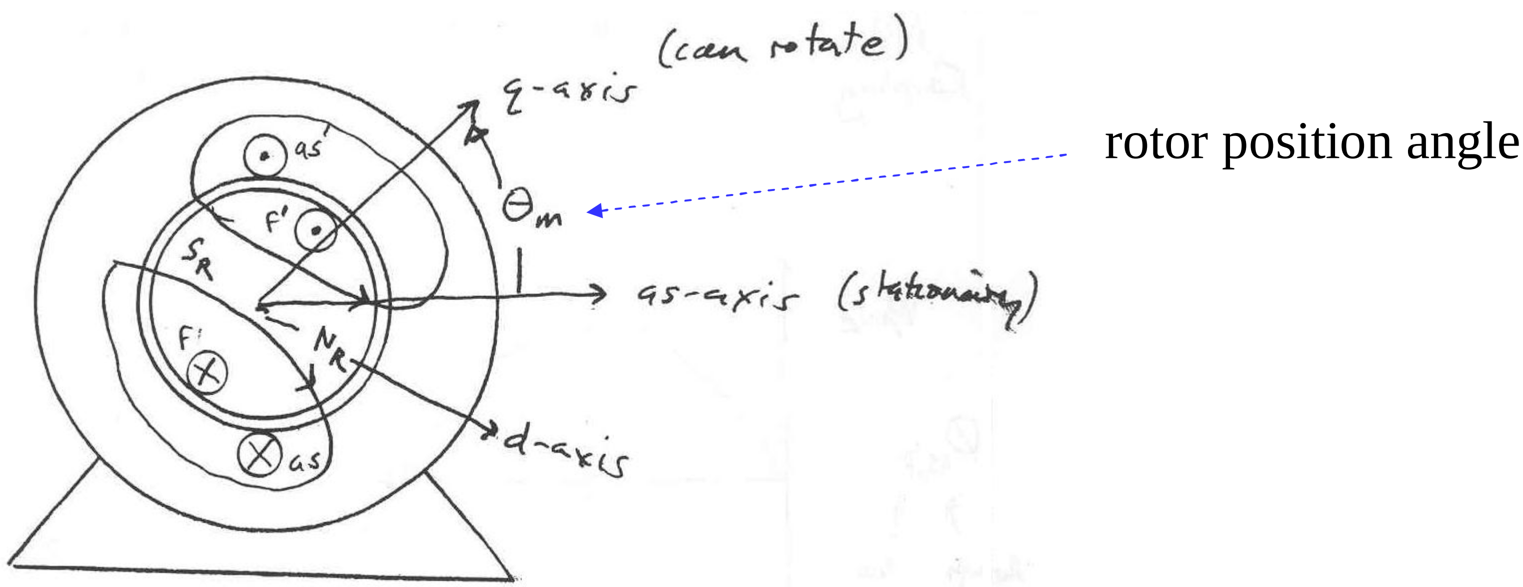
- Notice that the lines of magnetic flux from the rotor winding partially enclose the stator windings.
- The amount of this coupling (flux linkage) depends upon the position of the rotor.

coupling between
rotor and stator
windings

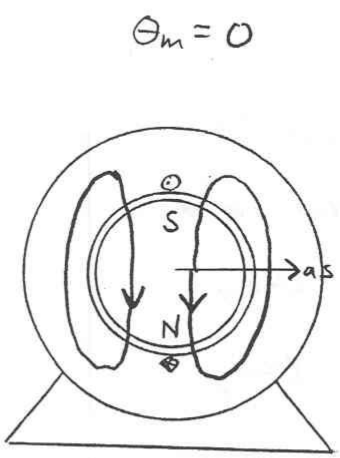


Rotor position angle

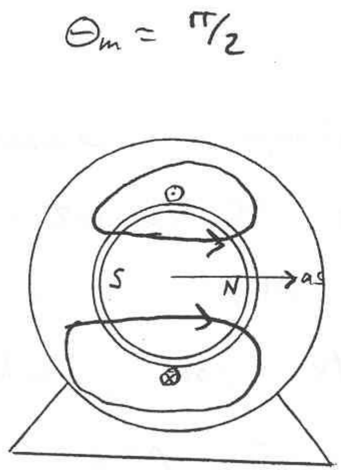
- We define angle θ_m as the rotor position angle between q-axis and as-axis.
- We would like to relate the amount of flux coupling $\phi_{as,f}$ as a function of θ_m.



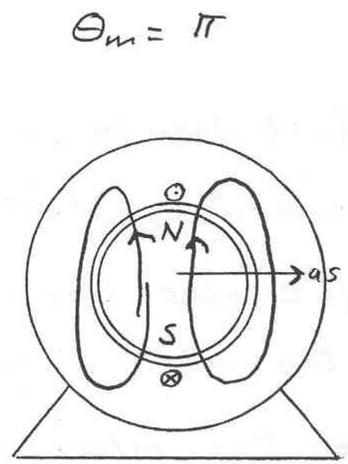
Flux coupling



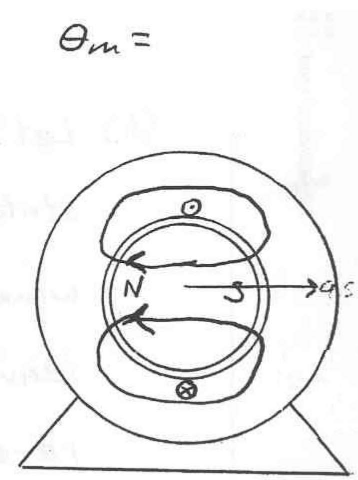
no coupling



maximum positive coupling

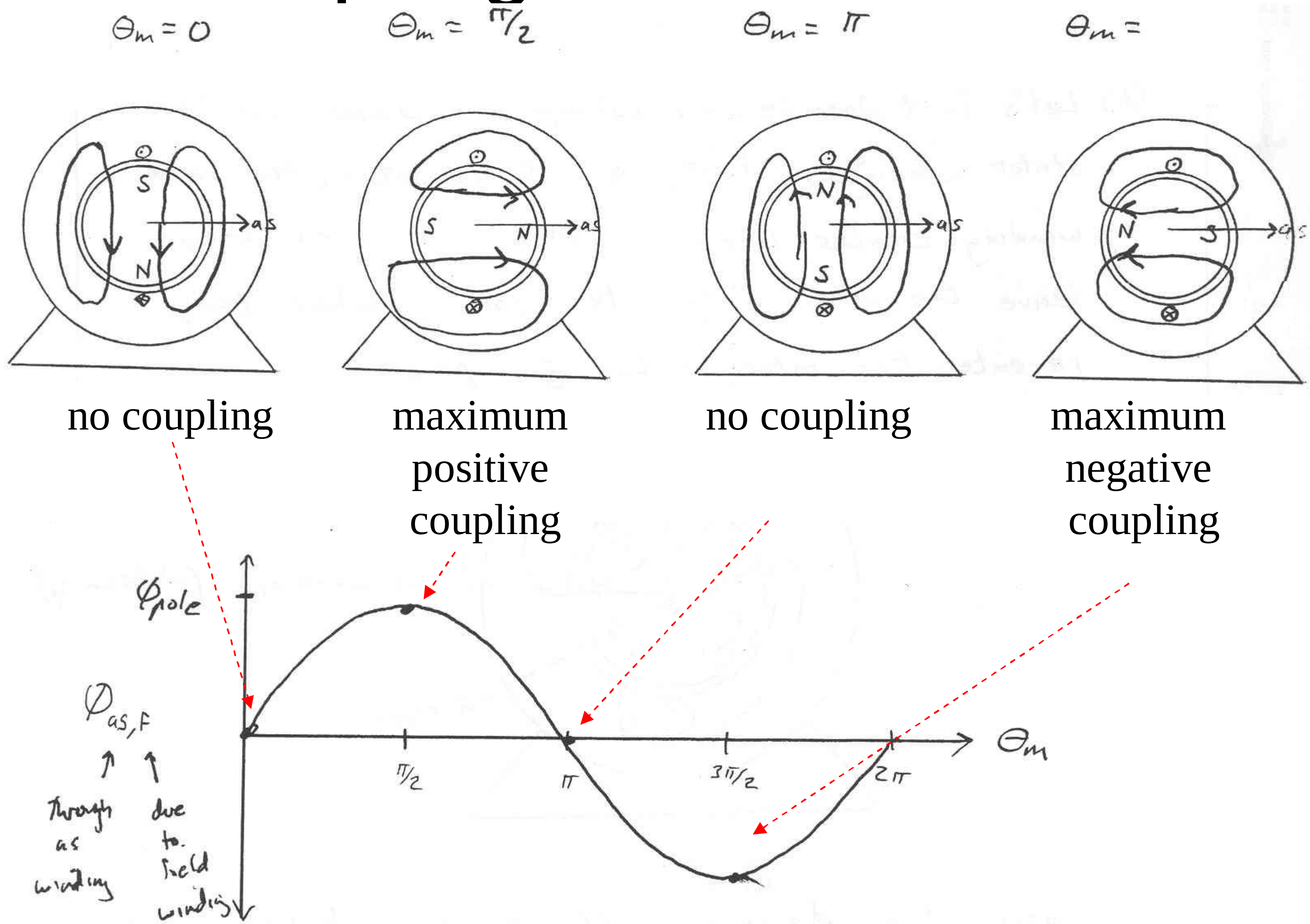


no coupling



maximum negative coupling

Flux coupling



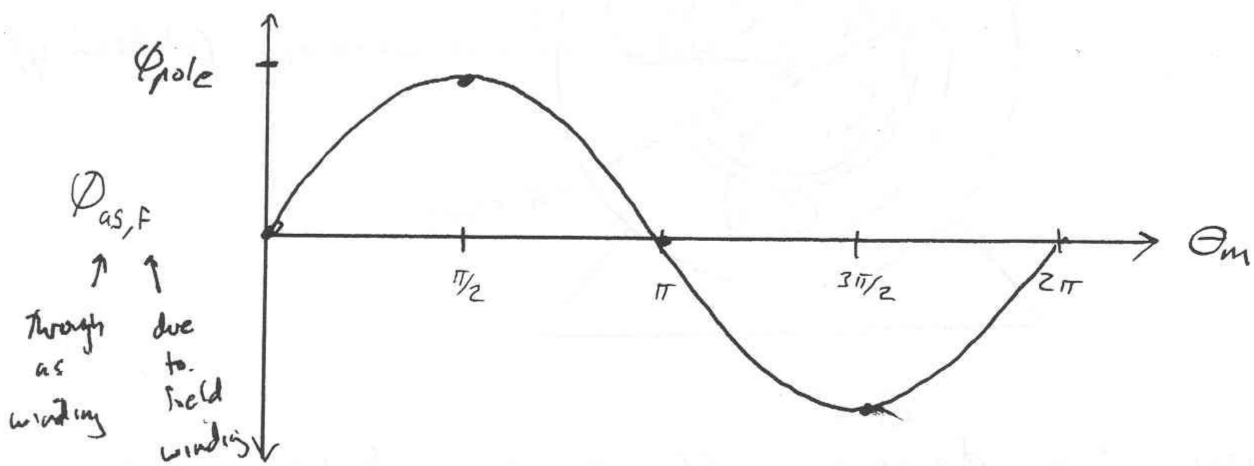
Flux coupling

- We represent the flux linkage ($\phi_{as,f}$) between *a*-phase of the stator and the rotor windings as

$$\phi_{as,f} = \phi_{\text{pole}} \sin \omega_m t$$

maximum coupling

mechanical angular speed (note: $\theta_m = \omega_m t$)



Induced voltage

- Since the flux is time-varying, it follows from Faraday's law that a voltage (e_{as}) will result

$$e_{as} = N_s \frac{d}{dt}\left(\phi_{as,f}\right)$$

- Since the flux is given $\phi_{as,f} = \phi_{\text{pole}} \sin \omega_m t$

it follows that $e_{as} = N_s \frac{d}{dt}\left(\phi_{\text{pole}} \sin \omega_m t\right)$

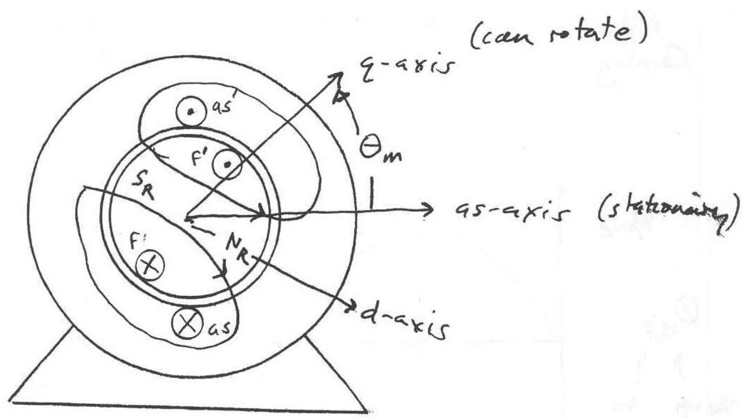
$$= \underbrace{N_s \phi_{\text{pole}} \omega_m}_{\text{peak phase voltage is function of}} \cos \omega_m t$$

peak phase voltage is function of

1. number turns in stator
2. rotor flux
3. rotor angular velocity

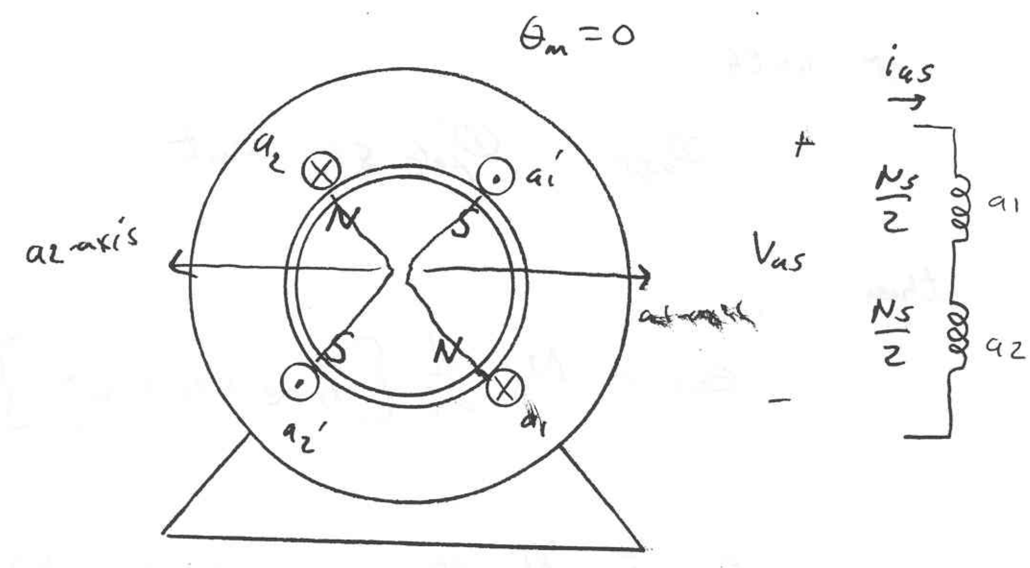
Rotor angular velocity

- In this machine, the frequency of the induced voltage (e_{as}) and the angular velocity of the machine (ω_m) are the same.
- If we desire f_e = 60-Hz ac, what must the rotor speed be in RPM?
- How might we change the rotor speed and still produce 60 Hz power?



Four pole machine

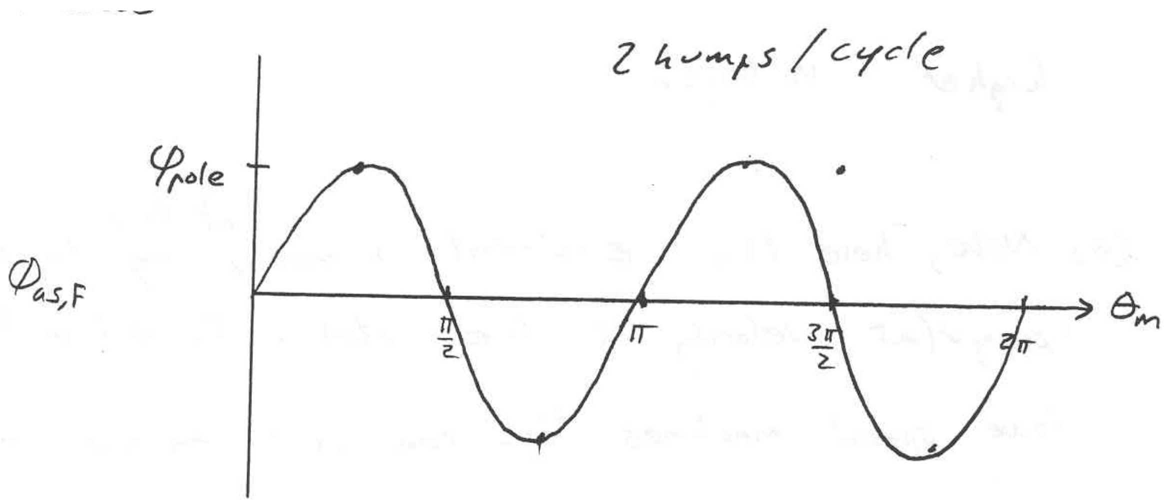
- Consider the four pole ($P = 4$) machine below.



- What does the flux linkage ($\phi_{as,f}$) look like now?

Four pole machine

- The flux linkage ($\phi_{as,f}$) is now twice the frequency.

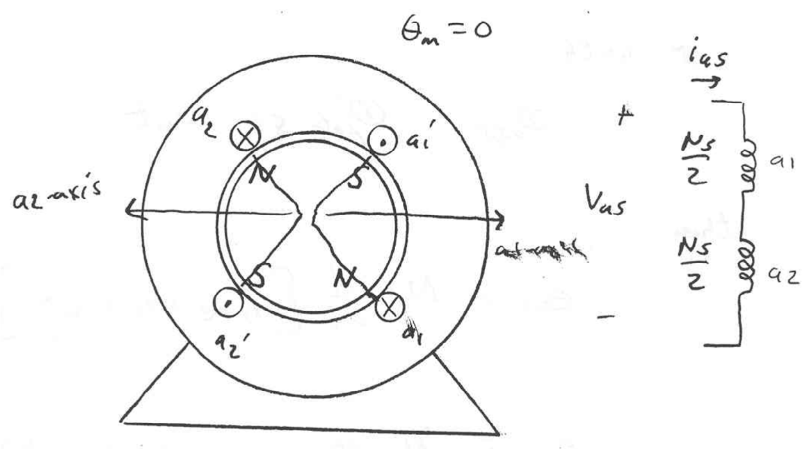


$$\phi_{as,f} = \phi_{\text{pole}} \sin 2\omega_m t$$

factor of two

Induced voltage (4 poles)

- The induced voltages ($e_{as,1}$,$e_{as,2}$) are given



$$e_{as,1} = e_{as,2} = \frac{N_s}{2}\frac{d}{dt}\left(\phi_{\text{pole}} \sin 2\omega_m t\right)$$

$$= \frac{N_s}{2}\phi_{\text{pole}} 2\omega_m \cos \omega_m t$$

- Because they are in series, the total voltage e_a is

$$e_{as} = e_{as,1} + e_{as,2} = 2N_s\phi_{\text{pole}}\omega_m \cos 2\omega_m t$$

Induced voltage (P poles)

- Generalizing this result to P poles (where P is an even integer ≥ 2), the total induced voltage (e_{as}) is given

$$
\begin{aligned}
e_{as} &= e_{as,1} + e_{as,2} + \cdots + e_{as,2P} \\
&= \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2} \omega_m t \right)
\end{aligned}
$$

Motor speed (multiple poles)

- The relation between rotor angular velocity (ω_m) and that of the induced voltage (ω_e) is given by

$$\omega_e = \frac{P}{2}\omega_m$$

- Converting to electrical frequency (f_e) in hertz and motor speed in RPM, we have

$$2\pi f_e = \frac{P}{2}\left(N_m \frac{2\pi}{60}\right) \quad \Rightarrow \quad N_m = \frac{120 f_e}{P} \quad \text{[rpm]}$$

Example Problem 1

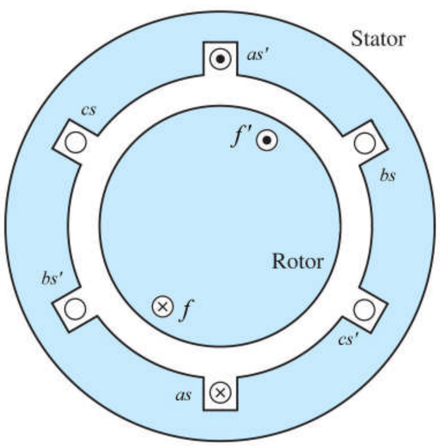
If reduction gear connected to a gas turbine spins a generator at 1200 rpm and we desire electricity generated at 60 Hz, how many poles P will our generator require?

Three-phase generator

- We have determined the induced voltage in just phase a.

$$e_{as} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2} \omega_m t \right)$$

- Since b and c are displaced by 120°, it follows that they will form a balanced set of voltages given



$$e_{bs} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2} \omega_m t - 120^\circ \right)$$

$$e_{cs} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2} \omega_m t - 240^\circ \right)$$

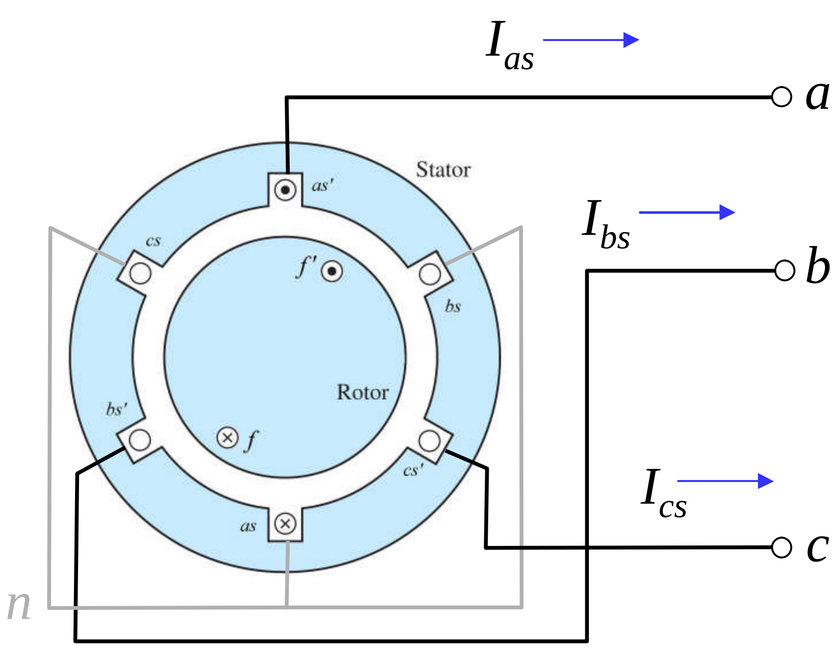
Three-phase generator

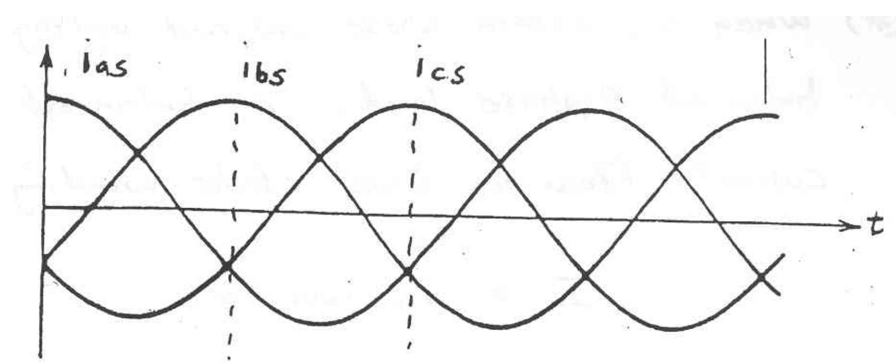
- Attaching these induced voltages to a balanced 3-phase load will produce balanced 3-phase currents given

$$I_{as} = I_m \cos \omega_e t$$

$$I_{bs} = I_m \cos(\omega_e t - 120°)$$

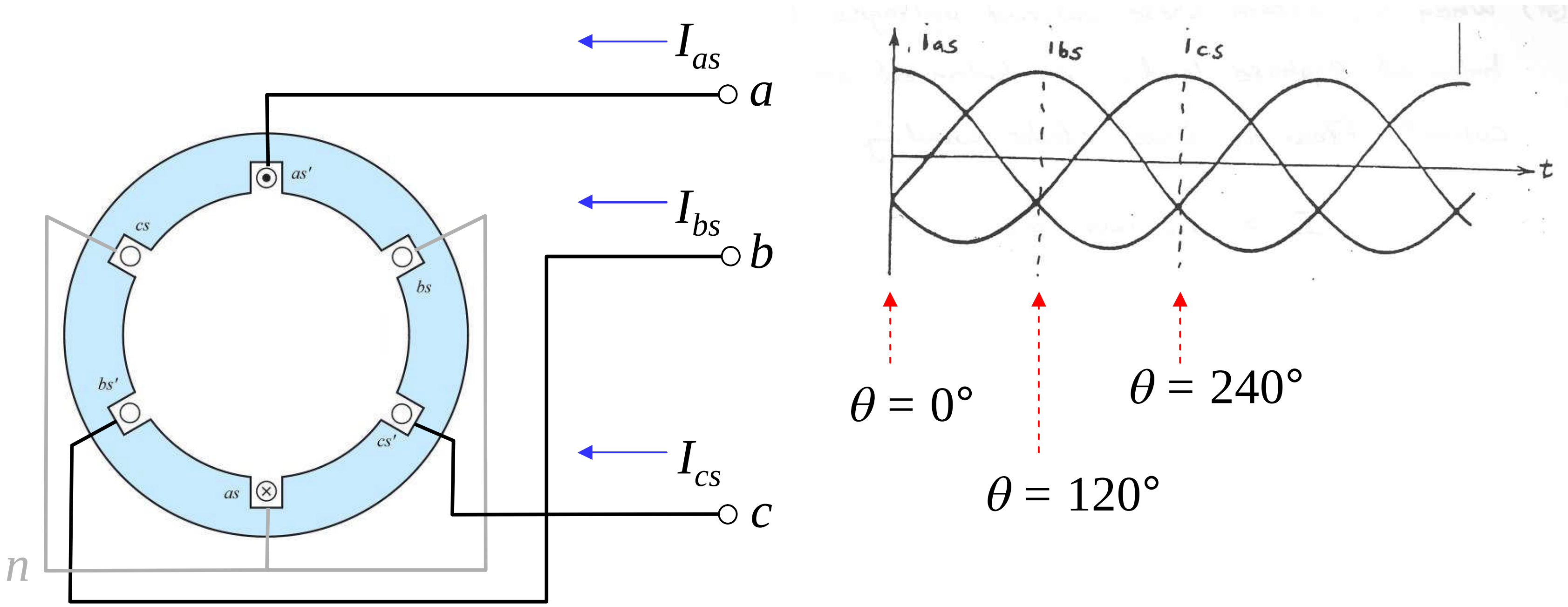
$$I_{bs} = I_m \cos(\omega_e t + 120°)$$



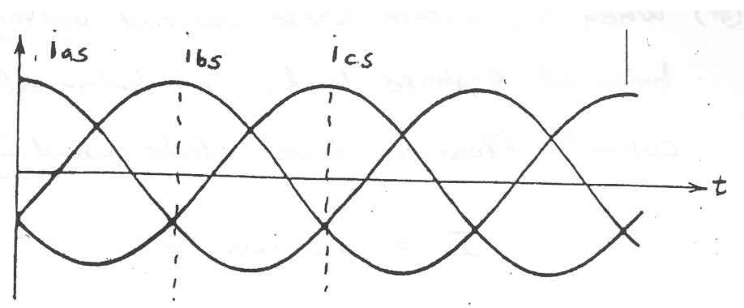


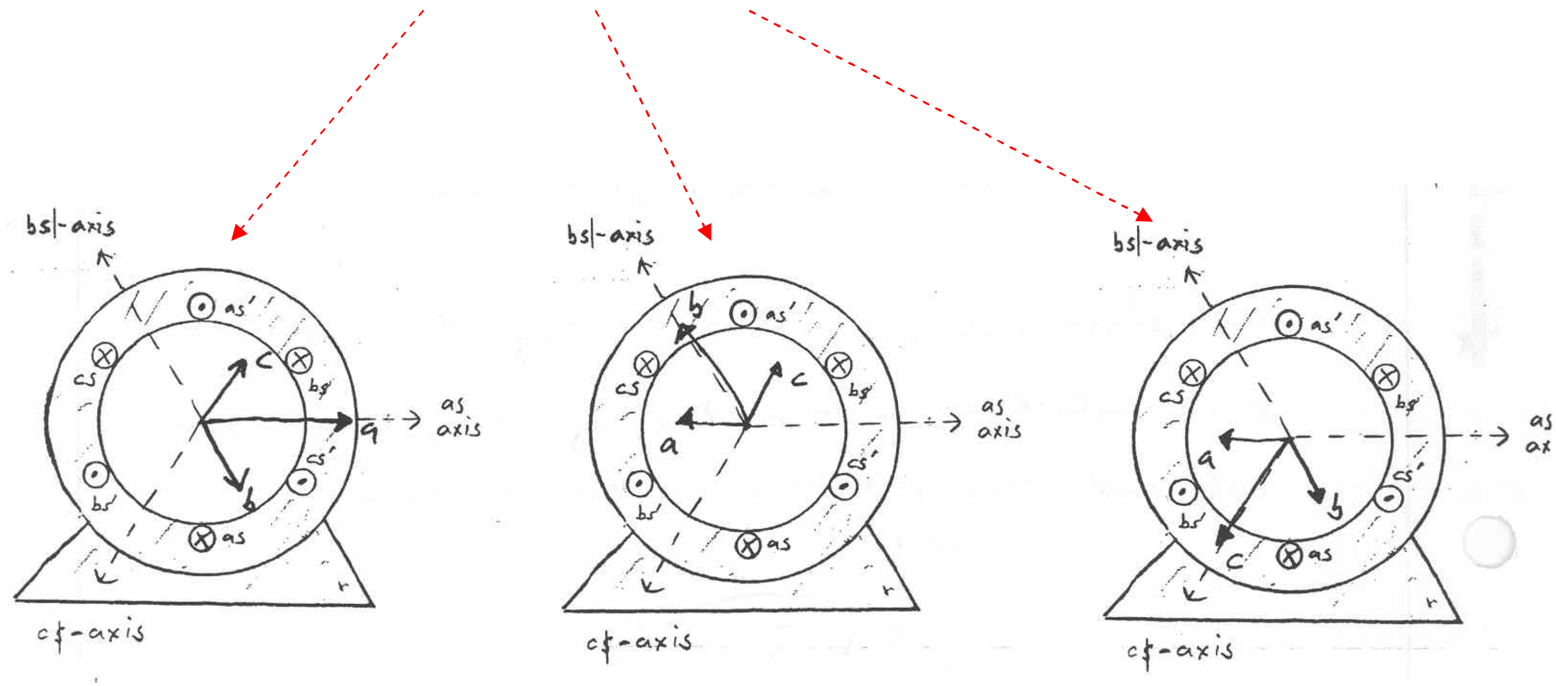
Stator magnetic field

- Now consider "driving" the stator windings with balanced 3-phase currents.
- What does the resulting stator field look like?
  - We will examine 3 instants in time.



Stator magnetic field





$\theta = 0°$

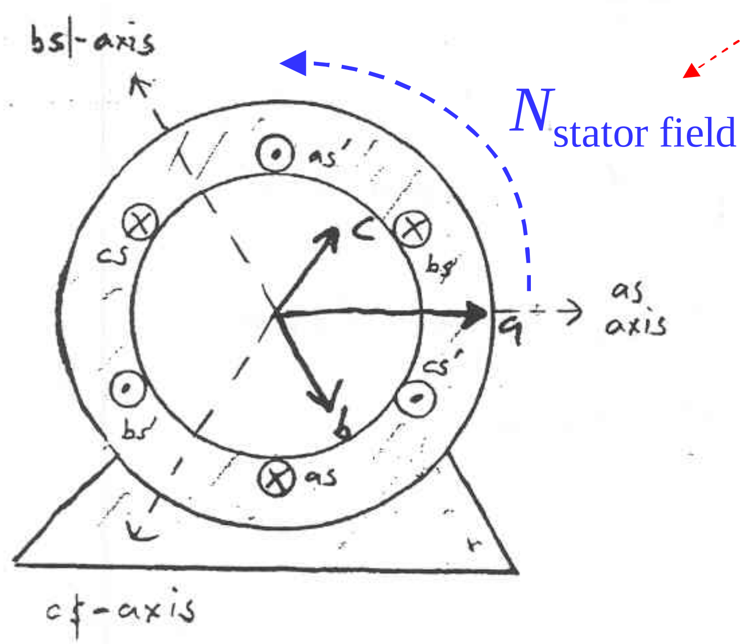
$\theta = 120°$

$\theta = 240°$

Stator magnetic field

- We observe that “driving” the stator windings with balanced 3-phase currents produces a rotating magnetic field.

the resultant stator magnetic field rotates

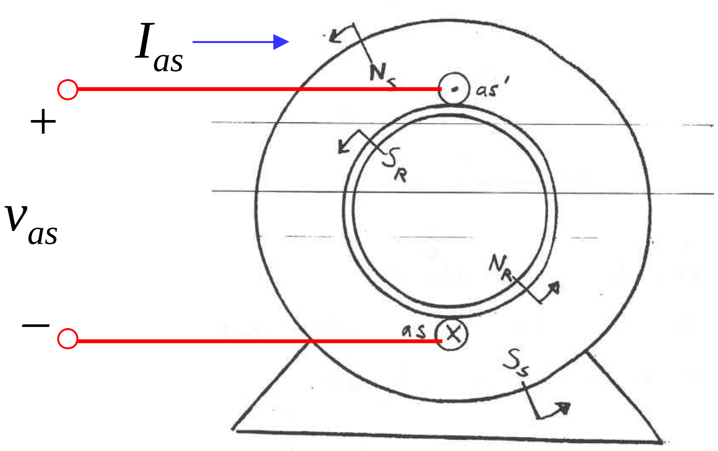


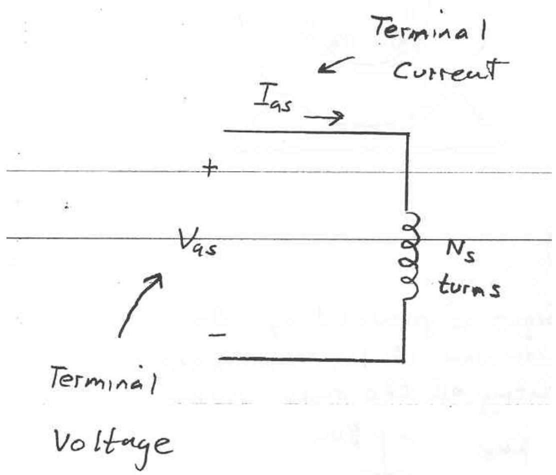
$$N_{\text{stator field}} = \frac{120 f_e}{P} \quad \text{[rpm]}$$

Per phase equivalent circuit

- Looking into the *a* phase of the coil we see
  - Coil resistance (r_s)
  - An induced voltage due to rotor magnetic field ($e_{as,f}$)
  - An induced voltage due to stator magnetic field ($e_{as,s}$)
- Thus, the terminal voltage (v_{as}) is given

$$v_{as} = r_s I_{as} + e_{as,f} + e_{as,s}$$





Induced voltage $e_{as,f}$

- We have previously developed an expression for the induced voltage due to the rotor magnetic field ($e_{as,f}$)

$$\begin{aligned} e_{as,f} &= \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2} \omega_m t \right) \\ &= N_s \phi_{\text{pole}} \omega_e \cos\left(\frac{P}{2} \omega_m t \right) \end{aligned}$$

- Since ϕ_{pole} is directly proportional to the rotor field current I_f, we can re-write $e_{as,f}$ as

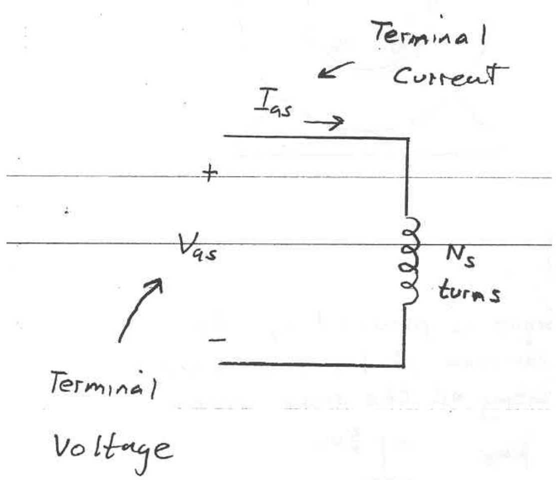
$$e_{as,f} = L_{sf} I_f \omega_e \cos\left(\frac{P}{2} \omega_m t \right)$$

Induced voltage $e_{as,s}$

- The induced voltage due to the stator magnetic field ($e_{as,s}$) is modeled as

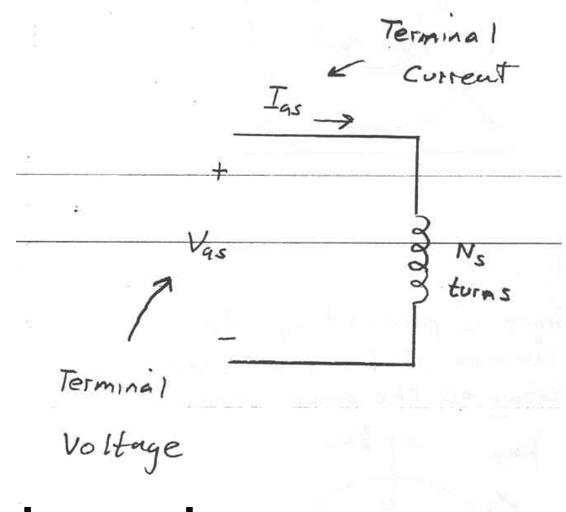
$$e_{as,s} = L_s \frac{d}{dt}(I_{as})$$

where inductance L_s takes into account the effect of the b and c phase currents.



Terminal voltage v_{as}

- Combining the terms, we have



$$v_{as} = r_s I_{as} + e_{as,f} + e_{as,s}$$
$$= r_s I_{as} + L_{sf} I_f \omega_e \cos\left(\frac{P}{2}\omega_m t\right) + L_s \frac{d}{dt}(I_{as})$$

- Transforming into the frequency domain
  - where d/dt is replaced by $j\omega_e$ and $X_s = L_s\omega_e$

$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \tilde{E}_a + L_s j\omega_e \tilde{I}_{as}$$
$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \tilde{E}_a + jX_s \tilde{I}_{as}$$

terminal voltage (line-to-neutral)

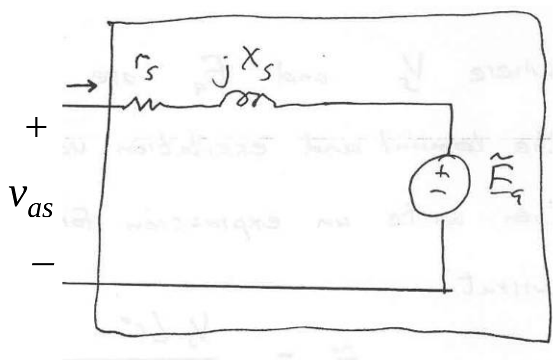
stator winding resistance

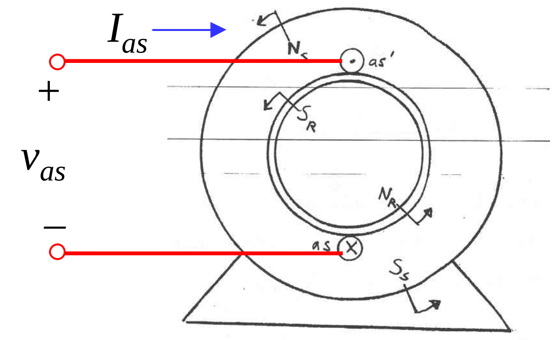
excitation or generated voltage

synchronous reactance

Per phase equivalent circuit

- The per phase equivalent can thus be drawn





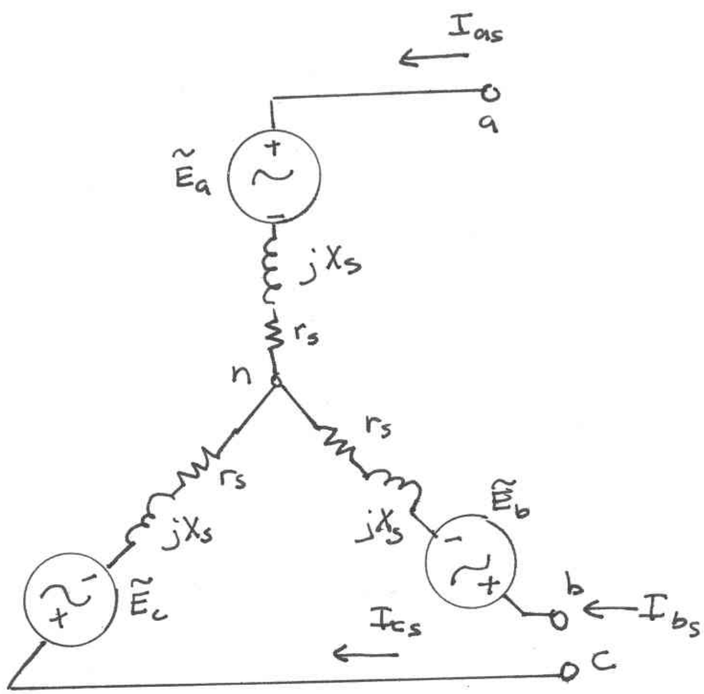
$$V_{as} = r_s I_{as} + \tilde{E}_a + jX_s \tilde{I}_{as}$$

terminal voltage (line-to-neutral)	stator winding resistance	excitation or generated voltage	synchronous reactance

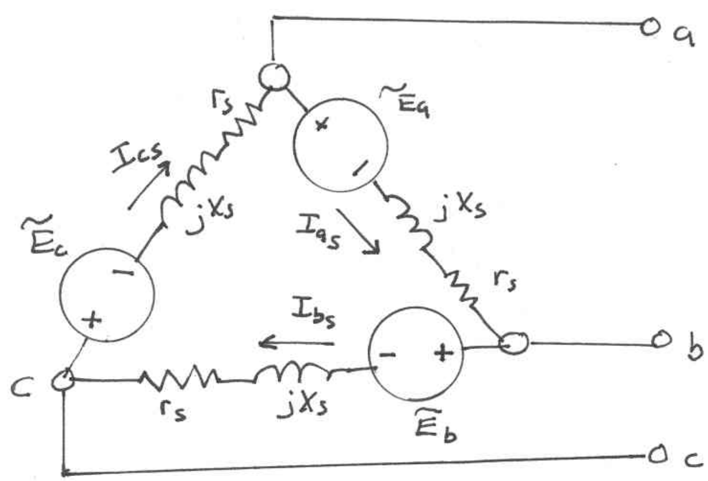
- This circuit would constitute one phase of a three-phase source (either Y or Δ connected).

Per phase equivalent circuit

- This circuit would constitute one phase of a three-phase source (either Y or Δ connected).



Y-connected source



Δ-connected source

Excitation voltage $\widetilde{E}_a$

- The excitation (or generated) voltage is derived from the term

$$L_{sf} I_f \omega_e \cos\left(\frac{P}{2}\omega_m t\right)$$

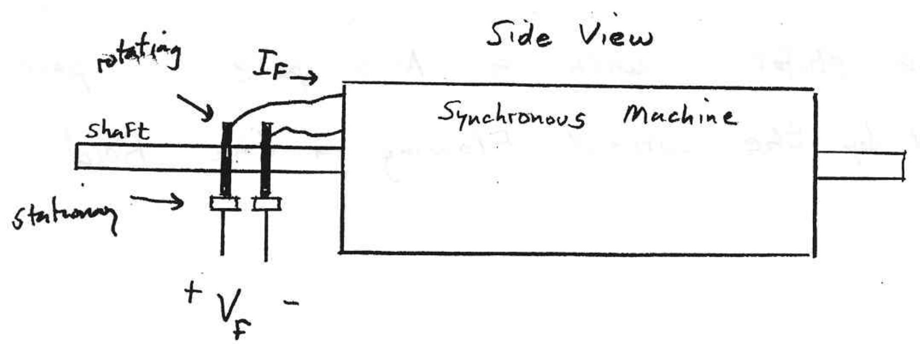
  so it has an rms value of $\dfrac{L_{sf} I_f \omega_e}{\sqrt{2}}$

- The angle of E_a is not necessarily 0° because it is related to the mechanical position, so we will call it δ, the power (or torque) angle such that

$$\tilde{E}_a = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} \angle \delta$$

Excitation voltage $\widetilde{E}a$

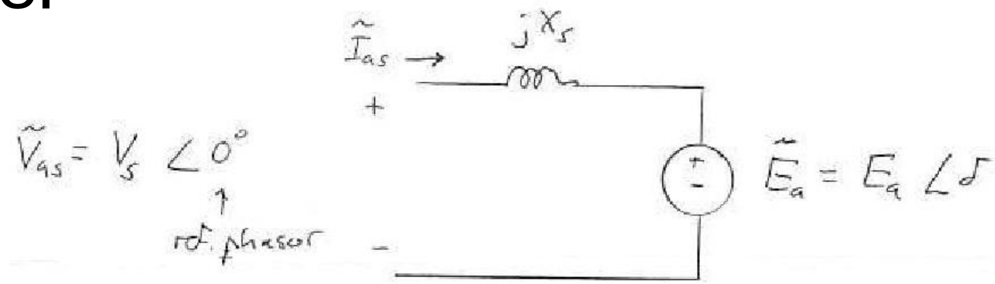
- Notice that the excitation voltage is a function of
  - the dc rotor field current I_f
  - inductance L_{sf} which models flux linkage between the rotor and stator windings (a fixed quantity)
  - the electrical frequency ω_e (a fixed quantity)
- Therefore if we wish to control the excitation voltage (in turn the generator output voltage) we can do so by controlling the dc field current I_f.



$$\tilde{E}_a = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} \angle \delta$$

Power vs. angle δ

- For large machines ($\eta > 99\%$), we can ignore r_s simplifying our model



- We can solve for I_{as}

$$\tilde{I}_{as} = \frac{\tilde{V}_{as} - \tilde{E}_a}{jX_s} = \frac{V_s \angle 0° - E_a \angle \delta}{jX_s} = \frac{V_s \angle 0°}{jX_s} - \frac{E_a \angle \delta}{jX_s}$$

$$= \frac{V_s}{X_s} \angle -90° - \frac{E_a}{X_s} \angle (\delta - 90°)$$

- Taking the conjugate

$$\tilde{I}_{as}^{*} = \frac{V_s}{X_s} \angle 90° - \frac{E_a}{X_s} \angle (90° - \delta)$$

Complex power

- The complex power for one phase ($\mathbf{S}_{1\mathrm{P}}$) is thus

$$\mathbf{S}_{1\mathrm{P}} = \tilde{V}_{as}\tilde{I}_{as}^{*} = V_s\angle 0^\circ\left[\frac{V_s}{X_s}\angle 90^\circ - \frac{E_a}{X_s}\angle(90^\circ - \delta)\right]$$

$$= \frac{V_s^2}{X_s}\angle 90^\circ - \frac{V_s E_a}{X_s}\angle(90^\circ - \delta)$$

- Applying Euler's identity $A\angle\theta = A\cos\theta + jA\sin\theta$

$$\mathbf{S}_{1\mathrm{P}} = P_{1\mathrm{P}} + jQ_{1\mathrm{P}}$$

$$= \frac{V_s^2}{X_s}\left[\cos 90^\circ + j\sin 90^\circ\right] - \frac{V_s E_a}{X_s}\left[\cos(90^\circ - \delta) + j\sin(90^\circ - \delta)\right]$$

$$= j\frac{V_s^2}{X_s} - \frac{V_s E_a}{X_s}\left[\sin(\delta) + j\cos(\delta)\right]$$

Real power

- Grouping the real and imaginary components

$$\mathbf{S}_{1\mathrm{P}} = P_{1\mathrm{P}} + jQ_{1\mathrm{P}} = -\frac{V_s E_a}{X_s}\sin(\delta) + j\left[\frac{V_s^2}{X_s} - \frac{V_s E_a}{X_s}\cos(\delta)\right]$$

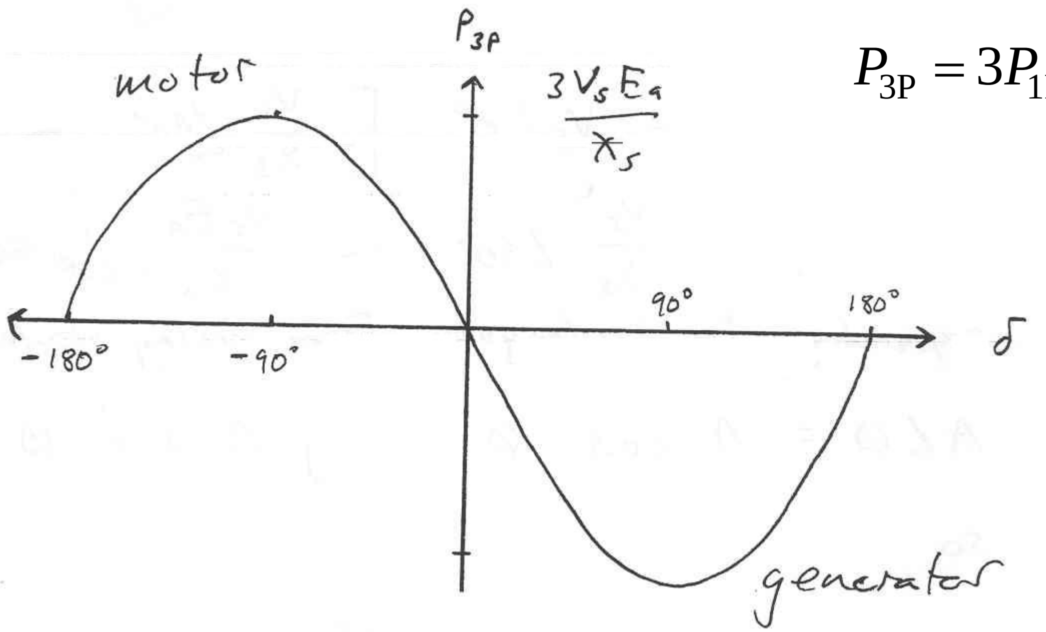
- The total real power P_T dissipated by our machine is given

$$P_{3\mathrm{P}} = 3P_{1\mathrm{P}} = -3\frac{V_s E_a}{X_s}\sin(\delta)$$

which is a function of the power angle δ.

Power vs. angle δ

- Plotting power as a function of δ we obtain the following

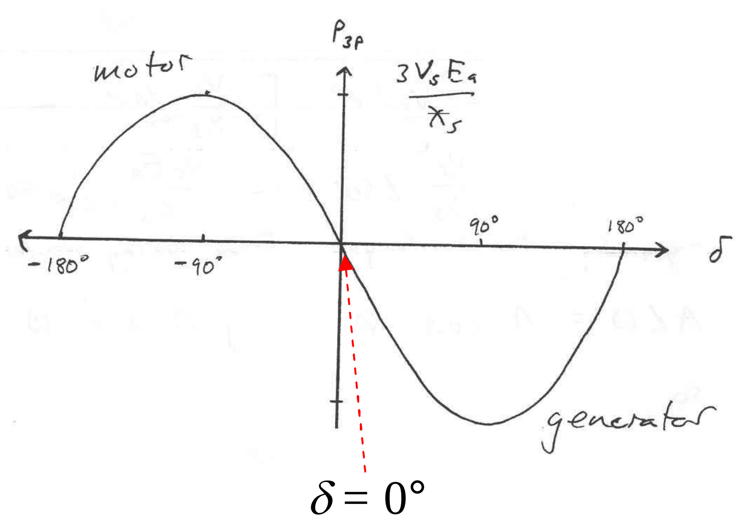


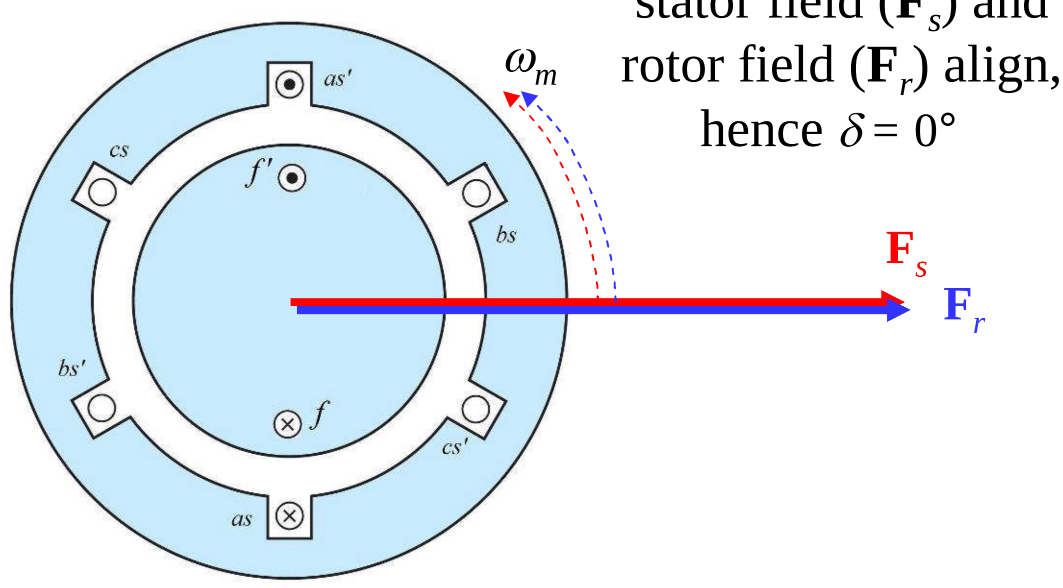
$$P_{3P} = 3P_{1P} = -3\frac{V_s E_a}{X_s}\sin(\delta)$$

- What does the power angle δ physically represent?

Power angle $\delta = 0°$

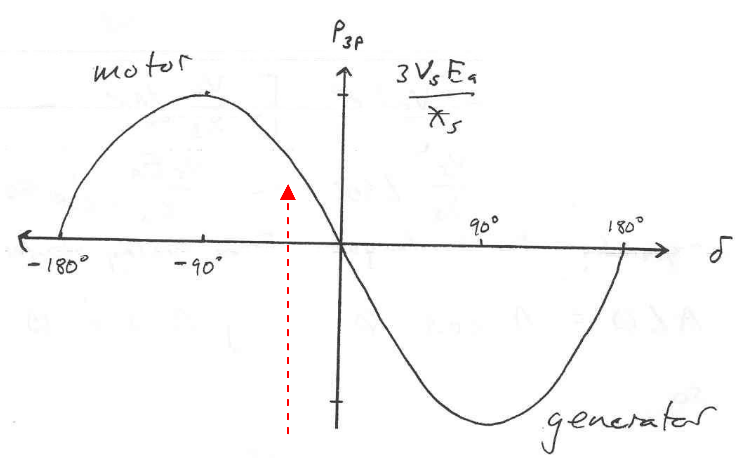
- With no load, the rotor's net magnetic field aligns exactly with the stator's rotating magnetic field.
  - With no load, no power is being dissipated and torque exerted on the rotor is zero.





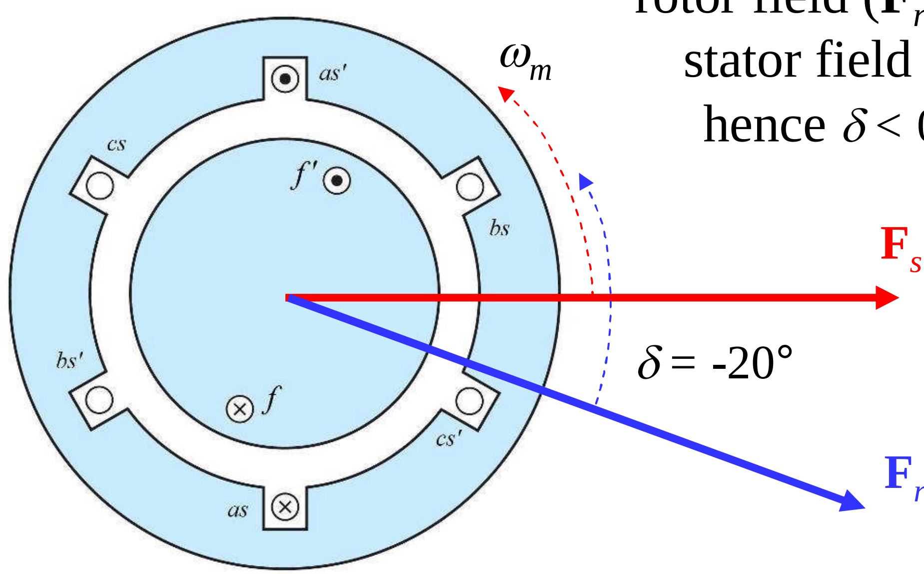
Power angle $\delta < 0^{o}$ (motor)

- With a load, the rotor's net magnetic field lags behind the stator's rotating magnetic field.
  - The angular difference results in a positive torque, hence positive power dissipation.



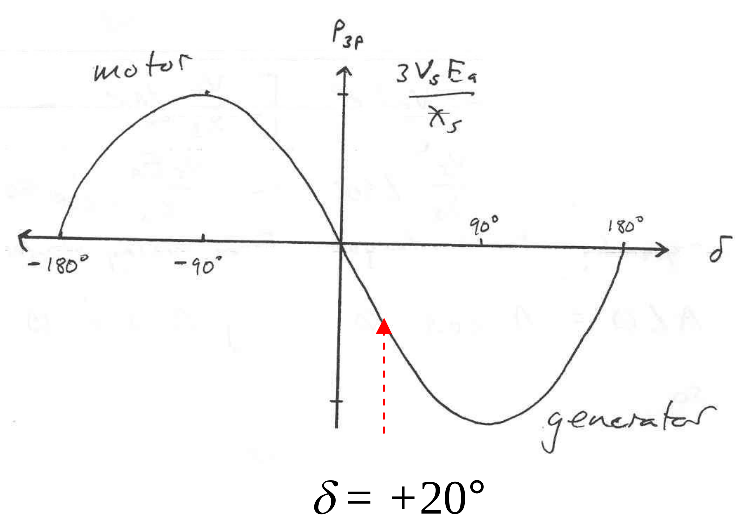
$\delta = -20°$

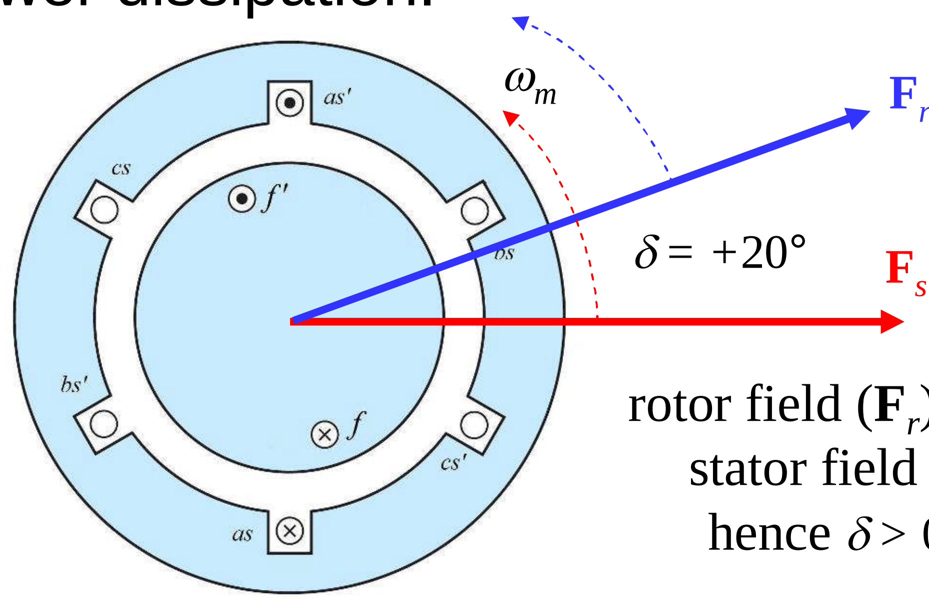
rotor field ($\mathbf{F}_r$) lags stator field ($\mathbf{F}_s$) hence $\delta < 0°$



Power angle $\delta > 0^o$ (generator)

- With a negative load (prime mover driving the rotor), the rotor's net magnetic field leads stator's rotating magnetic field.
  - The angular difference results in a negative torque, hence negative power dissipation.





rotor field ($\mathbf{F}_r$) leads stator field ($\mathbf{F}_s$) hence $\delta > 0°$

Torque

- Since we have assumed 100% efficiency the electrical power must equal the mechanical power at the shaft

$$P_{3P} = -3\frac{V_s E_a}{X_s}\sin(\delta) = P_{\mathrm{mech}} = T_e \omega_m$$

thus $T_{\mathrm{dev}} = \dfrac{P_{\mathrm{mech}}}{\omega_m}$

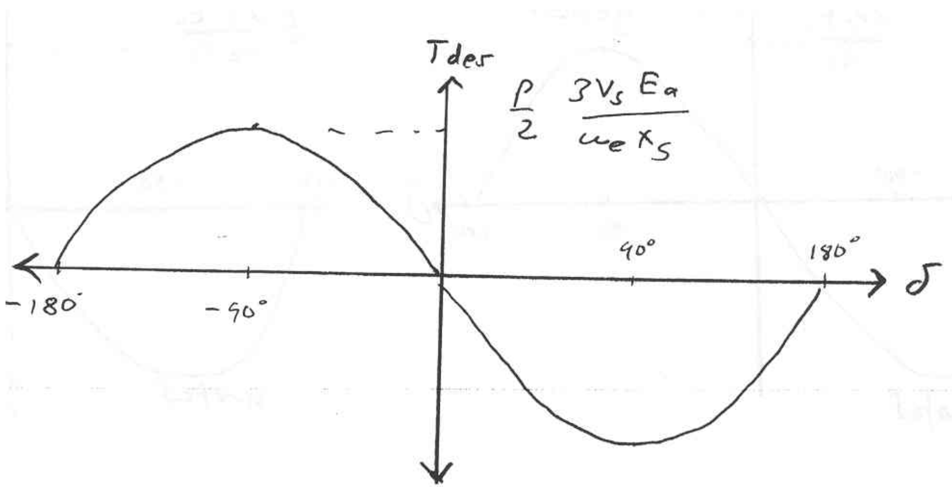
- Since $\omega_m = \dfrac{2}{P}\omega_e$

we conclude that $T_{\mathrm{dev}} = -\dfrac{P}{2}\dfrac{3V_s E_a}{\omega_e X_s}\sin(\delta)$

Torque vs. angle δ

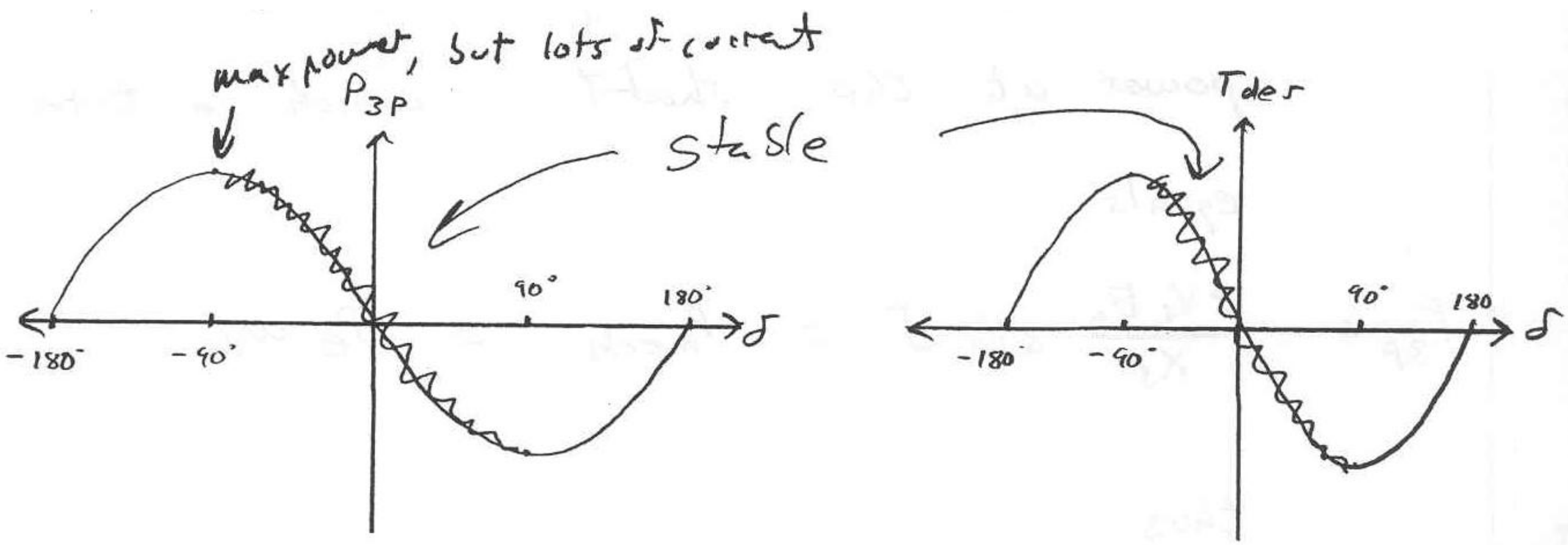
- Plotting torque as a function of δ we obtain the following

$$\frac{P}{2}\frac{3V_s E_a}{\omega_e X_s}\sin(\delta)$$



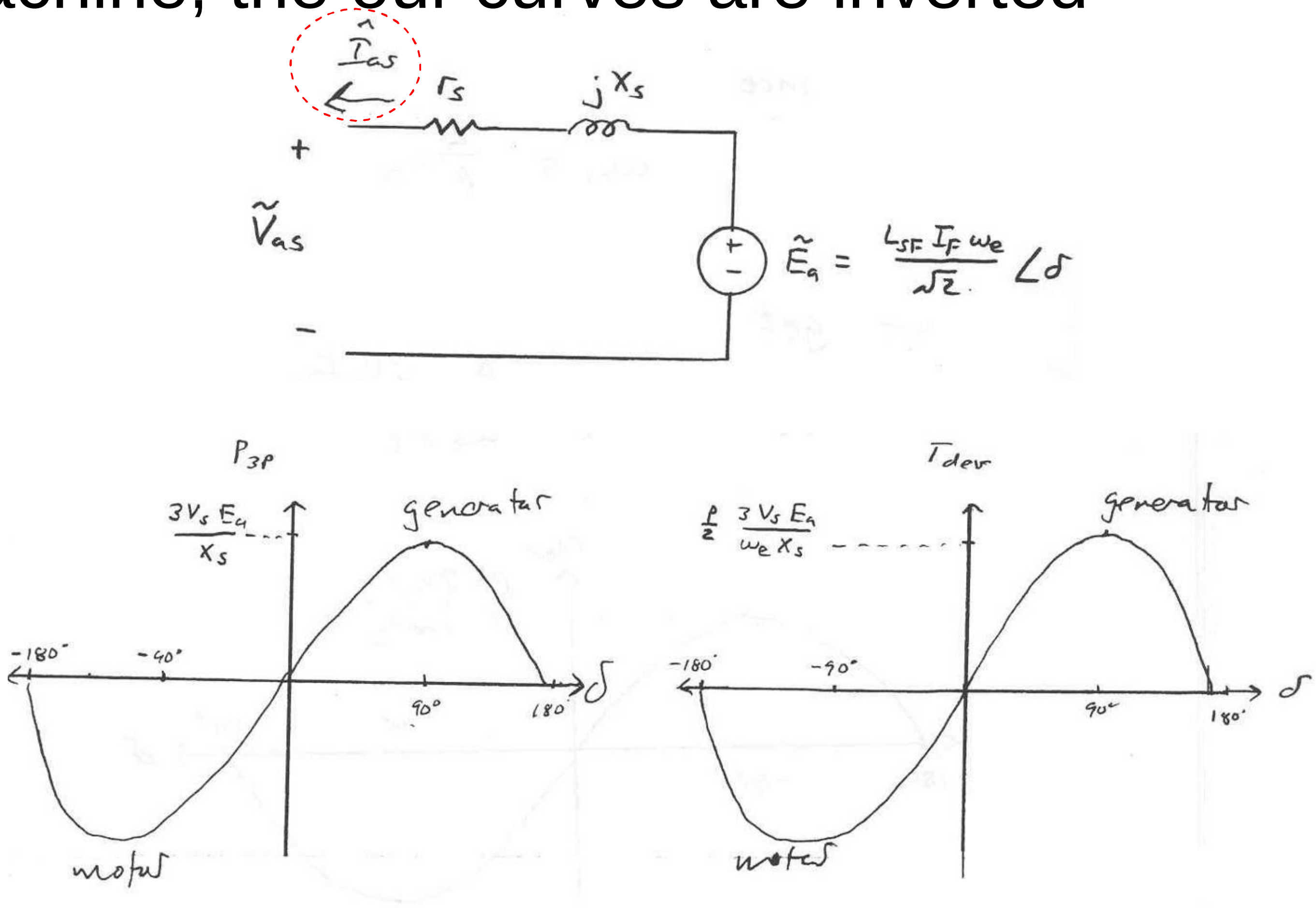
Operating points

- For both characteristics, the stable operating points reside on the negative sloped portions of the curves.



Changing reference current

- If we define the reference current as leaving the machine, the our curves are inverted



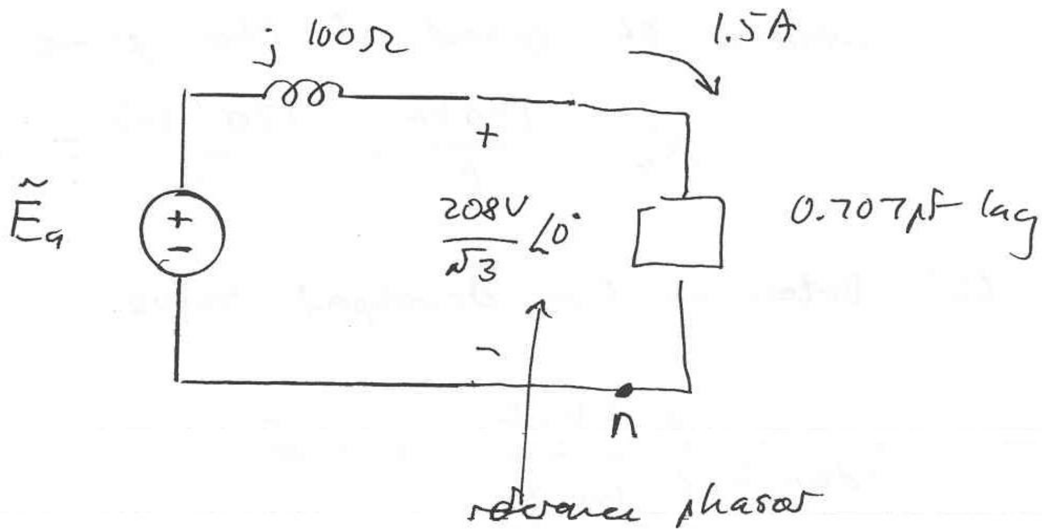
Example Problem 2

A 208-V_{rms} (line), 60 Hz, Y-connected generator has $X_s = 100\ \Omega$, $r_s = 0\ \Omega$. At rated terminal voltage, the machine delivers 1.5-A_{rms} (line) to a 0.707 pf lagging load.

a. Determine the machine current.

b. Determine the excitation (or generator) voltage E_a.

c. Determine the power (torque) angle δ.



Example Problem 2 (continued)

d. If the generator has 4 poles, what is the speed of the prime mover.

e. Determine the developed torque.

f. Suppose the load current increases to $3\text{-}A_{rms}$ at a 0.707 lagging pf. If we have the same terminal voltages, how must the field current change?