Lessons 30-31: Synchronous Machines

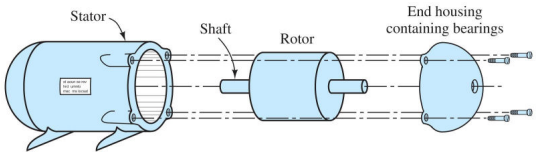
Electric motors

DC motors

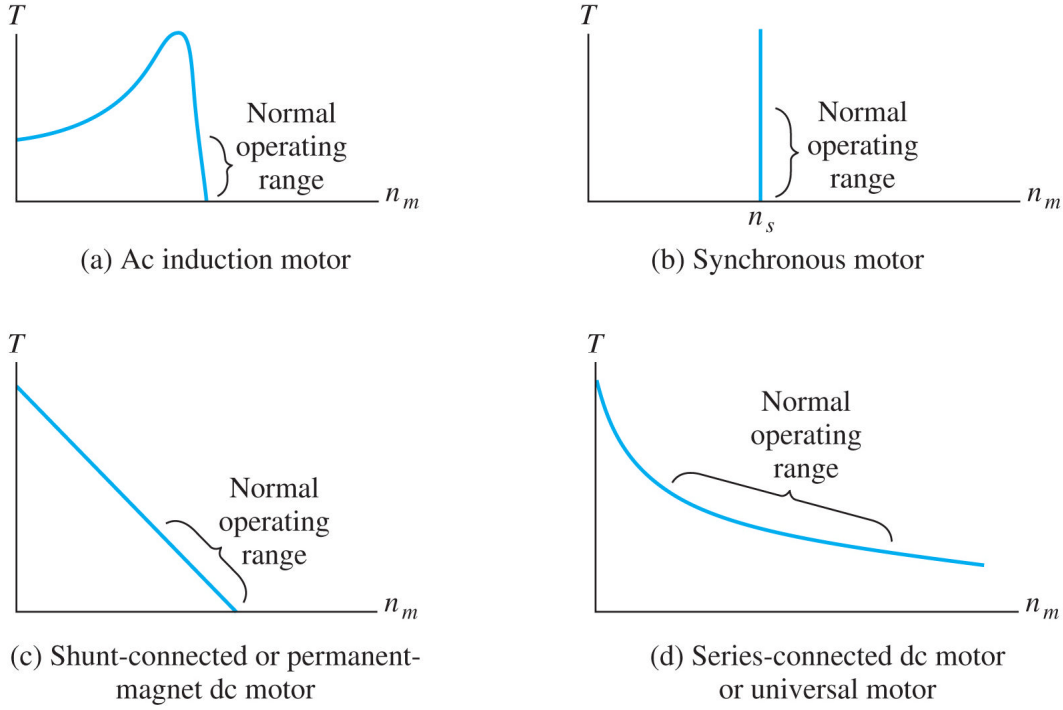
- High start torque, easy speed control
- Up to ~200 hp output, commutator wear

AC motors

- Most common (> 5 hp), up to ~50,000 hp output
- Less start torque



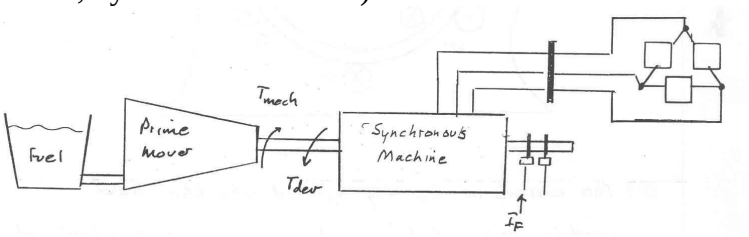
Motor torque-speed characteristics

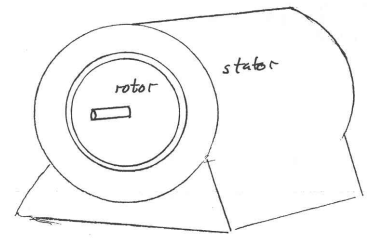


Synchronous AC machines

Used for nearly all electrical energy generation.
Used for _____-power, low-speed applications as motors.

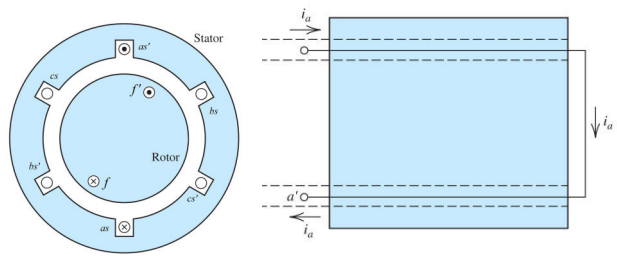
The machine can operate as a generator producing AC power if driven by a prime mover (gas, steam, hydroelectric turbine).





Goals
Understand how a rotating magnetic field is created in the stator
Understand the power (torque) angle δ
Use the equivalent circuit model to solve problems.

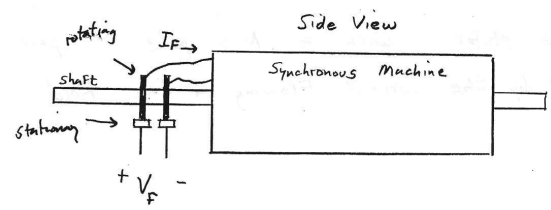
Stator
The stator (the _____ part) contains set of 3-phase windings to establish the stator field.
The number of slots in the stator is an integer multiple of ____. (why?)



Rotor
The rotor contains a winding called the field winding.

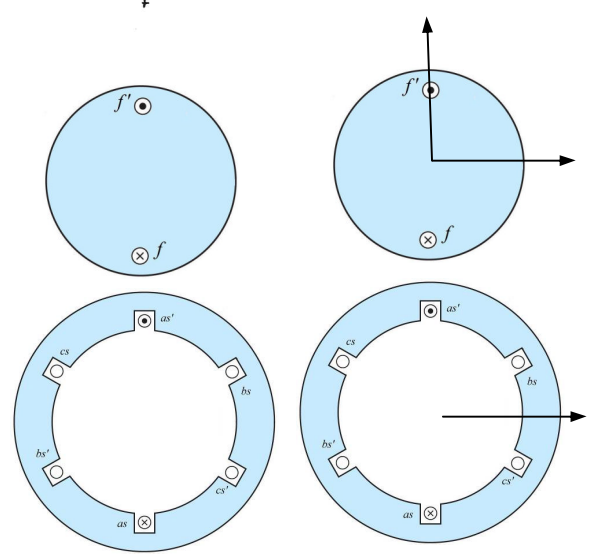
- Unlike the stator, it carries ____ current.

The dc current is often connected to the rotor via _____.
Because these contacts aren't commutating (changing _____ direction) wear is much less of an issue than in dc motors.



Rotor magnetic field
The dc current in the field windings produces a magnetic flux.

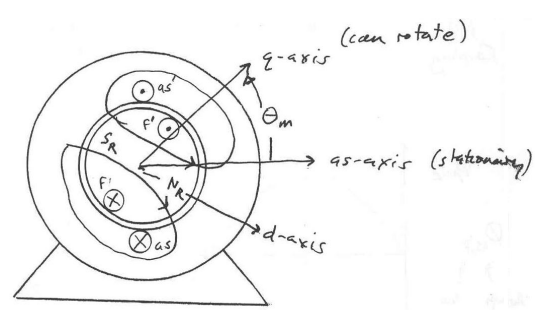
- We assign an axis to the direction of the flux labeled ___-axis (direct axis) and __-axis (quadrature axis, 90° ahead).



Stator magnetic field
If current were flowing in the stator windings, it too would create magnetic flux.

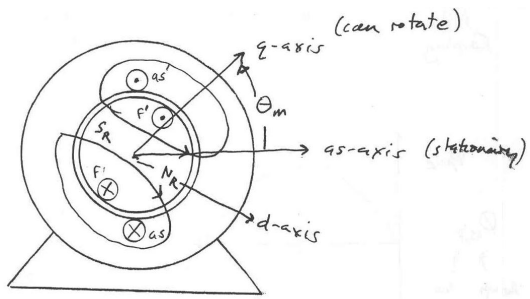
- We will assign the ___-axis to denote the direction of the magnetic field of the a-phase of the stator.

Magnetic fields
Recall that the rotor rotates at a speed of ω_m, therefore the rotor's *d*-axis and *q*-axis also rotate.
The stator is fixed, so *as*-axis the remains stationary.



Flux coupling
Notice that the lines of magnetic flux from the rotor winding partially enclose the stator windings.
The amount of this coupling (flux linkage) depends upon the position of the rotor.

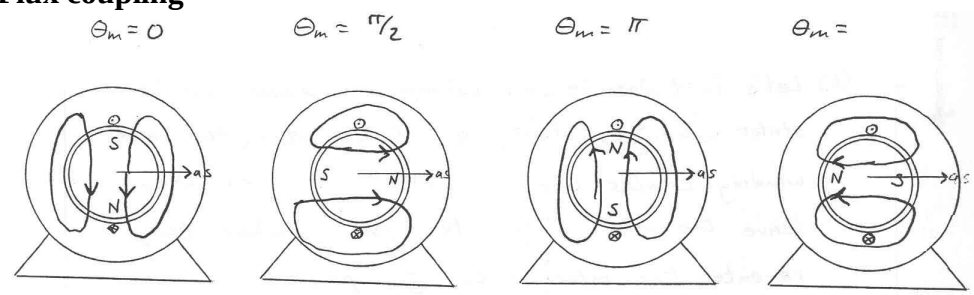
Rotor position angle



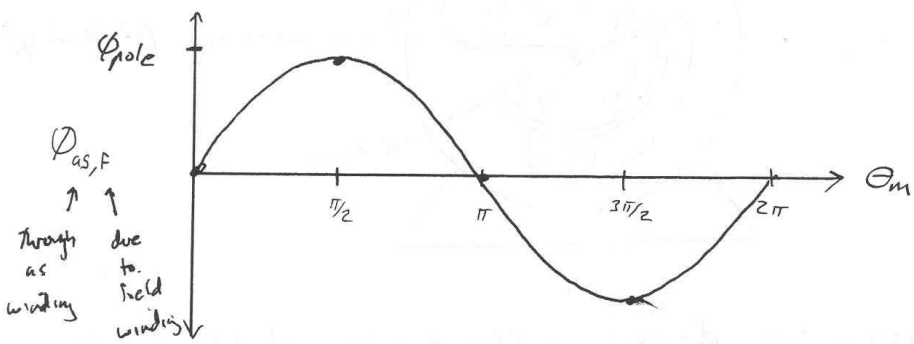
We define angle θ_m as the rotor position angle between q-axis and as-axis.

We would like to relate the amount of flux coupling $\phi_{as,f}$ as a function of θ_m.

Flux coupling



no coupling | maximum positive coupling | no coupling | maximum negative coupling



We represent the flux linkage ($\phi_{as,f}$) between a-phase of the stator and the rotor windings as

$\phi_{as,f} =$

Induced voltage

Since the flux is time-varying, it follows from Faraday's law that a voltage (e_{as}) will result

$$e_{as} = N_s \frac{d}{dt}\left(\phi_{as,f}\right)$$

Since the flux is given $\phi_{as,f} = \phi_{\text{pole}} \sin \omega_m t$ it follows that

$$e_{as} = N_s \frac{d}{dt}\left(\phi_{\text{pole}} \sin \omega_m t\right)$$

$=$

peak phase voltage is function of

1.
2.
3.

Rotor angular velocity

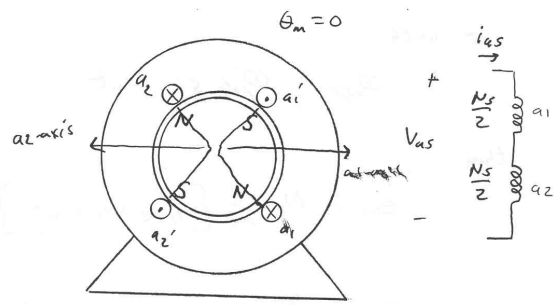
In this machine, the induced voltage (e_{as}) has a frequency (or) which is the same as the angular velocity of the rotor (ω_m).

If we desire f_e = 60-Hz ac, what must the rotor speed be in RPM (N_m)?

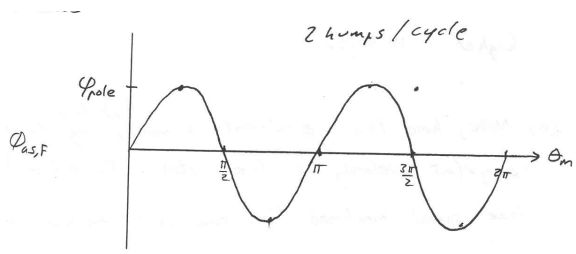
How might we change the rotor speed and still produce 60 Hz power?

Four pole machine

Consider the four pole (P =) machine below.



What does the flux linkage ($\phi_{as,f}$) look like now?



The flux linkage ($\phi_{as,f}$) is now _____ the frequency. $\phi_{as,f} = \phi_{\text{pole}} \sin 2\omega_m t$

Induced voltage (4 poles)

The induced voltages ($e_{as,1}$,$e_{as,2}$) are given $e_{as,1} = e_{as,2} = \frac{N_s}{2}\frac{d}{dt}\left(\phi_{\text{pole}} \sin 2\omega_m t\right) = \frac{N_s}{2}\phi_{\text{pole}} 2\omega_m \cos \omega_m t$

Because they are in series, the total voltage e_a is $e_{as} = e_{as,1} + e_{as,2} = 2N_s\phi_{\text{pole}}\omega_m \cos 2\omega_m t$

Induced voltage (*P* poles)

Generalizing this result to P poles (where P is an even integer ≥ 2), the total induced voltage (e_{as}) is given

$$e_{as} = e_{as,1} + e_{as,2} + \cdots + e_{as,2P} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2}\omega_m t\right)$$

Motor speed (multiple poles)

The relation between rotor angular velocity (ω_m) and that of the induced voltage (ω_e) is given by $\omega_e = \frac{P}{2}\omega_m$

Converting to electrical frequency (f_e) in hertz and motor speed in RPM, we have

$$2\pi f_e = \frac{P}{2}\left(N_m \frac{2\pi}{60}\right) \quad \Rightarrow \quad N_m = \qquad\qquad [\text{rpm}]$$

Three-phase generator

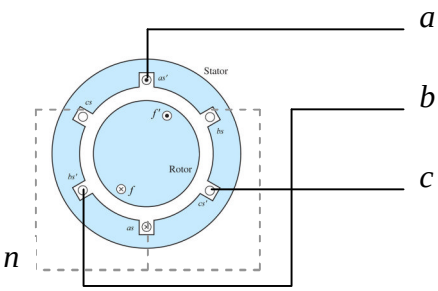
We have determined the induced voltage in just phase *a*. $e_{as} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2}\omega_m t\right)$

Since b and c are displaced by 120°, it follows that they will form a balanced set of voltages given

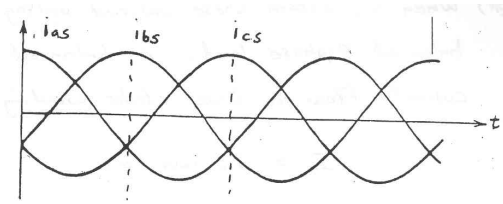
$$e_{bs} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2}\omega_m t - 120°\right)$$

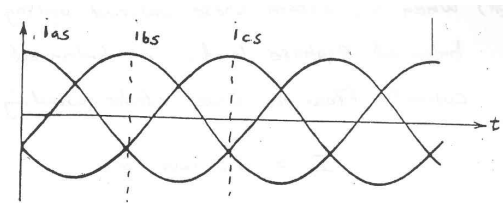
$$e_{cs} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2}\omega_m t - 240°\right)$$

Attaching these induced voltages to a balanced 3-phase load will produce balanced 3-phase currents given



$$I_{as} = I_m \cos \omega_e t$$
$$I_{bs} = I_m \cos(\omega_e t - 120°)$$
$$I_{bs} = I_m \cos(\omega_e t + 120°)$$



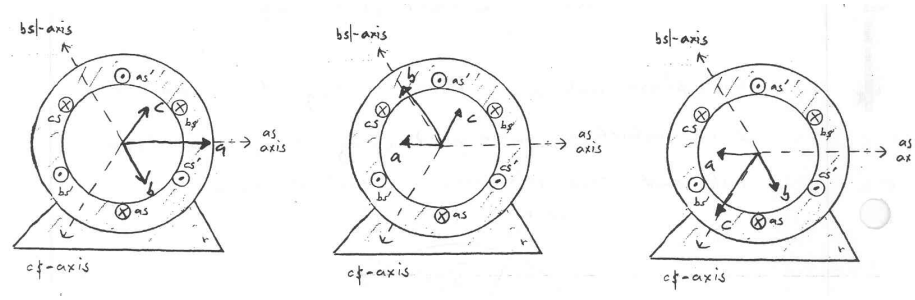


Stator magnetic field

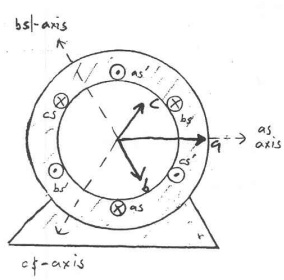
Now consider "driving" the stator windings with balanced 3-phase currents.

What does the resulting stator field look like?

- We will examine 3 instants in time.



We observe that "driving" the stator windings with balanced 3-phase currents produces a rotating magnetic field.



the resultant stator magnetic field rotates

$$N_{\text{stator field}} = \frac{120 f_e}{P} \quad \text{[rpm]}$$

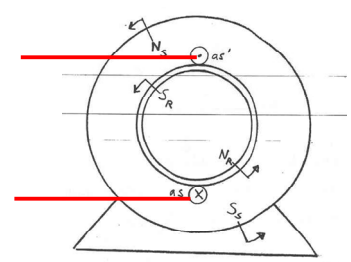
Per phase equivalent circuit

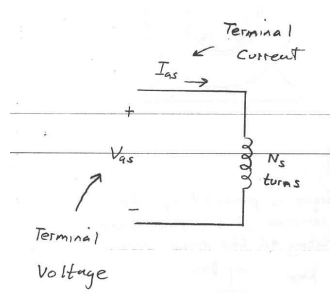
Looking into the *a* phase of the coil we see

- Coil resistance (r_s)
- An induced voltage due to rotor magnetic field ($e_{as,f}$)
- An induced voltage due to stator magnetic field ($e_{as,s}$)

Thus, the terminal voltage (v_{as}) is given

$$v_{as} = r_s I_{as} + e_{as,f} + e_{as,s}$$





Induced voltage $e_{as,f}$

We have previously developed an expression for the induced voltage due to the rotor magnetic field ($e_{as,f}$)

$$e_{as,f} = \frac{P}{2} N_s \phi_{\text{pole}} \omega_m \cos\left(\frac{P}{2}\omega_m t\right) = N_s \phi_{\text{pole}} \omega_e \cos\left(\frac{P}{2}\omega_m t\right)$$

Since ϕ_{pole} is directly proportional to the rotor field current I_f, we can re-write $e_{as,f}$ as

$$e_{as,f} = L_{sf} I_f \omega_e \cos\left(\frac{P}{2}\omega_m t\right)$$

Induced voltage $e_{as,s}$

The induced voltage due to the stator magnetic field ($e_{as,s}$) is modeled as $e_{as,s} = L_s \frac{d}{dt}(I_{as})$

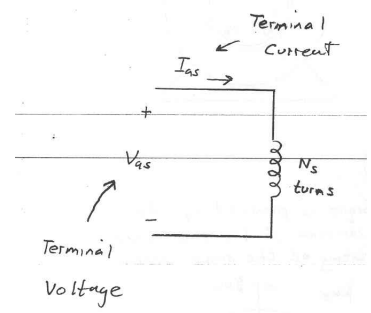
where inductance L_s takes into account the effect of the *b* and *c* phase currents.

Terminal voltage v_{as}

Combining the terms, we have

$$v_{as} = r_s I_{as} + e_{as,f} + e_{as,s}$$

$$= r_s I_{as} + L_{sf} I_f \omega_e \cos\left(\frac{P}{2}\omega_m t\right) + L_s \frac{d}{dt}(I_{as})$$



Transforming into the frequency domain

- where d/dt is replaced by $j\omega_e$ and $X_s = L_s\omega_e$

$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \tilde{E}_a + L_s j\omega_e \tilde{I}_{as}$$

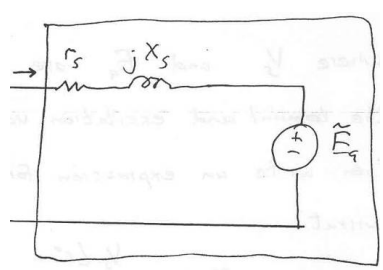
$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \tilde{E}_a + jX_s \tilde{I}_{as}$$

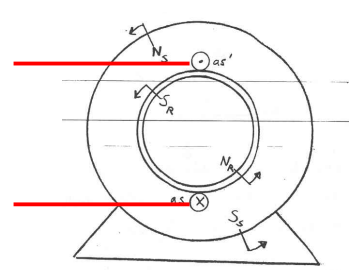
_____ voltage (line-to-neutral) _____ winding resistance _____or generated voltage synchronous reactance

Per phase equivalent circuit

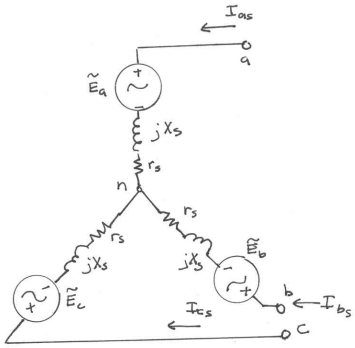
The per phase equivalent can thus be drawn

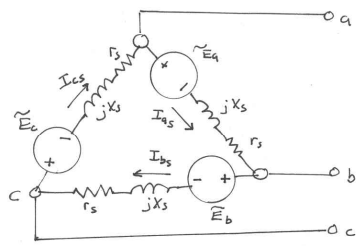
$$\tilde{V}_{as} = r_s \tilde{I}_{as} + \tilde{E}_a + jX_s \tilde{I}_{as}$$





This circuit would constitute one phase of a three-phase source (either Y or Δ connected).





Excitation voltage

The excitation (or generated) voltage is derived from the term $L_{sf} I_f \omega_e \cos\left(\frac{P}{2}\omega_m t\right)$

so it has an rms value of

$$\tilde{E}_a = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} \angle \delta$$

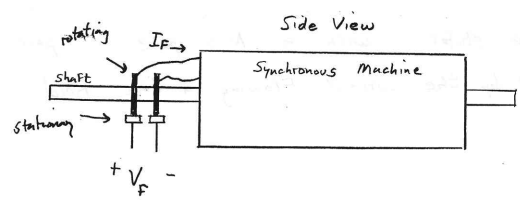
The angle of E_a is not necessarily 0° because it is related to the mechanical position, so we will call it δ, the _____ (or torque) angle such that

$$\tilde{E}_a = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} \angle \delta$$

Notice that the excitation voltage is a function of

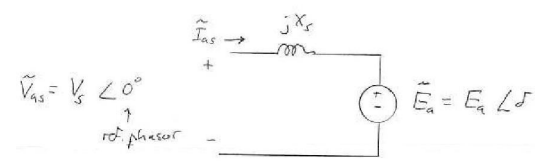
- the dc rotor field current I_f
- inductance L_{sf} which models flux linkage between the rotor and stator windings (a fixed quantity)
- the electrical frequency ω_e (a fixed quantity)

Therefore if we wish to control the excitation voltage (in turn the generator output voltage) we can do so by controlling the _____ I_f.



Power vs. angle δ

For large machines ($\eta > 99\%$), we can ignore ____ simplifying our model



We can solve for I_{as} $\quad \tilde{I}_{as} = \frac{\tilde{V}_{as} - \tilde{E}_a}{jX_s} = \frac{V_s\angle 0° - E_a\angle\delta}{jX_s} = \frac{V_s\angle 0°}{jX_s} - \frac{a}{jX_s} = \frac{s}{X_s}\angle -90° - \frac{a}{X_s}\angle(\delta - 90°)$

Taking the conjugate $\quad \tilde{I}_{as}^* = \frac{V_s}{X_s}\angle 90° - \frac{E_a}{X_s}\angle(90° - \delta)$

Complex power

The complex power for one phase (S_{1P}) is thus $\quad \mathbf{S}_{1P} = \tilde{V}_{as}\tilde{I}_{as}^* = V_s\angle 0°\left[\frac{V_s}{X_s}\angle 90° - \frac{E_a}{X_s}\angle(90° - \delta)\right]$

$$= \frac{V_s^2}{X_s}\angle 90° - \frac{V_s E_a}{X_s}\angle(90° - \delta)$$

Applying Euler's identity

$$\mathbf{S}_{1P} = P_{1P} + jQ_{1P} = \frac{V_s^2}{X_s}\left[\cos 90° + j\sin 90°\right] - \frac{V_s E_a}{X_s}\left[\cos(90° - \delta) + j\sin(90° - \delta)\right]$$

$$= j\frac{V_s^2}{X_s} - \frac{V_s E_a}{X_s}\left[\sin(\delta) + j\cos(\delta)\right]$$

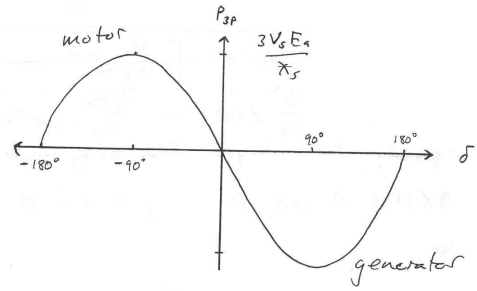
Real power

Grouping the real and imaginary components

$$\mathbf{S}_{1P} = P_{1P} + jQ_{1P} = -\frac{V_s E_a}{X_s}\sin(\delta) + j\left[\frac{V_s^2}{X_s} - \frac{V_s E_a}{X_s}\cos(\delta)\right]$$

The total real power P_T dissipated by our machine is given $P_{3P} = 3P_{1P} = -3\frac{V_s E_a}{X_s}\sin(\delta)$

which is a function of the power angle δ.



Power vs. angle δ

Plotting power as a function of δ we obtain the following

What does the power angle δ physically represent?

Power angle $\delta = 0°$

With no load, the rotor's net magnetic field aligns exactly with the stator's rotating magnetic field.

- With no load, no power is being dissipated and torque exerted on the rotor is _____.

Power angle $\delta < 0°$ (_____)

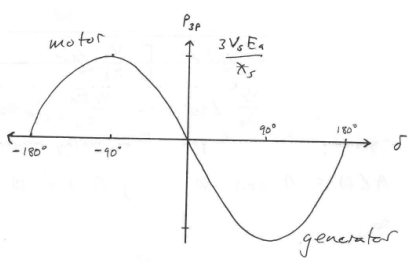
With a load, the rotor's net magnetic field _____ behind the stator's rotating magnetic field.

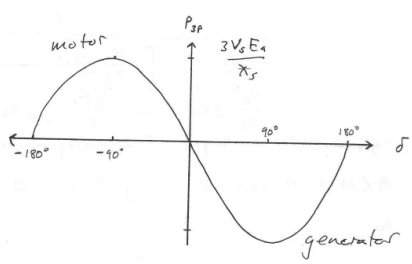
- The angular difference results in a positive torque, hence positive power dissipation.

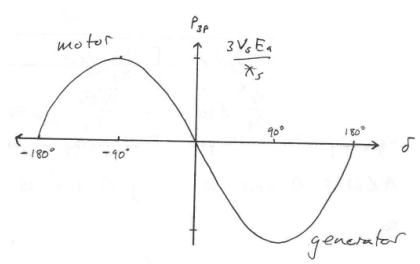
Power angle $\delta > 0°$ (_____)

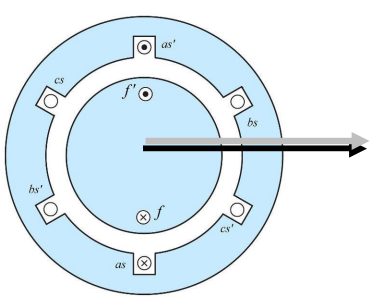
With a negative load (prime mover driving the rotor), the rotor's net magnetic field _____ stator's rotating magnetic field.

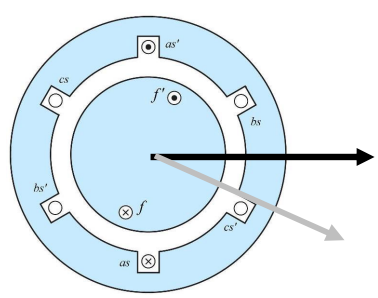
- The angular difference results in a negative torque, hence negative power dissipation.

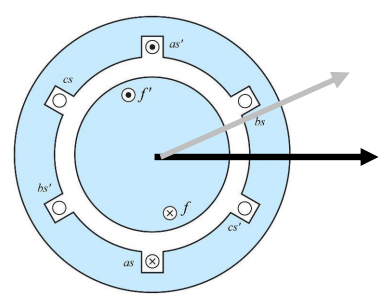










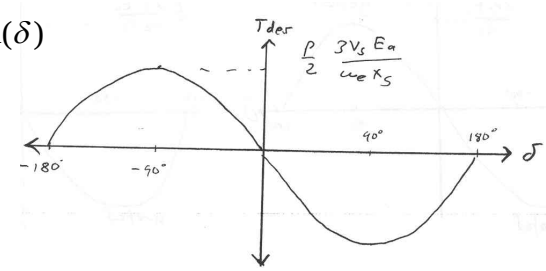


Torque

Since we have assumed 100% efficiency the electrical power must equal the mechanical power at the shaft

$$P_{3P} = -3\frac{V_s E_a}{X_s}\sin(\delta) = P_{\text{mech}} = T_e \omega_m \qquad \text{thus} \qquad T_{\text{dev}} = \frac{P_{\text{mech}}}{\omega_m}$$

Since $\omega_m = \frac{2}{P}\omega_e$ we conclude that $T_{\text{dev}} = -\frac{P}{2}\frac{3V_s E_a}{\omega_e X_s}\sin(\delta)$

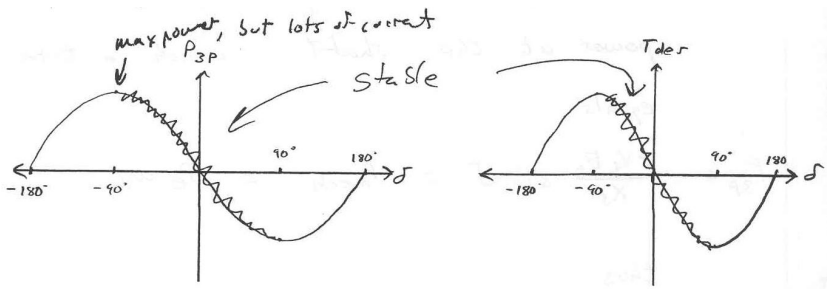


Torque vs. angle δ

Plotting torque as a function of δ we obtain the following

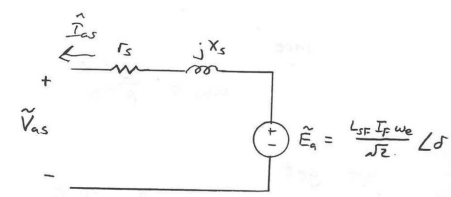
Operating points

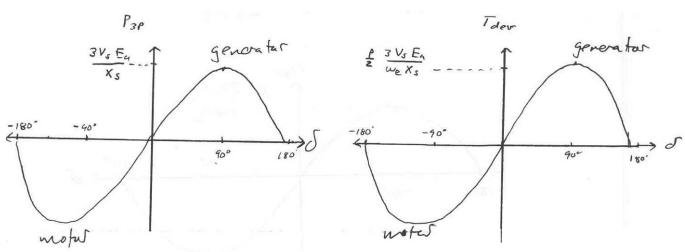
For both characteristics, the stable operating points reside on the negative sloped portions of the curves.



Changing reference current

If we define the reference current as leaving the machine, the our curves are inverted





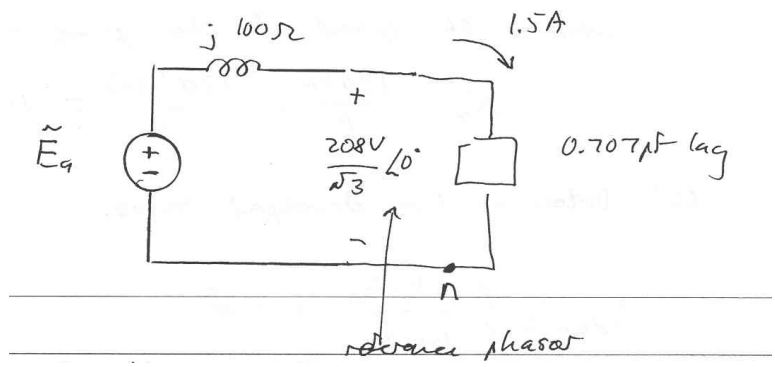
Example Problem 1

If reduction gear connected to a gas turbine spins a generator at 1200 rpm and we desire electricity generated at 60 Hz, how many poles *P* will our generator require?

Example Problem 2

A 208-Vrms (line), 60 Hz, Y-connected generator has Xs= 100 W, rs = 0 W. At rated terminal voltage, the machine delivers 1.5-Arms (line) to a 0.707 pf lagging load.

a. Determine the machine current.



b. Determine the excitation (or generator) voltage E_a.

b. Determine the power angle or torque angle δ.

d. If the generator has 4 poles, what is the speed of the prime mover.

e. Determine the developed torque.

f. Suppose the load current increases to 3-Arms at a 0.707 lagging pf. If we have the same terminal voltages, how must the field current change?