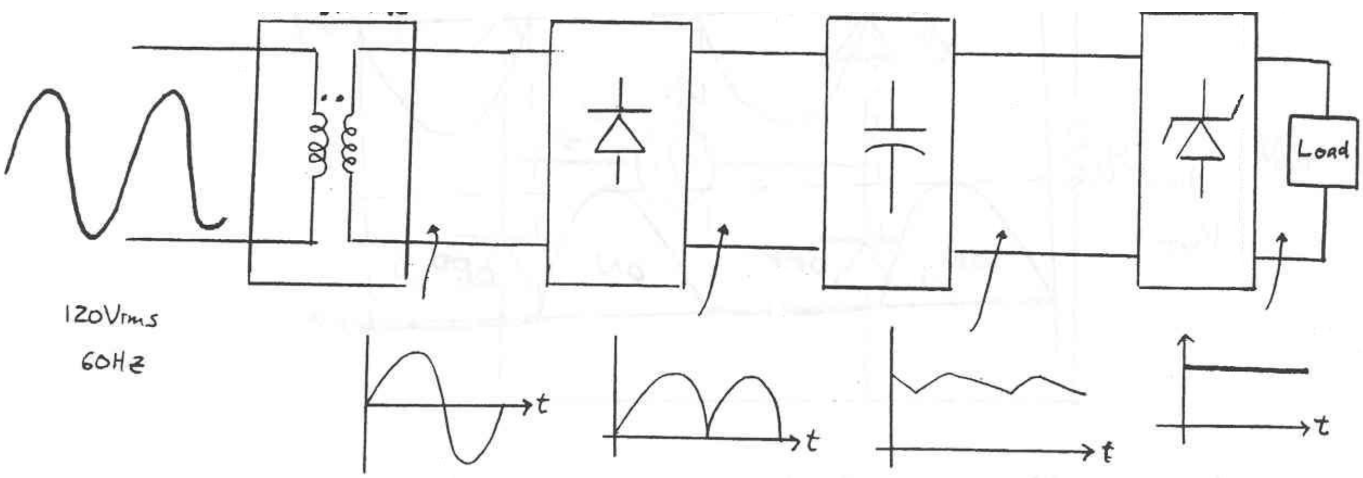
Lesson 33: Rectifiers and Filtering

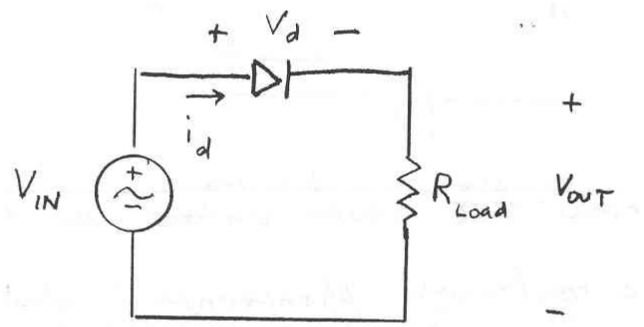
Overview

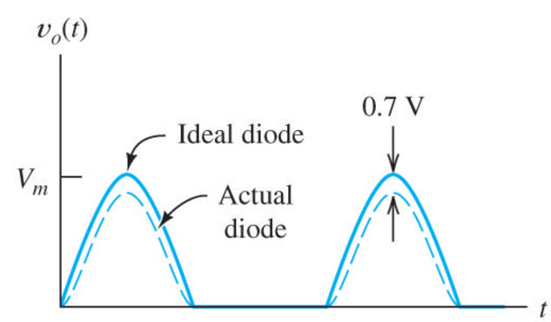
Our ultimate goal is to construct a dc power supply depicted by the block diagram below.



Half-wave rectifier review

Previously we examined the half-wave rectifier.





average output voltage $v_{\text{out,ave}} =$

rms output voltage $v_{\text{out,rms}} =$

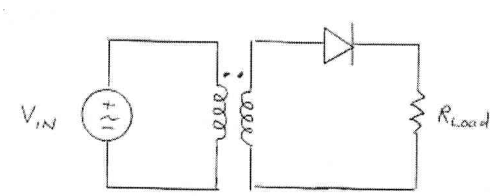
Non-ideal effects

We also coupled it with a transformer to create a more realistic model which included diode voltage drop and transformer resistance.

$$v_{\text{sec,pk}} = \left(v_{\text{sec,pk,NL}} - 0.7\right)\frac{R_{\text{Load}}}{R_{\text{Load}} + R_{\text{Tran}}}$$

$$v_{\text{out,ave}} = \frac{v_{\text{sec,pk}}}{\pi}$$

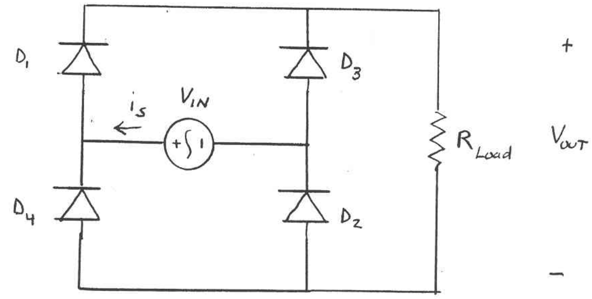
$$v_{\text{out,rms}} = \frac{v_{\text{sec,pk}}}{2}$$



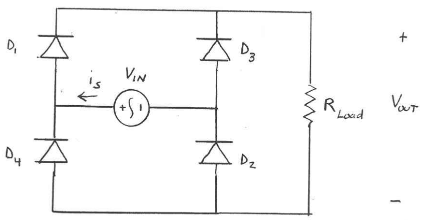
Full-wave rectifier

Let us consider the following rectifier circuit.

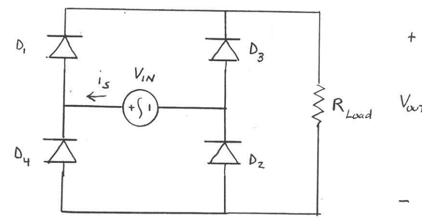
- Use the assumed diode states method of analysis.



There are two possible paths for the source current i_s.



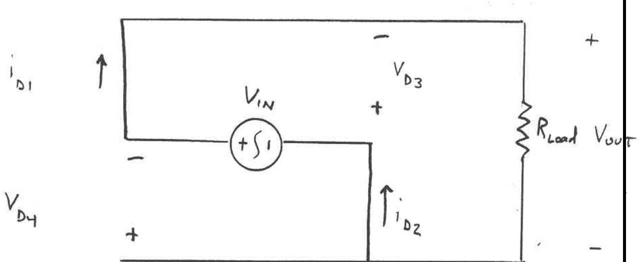
when i_s is positive



when i_s is negative

Step 1

Assume D_1 and D_2 are "on," D_3 and D_4 are "off."

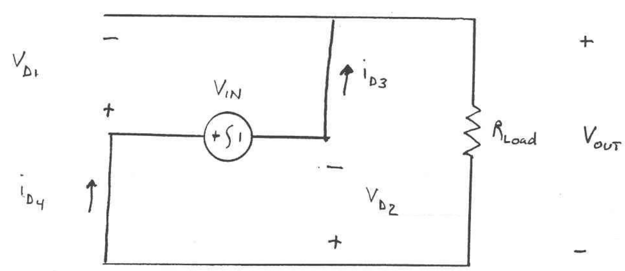


- Consider i_{D1}, i_{D2} and v_{D4}, v_{D3} when $v_{in} = v_{pk} \sin \omega t$.
- When are our assumptions true?

Our assumptions are true when v_{in} ________.

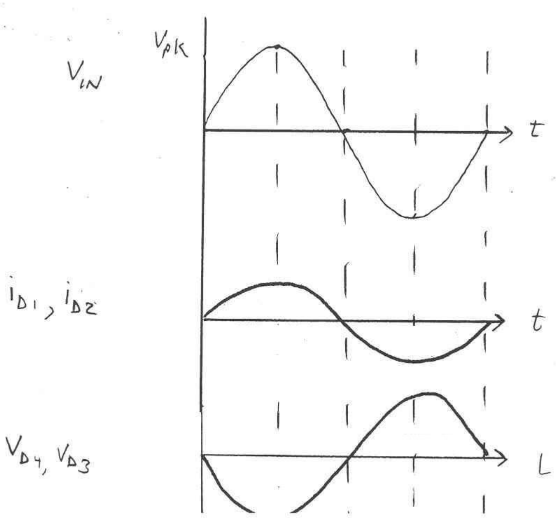
Step 2

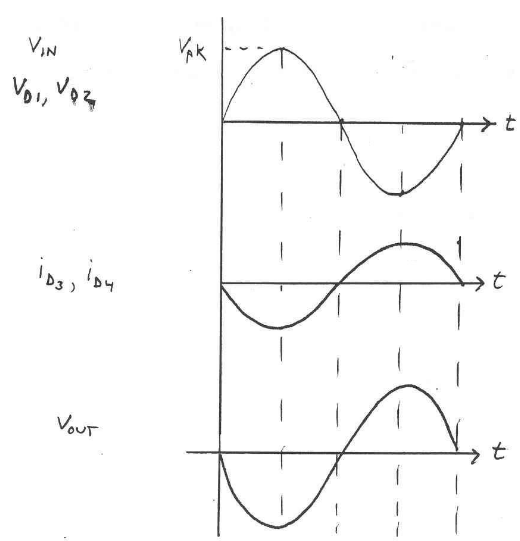
Assume D_3 and D_4 are "on," D_1 and D_2 are "off."



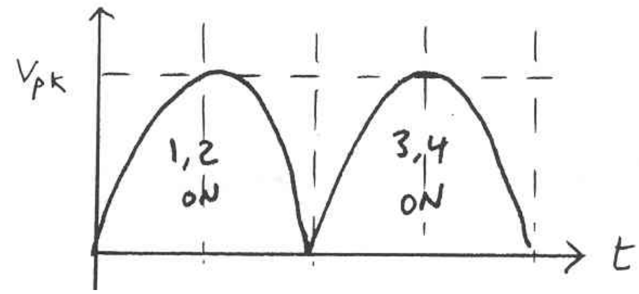
- Consider i_{D3}, i_{D4} and v_{D1}, v_{D2} when $v_{in} = v_{pk} \sin \omega t$.
- When are our assumptions true?

Our assumptions are true when v_{in} ________.





Step 3
Combining these results, we determine that v_{out} is given



What are $v_{out,ave}$ and $v_{out,rms}$?

Full-wave rectified sinusoid
The average and rms values for a full-wave rectified sinusoid are given

$$v_{\text{out,ave}} = \frac{1}{\pi}\int_0^{\pi} v_{\text{pk}} \sin\theta \, d\theta = \frac{v_{\text{pk}}}{\pi}\left(-\cos\theta\right)\Big|_0^{\pi} =$$

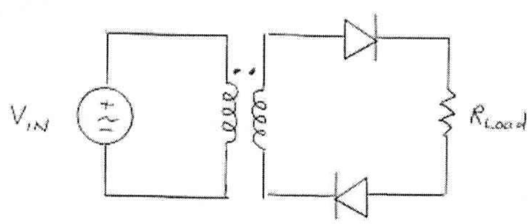
$$v_{\text{out,rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left(v_{\text{pk}} \sin\theta\right)^2 d\theta} =$$

The average power delivered to the load is given $p_{\text{load,ave}} = \dfrac{v_{\text{out,rms}}^2}{R_{\text{load}}} =$

Non-ideal effects
As we did previously, our model should include

- Voltage drops of the conducting both diodes (now totaling ~1.4 V)
- Output resistance of transformer coil.

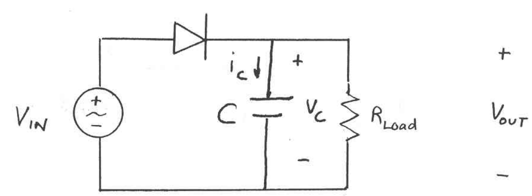
These two factors reduce the peak secondary voltage ($v_{\text{sec,pk}}$) from that of the ideal case.



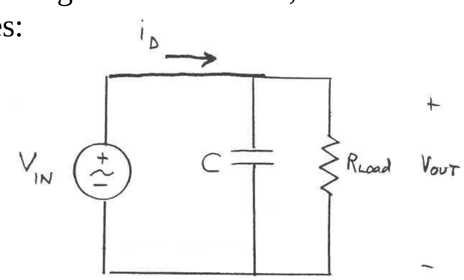
$$v_{\text{pk}} = \left(v_{\text{sec,pk,NL}} - 1.4\right)\frac{R_{\text{load}}}{R_{\text{load}} + R_{\text{tran}}}$$

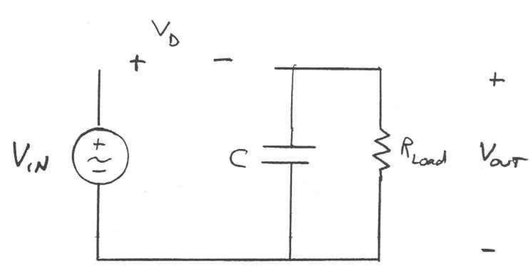
Filtering
The goal of our power supply is to convert an ac signal into _________ dc.
An unfiltered, rectified signal would not be tolerated by most devices expecting a dc voltage.

Let us consider a half-wave rectifier combined with a capacitor across the load.

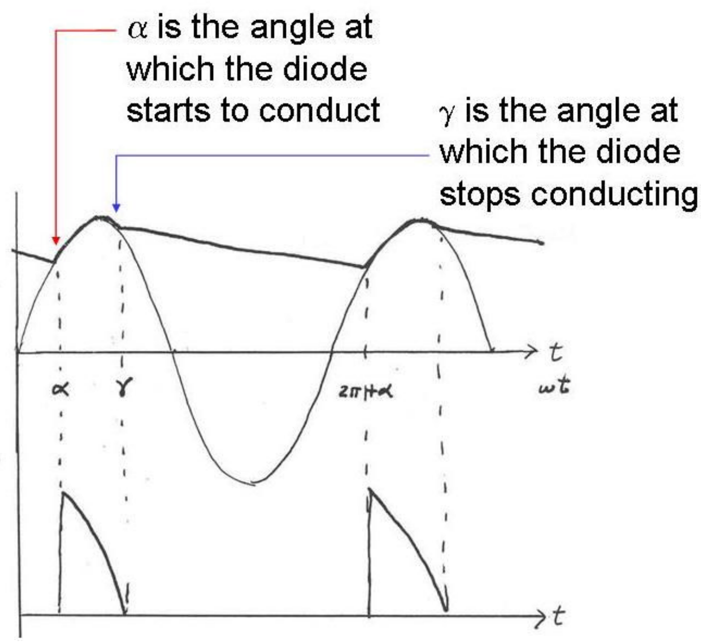


Assuming an "ideal" diode, the circuit has two possible states:





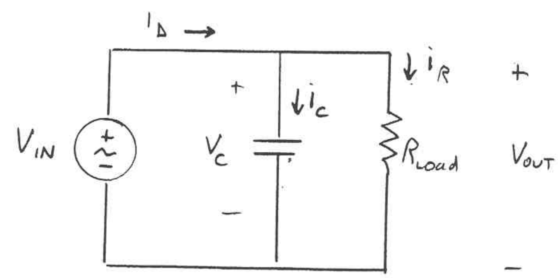
diode "on," capacitor _____________ diode "off," capacitor _____________



Step 1: diode “on”

Analyze the circuit when the diode is assumed “on” and determine when $i_d > 0$.

With diode “on” all three components are in parallel, so



$$v_{\text{in}} = v_{\text{pk}} \sin \omega t = v_{\text{pk}} \sin \theta$$

Determine an expression for i_d

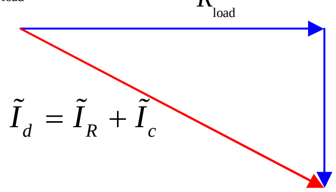
Analyze the circuit when the diode is assumed “on” and determine when $i_d > 0$.

time domain | phasor domain

$$i_R = \frac{v_{\text{in}}}{R_{\text{load}}} = \frac{v_{\text{pk}}}{R_{\text{load}}} \sin \omega t \quad \Rightarrow \quad \tilde{I}_R = \frac{v_{\text{pk}}}{R_{\text{load}}} \angle 0°$$

$$i_c = C \frac{dv_c}{dt} = v_{\text{pk}} C\omega \cos \omega t \quad \Rightarrow \quad \tilde{I}_c = v_{\text{pk}} C\omega \angle -90°$$

$$\theta = \tan^{-1}(\omega R_{\text{load}} C) \qquad \tilde{I}_R = \frac{v_{\text{pk}}}{R_{\text{load}}}$$



$$\tilde{I}_c = -v_{\text{pk}} C\omega$$

$$\tilde{I}_d = \tilde{I}_R + \tilde{I}_c$$

$$|\tilde{I}_d| = \sqrt{\left(\frac{v_{\text{pk}}}{R_{\text{load}}}\right)^2 + (v_{\text{pk}} C\omega)^2}$$

The time domain expression for i_d is given

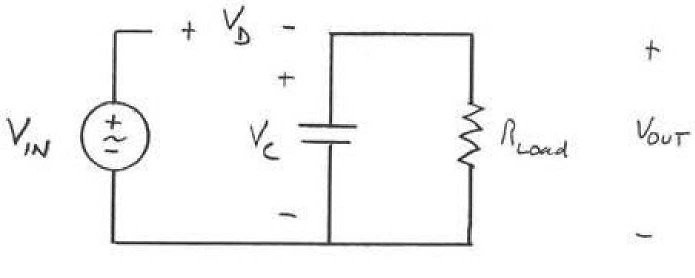
$$i_d = v_{\text{pk}} \sqrt{\left(\frac{1}{R_{\text{load}}}\right)^2 + (C\omega)^2} \sin\left[\omega t + \tan^{-1}(\omega R_{\text{load}} C)\right]$$

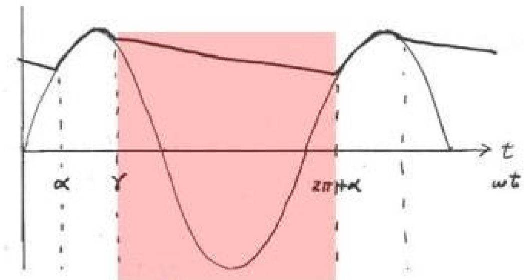
The diode conducts until this current goes to zero $\quad i_d = 0 \quad \rightarrow \quad \omega t + \tan^{-1}(\omega R_{\text{load}} C) = 0, \pi, 2\pi \ldots$

We defined this angle $\omega t = \gamma$, thus $\gamma =$

Step 2: diode “off”

Analyze the circuit when the diode is assumed “off” and determine when $v_d < 0$.





Determine an expression for v_c

- We need the initial voltage at $\omega t = \gamma$ when the diode turns off.

The initial voltage v_{i} is v_{i} = and the capacitor voltage v_c is given $v_c = v_{\text{i}} e^{-\left(\frac{t-t_\gamma}{RC}\right)} = v_{\text{i}} e^{-\left(\frac{\theta-\gamma}{RC}\right)}$

The diode voltage v_d is v_d =

The diode does not conduct until $v_d = 0$ when $\theta = 2\pi + \gamma$

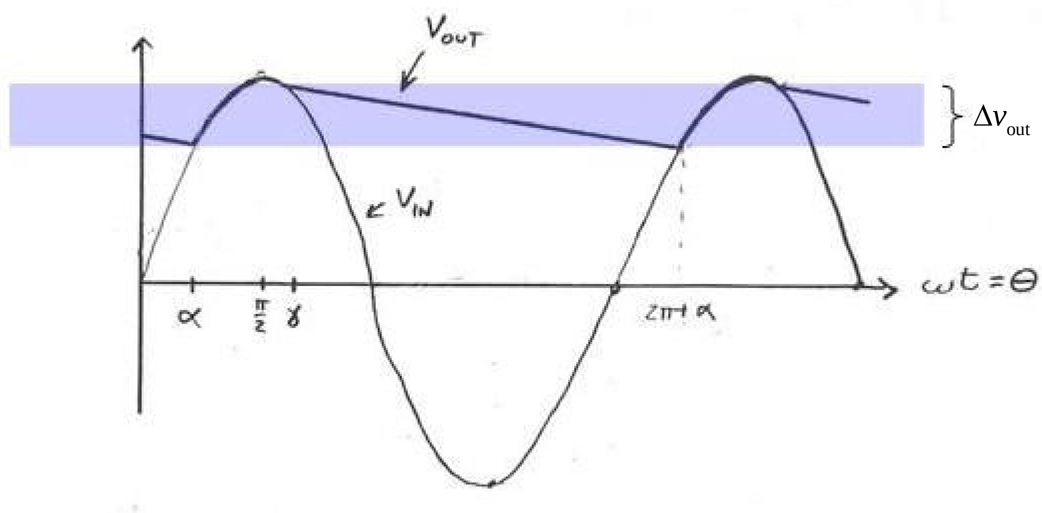
$$v_d = v_{\text{pk}} \sin(2\pi + \alpha) - v_{\text{pk}} \sin(\gamma) e^{-\left(\frac{2\pi+\alpha-\gamma}{\omega R_{\text{load}} C}\right)} = 0$$

This equation can be solved numerically (calculator) for α

Step 3

Once γ and α are know, the magnitude of the capacitor voltage ripple can be determined

$$\Delta v_{\text{out}} =$$

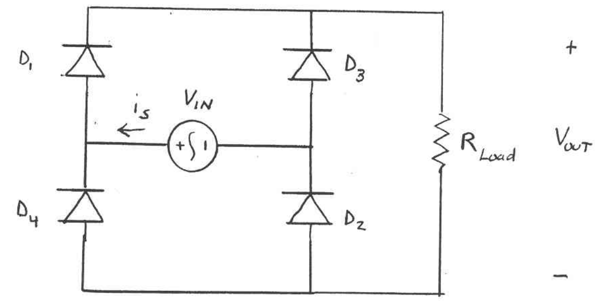


The average value is close to $\Delta v_{\text{out,ave}} = v_{\text{pk}} - \frac{v_{\text{out}}}{2}$

$$= \frac{1}{2}\left(v_{\text{pk}} + v_{\text{pk}} \sin\alpha\right)$$

Example Problem 1

We would like to deliver 2 W to a 20-Ω load through a transformer with a 2-Ω output resistance. Determine the no-load RMS value of the transformer secondary assuming a full-wave rectifier is used.



Example Problem 2

If R_{load} = 500 Ω, C = 100 μF, ω = 377 rad/sec and v_{pk} = 15 V, determine the ripple and average voltage.