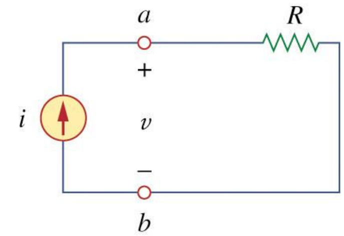
Lesson 7: Network Analysis 2: Thévenin's theorem



Linear circuits

Linear circuits are circuits composed of elements with a linear voltage-current (v-i) characteristic.
Resistors are an example of a linear element.

Requirements for linearity

Homogeneity (scaling) - Multiplying the excitation i by a constant k results in the response v also being scaled by k. if $v = iR$ then $kv =$

Additivity – the response to a sum of inputs is the sum of the responses to each input applied separately.

$$\text{if } i = i_1 + i_2 \text{ and } v_1 = i_1 R \text{ and } v_2 = i_2 R, \text{ then}$$
$$v = iR = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

Superposition

A direct result of additivity is principle of superposition.
Superposition states that the voltage across (or current through) an element in a linear circuit is the ________________________ of the voltages across (or currents through) that element due to each independent source acting alone.

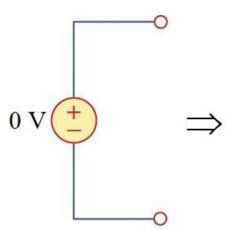
Applying superposition

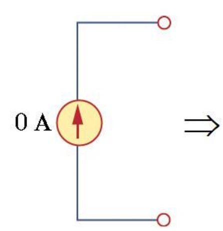
1. _______________ all but one __________________ sources and determine the output (voltage or current) due to that source (leave dependent sources intact).
2. Repeat for each of the other independent sources.
3. __________ the individual contributions to determine the total response.

"Turning off" sources

"Turning off" a source means setting its value equal to ________.

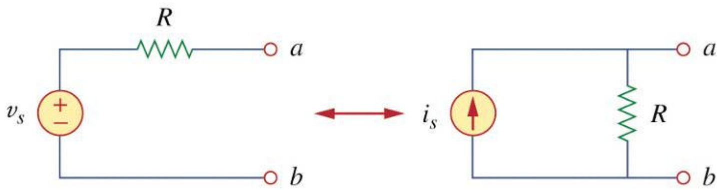
- Voltage sources – 0 V is equivalent to a _________________.
- Current sources – 0 A is equivalent to a _________________.





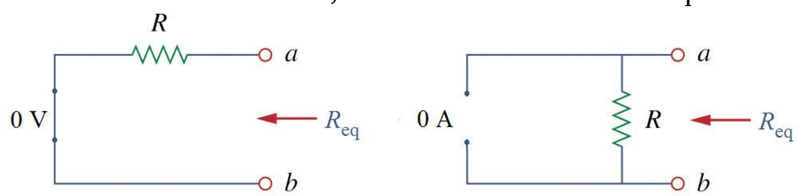
Source transformation

Source transformation is another circuit analysis technique that allows us to replace a voltage source v_s in ____________ with a resistor R by a current source i_s in ___________ with a resistor R, or vice versa.

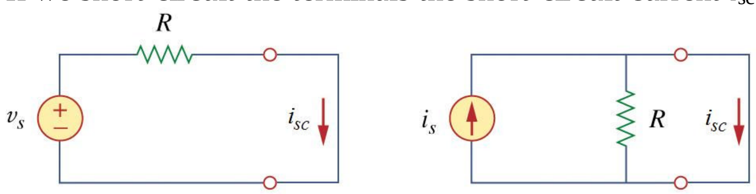


The two circuits are equivalent provided they have the same voltage-current characteristic at terminals *a*-*b*.

If we turn off each source, note the both circuits an equivalent resistance R_{eq}.



If we short-circuit the terminals the short-circuit current $i_{sc} = v_s/R$.



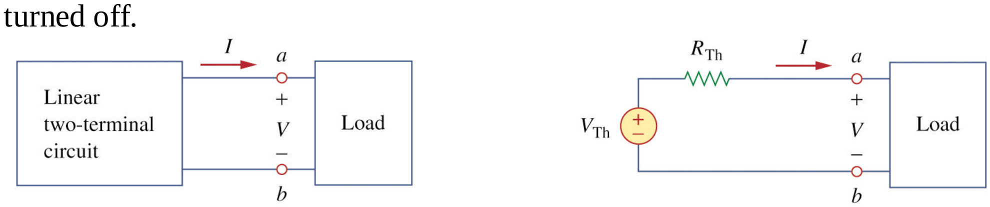
Thus, if _____________ the two circuits are equivalent.

Thévenin's theorem

Thévenin's theorem greatly simplifies analysis of complex linear circuits by allowing us to replace all of the elements with a combination of just one voltage source and one resistor. Thévenin's theorem is another power tool.

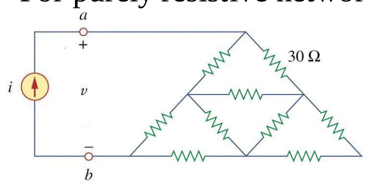
Thévenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a ____________________ V_{Th} in series with a ______________ R_{Th} where

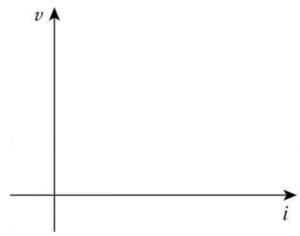
- V_{Th} is the open circuit voltage at the terminals, and
- R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

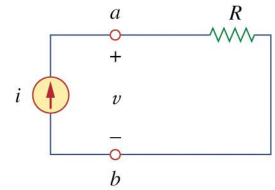


Equivalence

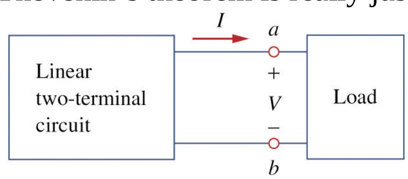
Two circuits are said to be equivalent if they have the same voltage-current (v-i) relationship. For purely resistive networks we've found R_{eq}.

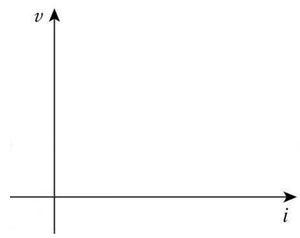


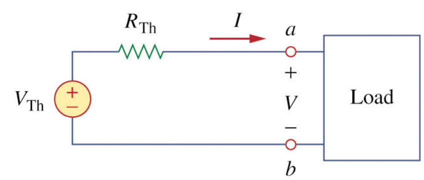




Thévenin's theorem is really just an expansion to include networks with sources.

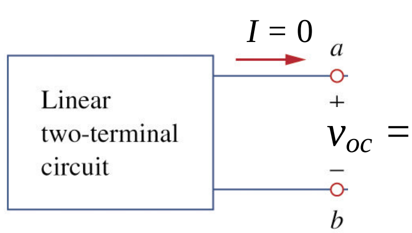


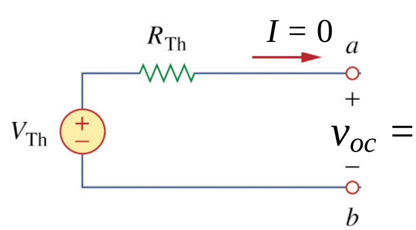




Determining V_{Th}

Referring to the plot, V_{Th} is voltage response when i = 0, therefore we just need to ____________ _________________ (open-circuit) and measure the resulting voltage.

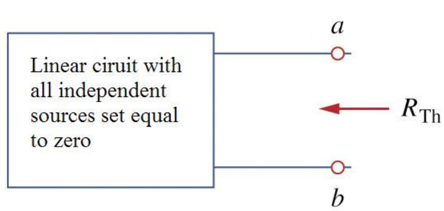


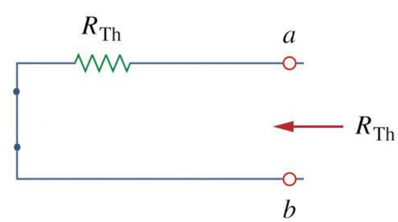


Determining R_{Th}

With the load disconnected, turn off all independent sources.

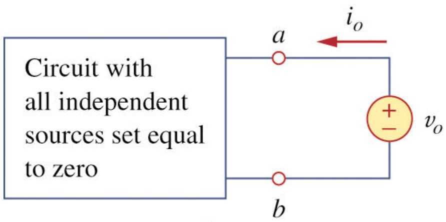
R_{Th} is equivalent resistance looking into the "dead" circuit through terminals a-b.



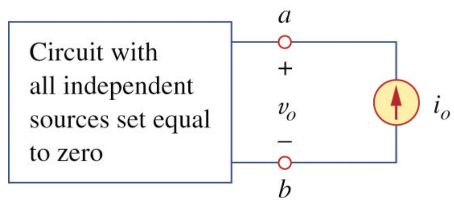


Case 1: If no ____________________ sources exist, find R_{Th}.

Case 2: If the network contains dependent sources, apply any voltage at v_o and determine resulting i_o (or vice-versa). Thus, $R_{Th} = v_o / i_o$.



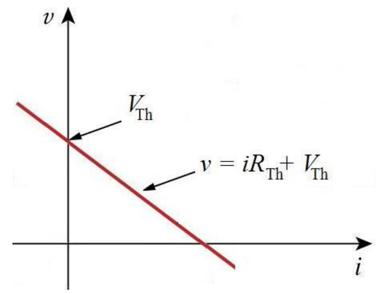
$$R_{Th} = \frac{v_o}{i_o}$$



$$R_{Th} = \frac{v_o}{i_o}$$

Negative R_{Th}

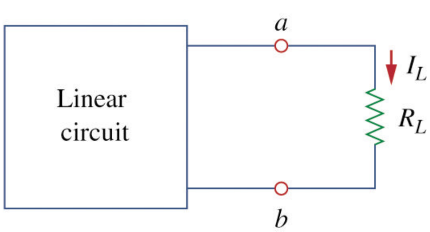
With dependent sources present, it is possible that R_{Th} will be negative, implying that the circuit is providing power.

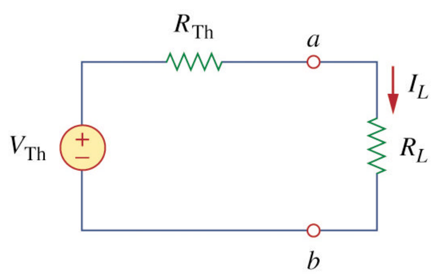


Applying Thévenin equivalent

Once V_{Th} and R_{Th} have been found, the original circuit is replaced by its equivalent and solving for I_L and V_L becomes trivial.

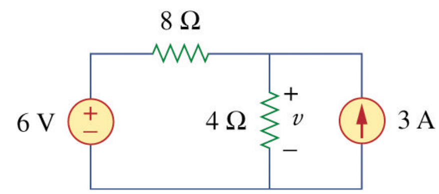
$I_L =$ $V_L =$





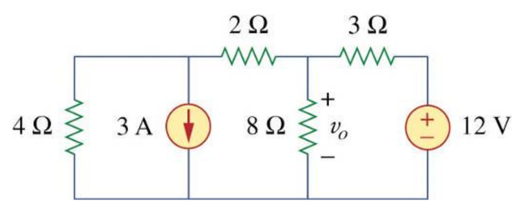
Example Problem 1

Use superposition to find v in the circuit.



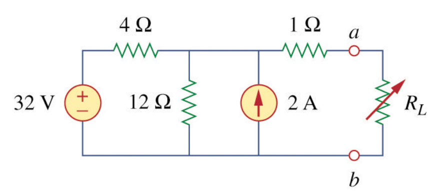
Example Problem 2

Use source transformation to determine v_o.



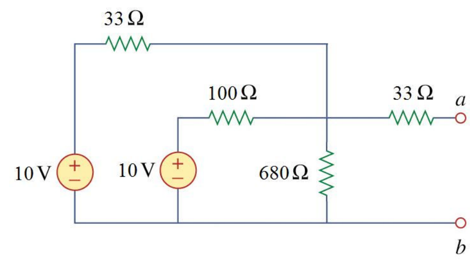
Example Problem 3

Find the Thévenin equivalent circuit to the left of terminals *a*-*b*. Then find the current through R_L = 6, 16, and 36 Ω.



Example Problem 4

Find the Thévenin equivalent of the circuit at terminals *a*-*b*.



Example Problem 5

Find the Thévenin equivalent of the circuit at terminals *a*-*b*.