Lesson 8:
Network Analysis 3:
Maximum Power Transfer

Circuit analysis toolbox

Ohm's law

Series resistance

Voltage divider rule

Nodal analysis

Thévenin's theorem

KVL

Meters

Circuit analysis toolbox

KCL

Parallel resistance

Current divider

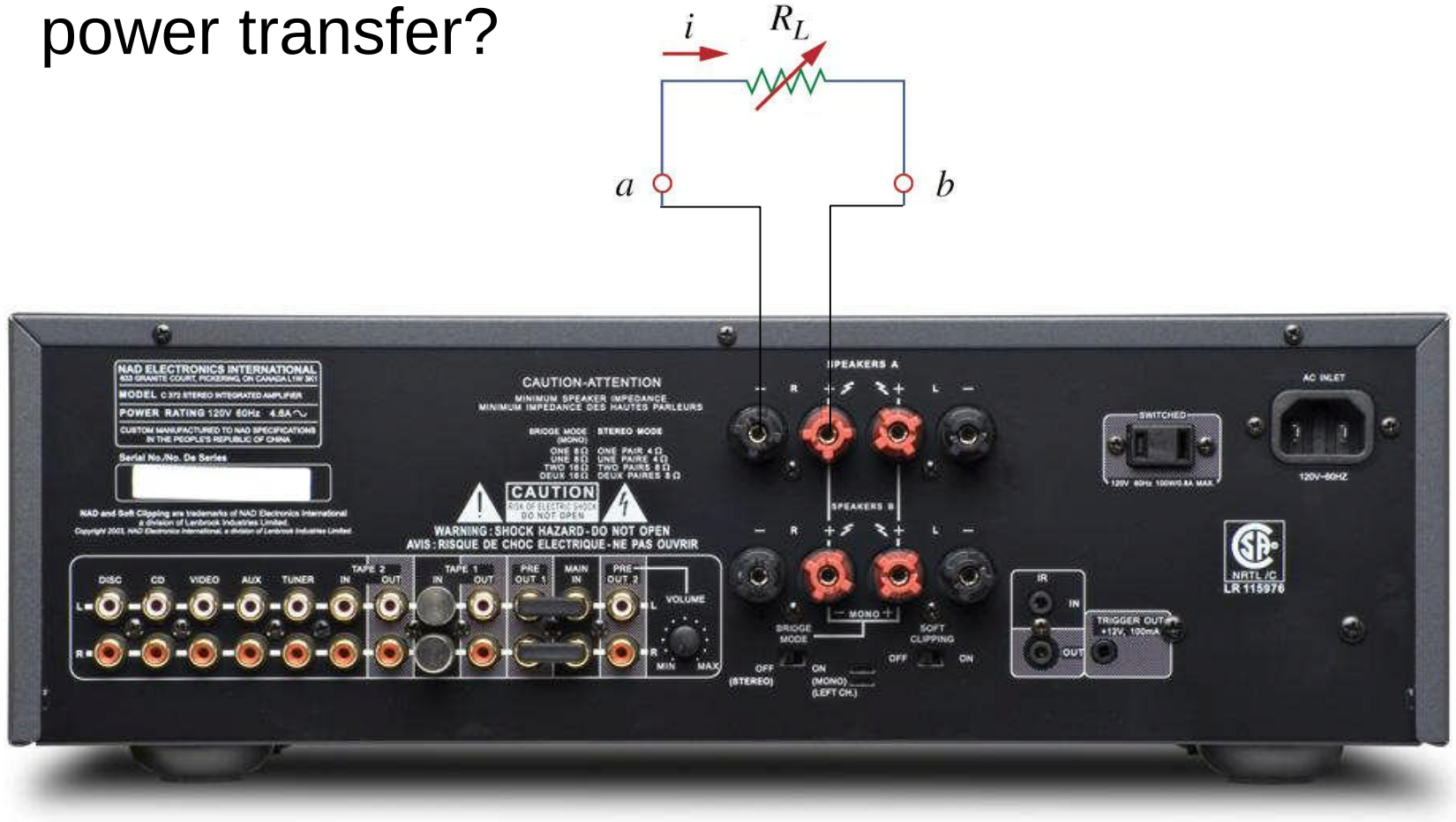
Source transformation

Maximum power transfer

- In some applications, the purpose of a circuit is to provide maximum power to a load.
  - Stereo amplifiers
  - Radio transmitters
- While analysis of the amplifier circuit may be daunting, Thévenin's theorem provides a simple method to examine this problem.

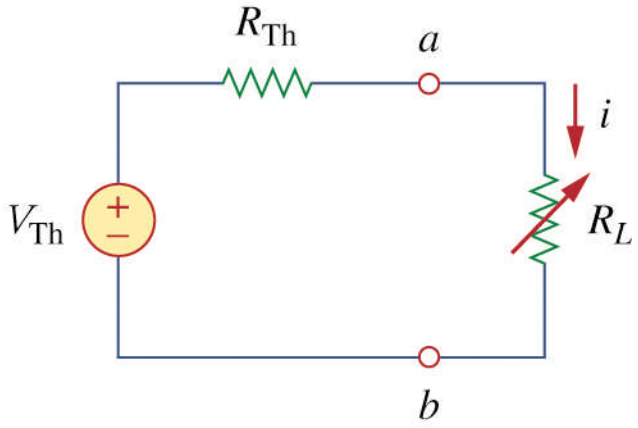
NAD C372 Stereo Integrated Amplifier

- What load should be connected to maximize power transfer?



Power to the load

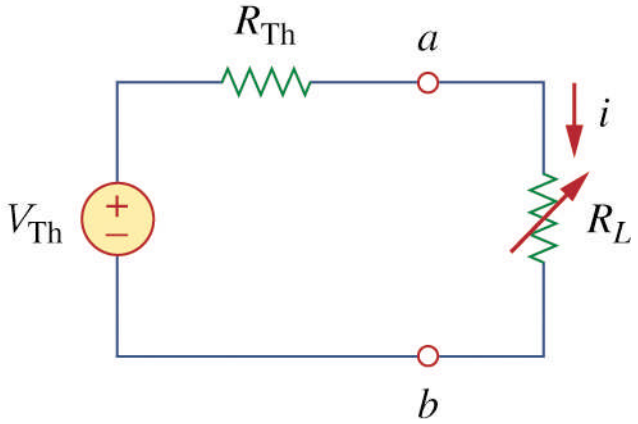
- We will replace the amplifier with a Thévenin equivalent.
- Consider the case that V_{Th} and R_{Th} are fixed and the load resistance R_L is variable.



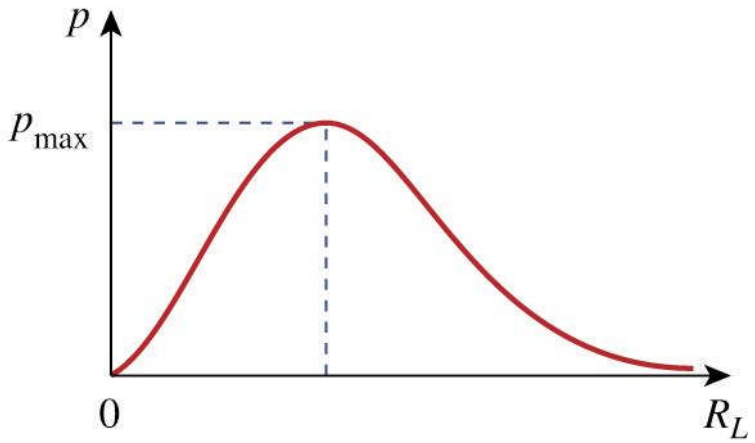
- What is the power p_L delivered to the load?

Maximizing p_L

- How might we determine R_L such that is p_L maximized?



$$p_L = i^2 R_L = \left(\frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} \right)^2 R_L$$



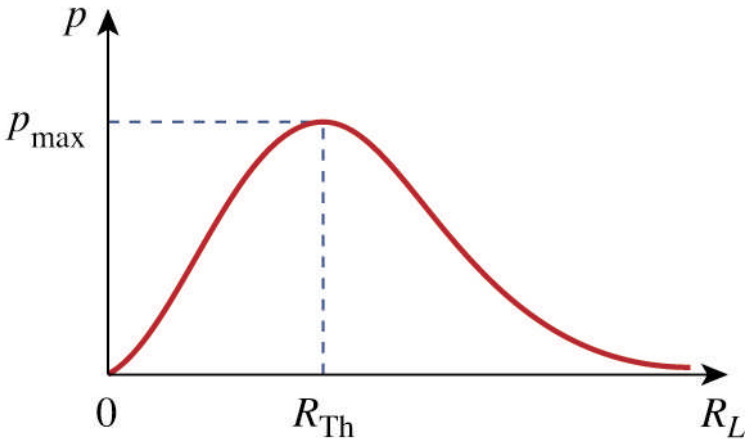
Maximizing p_L

- Taking the derivative of p_L and setting it equal to zero, we find that

$$\frac{dp_L}{dR_L} = \frac{d}{dR_L}\left[\left(\frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}\right)^2 R_L\right] = V_{\text{Th}}^2\left[\frac{R_{\text{Th}} - R_L}{\left(R_{\text{Th}} + R_L\right)^3}\right] = 0$$

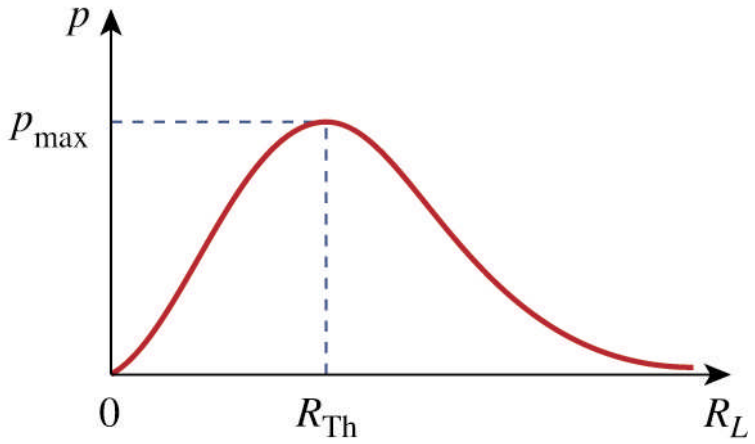
This implies $R_{\text{Th}} - R_L = 0$

which yields $R_L = R_{\text{Th}}$



Maximum power transfer theorem

- **Maximum power** is transferred to the load when the load resistance equals the Thévenin resistance as seen from the load ($R_L = R_{\text{Th}}$).
  - When $R_L = R_{\text{Th}}$, the source and load are said to be **matched**.



- Given $R_L = R_{\text{Th}}$, what is $p_{\max}$?

Maximum power

- The power delivered when $R_L = R_{\text{Th}}$ is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

- Note that this is not true when $R_L \neq R_{\text{Th}}$.
- What does this say about efficiency (η)?

$$\eta = \frac{p_{\text{out}}}{p_{\text{in}}} \times 100\%$$

Efficiency

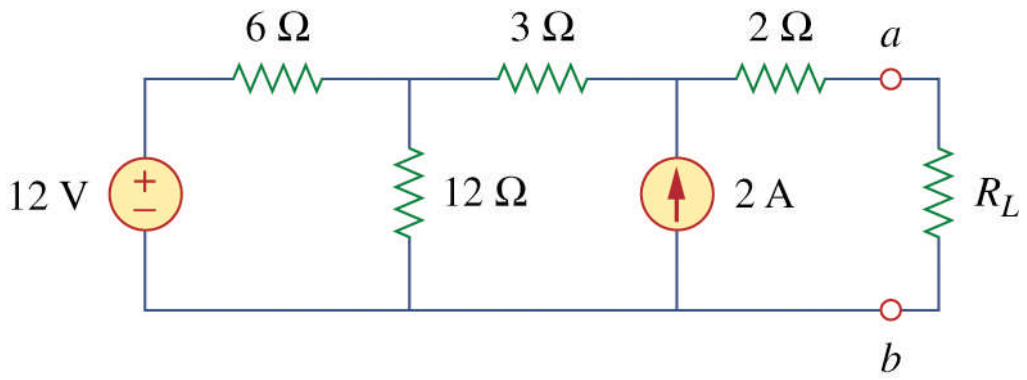
- When maximum power is delivered to R_L, the efficiency is a mere 50%.

$$\eta = \frac{p_{\text{out}}}{p_{\text{in}}} = \frac{i^2 R_L}{i^2 R_{\text{Th}} + i^2 R_L} \times 100\% = \frac{i^2 R_{\text{Th}}}{i^2 R_{\text{Th}} + i^2 R_{\text{Th}}} \times 100\% = 50\%$$

- Efficiency is maximized as $R_L \to \infty$.

Example Problem 1

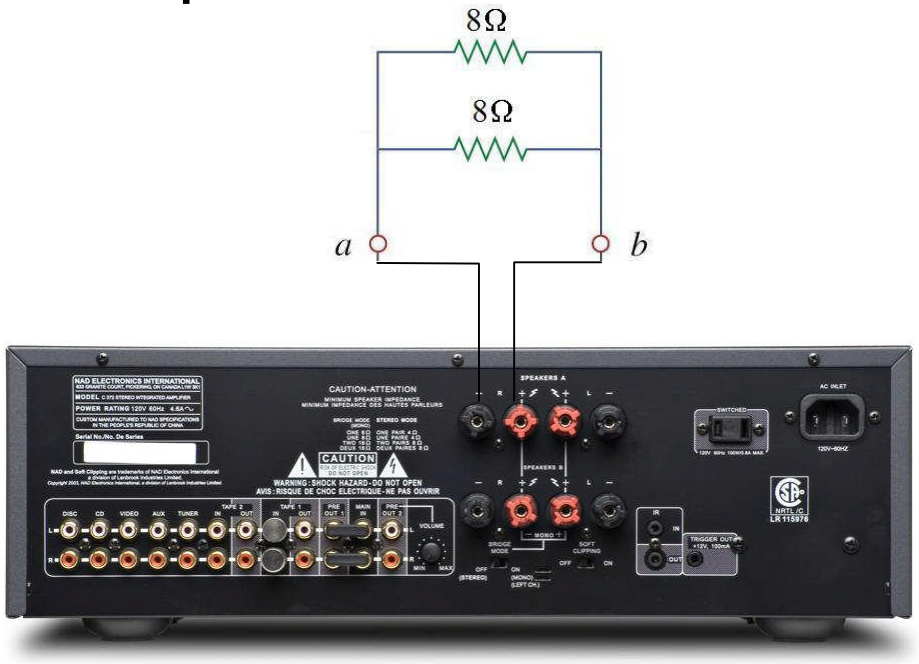
Find the value of R_L for maximum power transfer in the circuit below. Find the maximum power.



Example Problem 2

According to the NAD C372 specs, maximum output power is 150 W per channel when $R_L = 8\ \Omega$. What would the output power be if two $8\ \Omega$ speakers were connected in parallel to one of the channels?

PSB Image T65



NAD C372 specifications

Minimum impedance warning

- Note the caution about “minimum speaker impedance.”
- In stereo mode, the minimum impedance for one pair is 4-Ω and for two pairs is 8-Ω .