Models of malicious behavior in sponsored search

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Abstract

Search engines such as Google, Yahoo, and MSN now auction off search terms to potential advertisers. The potential advertisers place their bids on each search term of interest, as well as specifying a daily budget. Each search on this term displays an advertisement that is linked to the advertiser's website, and the advertiser pays the search engine every time the link is activated. When an advertiser's budget is reached, the search engine stops displaying their ad. This kind of advertising is extremely popular – the combined revenue of Yahoo and Google in 2005 was estimated at over 4.5 billion dollars. We develop small models which still have the property that malicious behavior such as bid-jamming still occurs as a rational best-response strategy. Such malicious behavior occurs frequently in practice. We are able to derive bidding strategies which are the best-responses when the budget of the bidder is low relative to her competitors, as well as strategies which protect against bid-jamming.

INTRODUCTION

Search engines such as Google, Yahoo, and MSN generate revenue by auctioning keywords or *adwords* to advertisers. Each search engine uses a slightly different mechanism for adword auctions. However, at a high level all the mechanisms are similar and can be abstracted and summarized as follows:

- For each adword the advertisers enter a *bid* which is the maximum amount they are willing to pay per click and the *daily budget*[1]. The bid can be updated, more or less continuously.
- The search engine uses some ranking function on the bids and (possibly) other bidder specific information to rank order of the advertisements to be displayed when the adword is queried.
- The payment that each advertiser pays the search engine each time its advertisement is clicked- is determined by some form of **next-price** scheme, i.e. the advertiser pays what the next highest ranked advertiser had bid.

One of the fundamental prescriptions guiding effective auction design is that they be truth-revealing, i.e. in equilibrium all bidders in the auction have the incentive to reveal truthfully their value for the goods [17]. Truth-revealing auctions have many nice properties. First, a bidder's optimal strategy is to bid her valuation. Second, the auctioneer is able to generate accurate information about the distribution of the bidder valuation. Unfortunately, adwords auctions as currently implemented by the search engines are not truth-revealing [11, 1]. However, under certain technical assumptions, including the restriction that the auction is single-round, Edelmann et. al. [11] and, independently, Varian [23] have shown the existence of a type of Nash Equilibrium. This work is elegant, and seems useful when bidders do not adjust their bid frequently. Goel and Motwani [1] and Lahaie [15] have extended this work to more general mechanisms in the single-round case.

Since the advertisers can dynamically change the bids, adword auctions are *multi*-round auctions. Moreover, the daily budget constraint ensures that the rounds are not *independent*. The multi-round nature of adword auctions and the fact that the single round auction is *not* truth-revealing lead to many malicious bidding strategies. **We develop simple multi-period models in which optimal bidder strategies exhibit some of these malicious behaviors.** In our setting, a Nash Equilibrium will be a bidding strategy for each bidder such that no bidder has a profit incentive to change their strategies. We refer to the Nash Equilibrium strategy for a particular bidder as the *weakly dominant* bid for that bidder (see Definition 1 below). An equivalent way to define Nash Equilibrium is the situation when every bidder is playing a weakly dominant strategy. We focus on a particular bidding behavior which we call *bid jamming*, and say that a bidder *jams* when she bids just below the next highest bidder. In a next-price setting, this forces the next highest bidder to pay the maximum possible amount and consequently her daily budget is consumed at the fastest possible rate. Thus, by jamming a bidder can very quickly force the next highest bidder to leave the auction and then win future clicks while bidding less as he no longer has to compete with the higher bidder. This exposes the chief weakness of second-price payments in a multi-period budgeted setting: ones bid does not determine

[1] Budgets are more typically given over sets of keywords, but in our model, we consider the case where the set has just one element.

what one pays, and, because of budgets, bidders have an incentive to force their opponents to pay as much as possible.

Another common malicious behavior associated with adword auctions is a "false" click. A "false" click occurs when a competitor clicks on a given bidder's advertisement only to drain out her budget. (There are now companies who are willing to take on this task for a given payment.) This behavior is often detectable – if caught in real time, the clicks are not charged and if caught after the fact, the search engine compensates the advertiser. Thus, advertisers are, in a sense, protected from "false"clicks. Bid jamming creates a greater challenge. First, the jamming behavior may be indistinguishable from normal behavior. Second, when jamming occurs, it is not clear how to compensate the bidder who was the victim of jamming. The search engine could partly compensate the victim by charging less for the clicks they won. However, jamming often forces the victim to drop out of auctions and in such a situation the appropriate compensation is not clear. Clearly, jamming and generating false clicks simultaneously generates a highly effective way of pushing a competitor out of the bidding.

Bid jamming has become a known and controversial part of sponsored search. There are software tools that include bid jamming as a part of their automated features [2, 13, 3], as well as advertising "anti-jamming" features [8]. Various online magazines, company websites, web "blogs,"and forums define bid jamming [21, 24], discuss strategies against it [16], and even how effective it can be [9]. Some "bidding experts" will advise that bid jamming is "risky" and not recommended [19], but others would say bid jamming is an essential strategy in a competitive bidding situation especially since "your competitors are using bid jamming against you" [10].

There has been some previous work on modeling the phenomenon akin to bid jamming. Brandt et. al. [5] examine a model, which they call *spiteful bidding*, where bidders lose utility equal to the sum of the utility other bidders receive, scaled by a parameter. Zhou and Lukose [25] work with a *vindictive bidding* model where they assume that some bidders choose to be *vindictive* and bid one cent below the next highest bidder. Both of these models are single period models and neither of these articles argue why bidders who are simply interested in maximizing their utility might engage this vindictive behavior. **We argue that jamming behavior naturally occurs in multi-period budget-constrained auctions with self-interested rational bidders.**

To the best of our knowledge, there has been little prior work on bidding strategies in multi-round budget-constrained auctions. Oren and Rothkopf [18] consider a multi-period model without budget constraints where each bidder is assumed to have a function that describes their beliefs about the bidding strategies of their competitors. Consequently, the problem of computing an equilibrium can be reformulated as a dynamic program. Recently, Borgs et. al. [4] describe heuristics for optimizing bids in the keywords setting, under the assumption that bidders only change their bids once every set time period.

A difficulty of the auction situation we model, is that bidders do not place bids simultaneously or obliviously, as is assumed in previous models ([14]). Rather, bidders place bids in a sequential fashion, and the auctioneer must resolve bids as they occur.

A simple model with jamming

Based on the previous literature, a complete analysis of multi-round budget-constrained auctions is a difficult task. We make the first steps towards a rigorous understanding of these auctions, with a particular emphasis on understanding strategic decisions related to jamming.

(a) We propose a two-round two-person model for budget-constrained multi-period auctions that exhibits some of the malicious behaviors seen in practice. In particular, we show that in this model it is rational for self-interested bidders to jam their competitors.

(b) Since computing Nash equilibria in multi-round budget constrained auctions are known to be computationally hard, we use a model where the player is competing against an adversary who has superior information. An equivalent game theoretic description is that our model is an instance of a *Stackelberg game* (see von Stackelberg [22]) where one of the bidders has an information advantage. Such models have been used in military applications, specifically the network interdiction problem (see Brown et al [6]), for which some complexity results are known (see Brown et al [7]). However, in these contexts, the participants in each of the games have inherent asymmetries in what actions are possible, as opposed to our model, where we impose asymmetries upon symmetric participants in order to attain tractability in a complicated theoretical problem. In doing so, we are also able to generate worst-case like results. We believe that the use of asymmetric information to approximate worst-case strategies in games is new and may have implications for other game-theoretic problems.

(c) We give closed form solutions for the optimal player strategy when the adversary's budget is no more than 150% that of the player budget.

NOTATION

The following details are shared by both of our models.

(a) 2 bidders;

Bidder 1: The player; and

Bidder 2: The adversary; (who is assumed to have more information. see below.)

(b) 2 rounds of bidding;

(c) 2 identical objects to be sold each round;

(d) Second price bidding resolution at the end of each round;

(e) Ties are resolved in favor of the player.

We borrow the following conventions from economics: Suppose the n-dimensional vector

$$\mathbf{x}^\top = (x_1, x_2, \ldots, x_n),$$

is given. Then, as usual, x_i will denote the ith vector. We let $\mathbf{x}_{-i}$ denote the $n-1$ vector identical to $\mathbf{x}$ but missing the ith component. Further, we let

$$(y, \mathbf{x}_{-i}^\top) = (x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n),$$

which is the vector $\mathbf{x}$ with the ith component replaced by the value y. For the case of $n = 2$, we will use x_{-i} to denote the $[2-(3-i)]$th component. Thus, x_{-i} is the "other" component with respect to x_i.

For a boolean statement, $\mathcal{S}$, we will let $\mathbf{1}$ denote the indicator function for this statement. Thus,

$$\mathbf{1}(\mathcal{S}) = \begin{cases} 1 & \mathcal{S} \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

We use the following notation:

$$\begin{aligned}
w_i, W_i &= \text{the budget (or wealth) of bidder } i, \\
&\quad W_1 = 1 \\
&\quad W_2 \sim F, \text{ cumulative distribution function on } [0, \omega], \\
v &= \text{the common value of the two objects}, \\
x_i^t &= \text{the bid of bidder } i \text{ in round } t, \\
u_i^t &= \text{the profit function to bidder } i \text{ in round } t, \\
&= \begin{cases} v - x_{-i}^t & \text{if } x_i^t \geq x_{-i}^t \\ 0 & \text{otherwise.} \end{cases} \\
u_i &= \text{the total profit function to bidder } i \\
&= u_i^1 + u_i^2 \\
r_i &= \text{remaining budget of bidder } i \text{ after round 1} \\
&= w_i - x_i^1
\end{aligned}$$

When we discuss a sample path realization of the adversary's budget we will use the lowercase w_2. The reason for this distinction is that the adversary knows his budget before choosing a bid, whereas the player knows only the distribution of the adversary's budget before choosing a bid or strategy. For $a \in \mathbb{R}$, define the following operator:

$$(a)_+ = \max\{0, a\}. \tag{1}$$

A MOTIVATING EXAMPLE

We now give an extended example which demonstrates situations in which jamming occurs and other situations in which it does not. Suppose the object value 2, and the adversary's budget $W_2 = 0.2$ with probability 0.1 and 1.5 with probability 0.9. Recall the player's budget is always 1.

When the adversary should *not* bid jam Suppose the player bids 0.3 with probability one in the first round. The adversary faces one of the following two cases: (1) $w_2 = 0.2$: The adversary does not have any real choice. One possibility is to bid 0.2 to try and reduce the player's budget, but he will still end up losing both rounds. (2) $w_2 = 1.5$: This time the adversary has to choose between jamming or winning. If he jams by bidding 0.3 in the first round, the player wins the first object at 0.3 and the adversary wins the second object by paying $1 - 0.3 = 0.7$. Thus, the adversary's utility from jamming is $2 - 0.7 = 1.3$. On the other hand, if the adversary were to win both rounds his utility would be $(2 - .3) + (2 - 1) = 2.7$. Thus, his best strategy is to win both rounds. The adversary's expected payoff is $0.9(2.7) = 2.43$, the payer's expected payoff when she bids 0.3 (assuming the adversary contests even when he is indifferent) is $0.1(2 - .2 + 2 - .2) = 0.36$.

When the adversary *should* bid jam Now, suppose the player bid 0.7 instead. Again, the adversary can challenge the player only when $w_2 = 1.5$. If the adversary were to bid to win the first round, his remaining budget will be .8 and the player will win the second round. Thus, the adversary's payoff is $2 - 0.7 = 1.3$ if he wins the first round outright. On the other hand, if he were to jam in the first round, he reduces the player's budget to 0.3 for the second round, and wins at least $2 - 0.3 = 1.7$. Hence, he should jam. The adversary's expected winnings are $0.9(1.7) = 1.53$ and the player's expected winnings are $0.1(2 - 0.2 + 2 - 0.2) + 0.9(2 - 0.7) = 1.53$.

More generally, suppose the player bids $0.5 + \varepsilon$, where $0 < \varepsilon << 1$. As before, the adversary can only jam if he receives a budget of 1.5. If he jams, he receives $2 - (1 - 0.5 - \varepsilon) = 1.5 + \varepsilon$. If he wins the first round, he cannot win the second round, and, therefore, receives $2 - .5 - \varepsilon = 1.5 - \varepsilon$. Thus, the adversary prefer jamming for all $\varepsilon > 0$. The player's payoff is $1.71 - \varepsilon$ if the adversary jams and $1.21 + \varepsilon$ if the adversary decides not to jam. Thus, the player can "force" the adversary to jam by bidding slightly above 0.5.

When the adversary is *indifferent* Suppose the player bids 0.5. As before, the adversary can only jam if he receives a budget of 1.5. If he jams, he receives $2 - (1 - .5) = 1.5$, and if he wins the first round, he receives $2 - .5 = 1.5$. Thus, the adversary is indifferent to whether or not he jams. The expected payoff for the adversary is then 1.35. The player's expected payoff is $0.1(2 - 0.2 + 2 - 0.2) + 0.9(2 - 0.5) = 1.71$ if the adversary decides to jam in the first round, but $).1(2 - 0.2 + 2 - 0.2) + .9(2 - 1) = 1.21$ if the adversary de-

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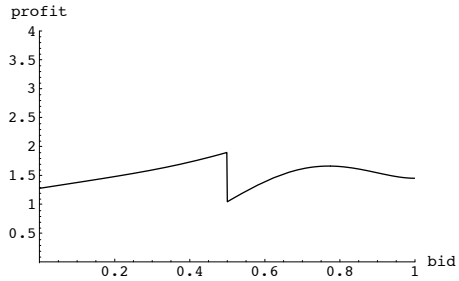
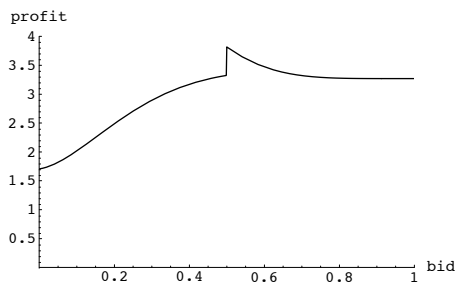


(a) $W_2 \sim Beta(5,2)$



(b) $W_2 \sim Beta(2,5)$

Figure 1. Player expected profit versus 1st round bid with common value $v = 2$.

cides to win outright. Thus, the player is *not* indifferent and, in fact, would prefer to be jammed!

A LEMMA ABOUT THE SECOND-ROUND

We are interested in finding *weakly dominant strategies* which are defined as follows:

Definition 1 (Weakly dominant strategy) *Let $m : \mathcal{X} \to \mathbb{R}$ be some given function representing the expected profit of a bidder playing a strategy from $\mathcal{X}$. $\bar{x}$ is a weakly dominant strategy for the player, if, for all $\mathbf{y} \in \mathcal{X}$, $m(\bar{x}) \geq m(\bar{y})$.*

In a multi-round auction, the strategy for the last round is often simpler.

Lemma 1 (Krishna [14]) *Suppose that round 1 has been resolved. Then a weakly dominant strategy in round 2 for each bidder i is:*

$$x_i^2 := \min\{v, r_i\}. \tag{2}$$

Lemma 1 is true whether or not the bidder is aware of the other bidder's bid or not. Also, Lemma 1 indicates that the only decision the Adversary and Player need to make is their bid in round 1. Thus, a strategy is completely determined by the bids in round 1. We will, therefore, suppress the superscript 1 and refer to the round 1 bid x_i^1 as simply x_i.

FIXED PLAYER STRATEGIES

In this section, we consider the special case when the play bids a fixed (non-randomized) amount in Round 1. Recall that W_2 is the wealth of the adversary, and is a random variable distributed according to the distribution F. If this is the case, then the adversary will bid according to Lemma 1, and $u_i(x_1,x_2), i = 1,2$ are well defined. In particular, if $x_1 \geq x_2$ and $r_1 := 1 - x_2 \geq W_2$ then the player wins both rounds, whence $u_1(x_1,x_2) = 2v - x_2 - W_2$ and $u_2(x_1,x_2) = 0$. If $x_1 \geq x_2$ but $r_1 < W_2$ then the player wins round one and the adversary wins round two whence $u_1(x_1,x_2) = v - x_2$ and $u_2(x_1,x_2) = v - r_1 = v - W_1 + x_2$. If $x_1 < x_2$ and $r_2 := W_2 - x_1 > W_1$ then the player loses both rounds whence $u_1(x_1,x_2) = 0$ and $u_2(x_1,x_2) = 2v - x_1 - W_1$. If $x_1 < x_2$ but $r_2 \leq W_1$, then the player wins the second round and the adversary the first, whence $u_1(x_1,x_2) = v - r_2 = v - W_2 + x_1$ and $u_2(x_1,x_2) = v - x_1$. Note that the u_i are not smooth, and not necessarily continuous. Moreover, note that there are diminishing returns for bidding too much, but a penalty for bidding too little. In particular, we can determine the best bids for the adversary:

Theorem 1 *Given that the player bids x_1 in round 1,a weakly dominant strategy for the adversary is to bid*

$$x_2 := \begin{cases} w_2 & w_2 \leq x_1; \\ \hline x \in (x_1, w_2] & \begin{array}{l} x_1 < w_2 \leq 1 + x_1, x_1 \leq 1/2, \\ w_2 > 1 + x_1, x_1 \leq v/2 \text{ or } v \leq 1 - x_1; \end{array} \\ \hline x_1 & \begin{array}{l} x_1 < w_2 \leq 1 + x_1, x_1 > 1/2, \\ w_2 > 1 + x_1, x_1 > v/2 \text{ and } v > 1 - x_1. \end{array} \end{cases} \tag{3}$$

Proof: We can assume without loss of generality that $x_1 \leq v$. If not, the adversary would also bid x_1 causing the player to incur negative profit, and lose budget for the second round. By bidding a smaller amount, the player would be able to decrease the negative profit incurred and increase the chance of winning in the second round. Thus,

$$x_1 \leq \min\{W_1, v\}. \tag{4}$$

We will prove the theorem using case analysis.

(a) $w_2 \leq x_1$: By (4), $w_2 \leq x_1 \leq v$. If $w_2 > 1 - x_2$ then the adversary profit is $u_2(x_1,x_2) = (v + x_2 - 1)_+$, otherwise the adversary profit is zero. u_2 is clearly maximized when $x_2 = w_2$. In this case, the adversary profits is:

$$u_2 = u_2(x_1, w_2) = \mathbf{1}\{w_2 > 1/2\}(v - 1 + w_2),$$

and the player profit is:

$$u_1(x_1, w_2) = (1 + \mathbf{1}\{w_2 \leq 1/2\})(v - w_2).$$

(b) $x_1 < w_2 \leq 1 + x_1, x_1 \leq 1/2$: In this case, if $x_2 > x_1$, then the adversary cannot win round 2, and his profit is $u_2(x_1,x_2) = (v - x_1)_+$. On the other hand, if $x_2 \leq x_1$, then the adversary will win round 2 if $w_2 > 1 - x_2$, and receive $u_2(x_1,x_2) = (v - 1 + x_2)_+$. If $w_2 \leq 1 - x_2$, the adversary

receives $u_2(x_1,x_2) = 0$. However, since $x_1 \leq 1/2, -x_1 \geq x_1 - 1$, and, if $x_2 \leq x_1$,

$$v - x_1 \geq v - 1 + x_1 \geq v - 1 + x_2.$$

This is an upper bound on the profit obtained by the adversary bids $x_2 \leq x_1$. Thus, the adversary's best strategy is to bid some value greater than x_1 and the adversary and player profits are:

$$u_2(x_1,x_2) = v - x_1, u_1(x_1,x_2) = (v - w_2 + x_1)_+.$$

(c) $x_1 < w_2 \leq 1 + x_1, x_1 > 1/2$: In this case, the adversary must decide between winning either the first or second round, but not both. If he chooses to win the first round then, $u_2(x_1,x_2) = v - x_1$, and he must bid $x_2 > x_1$. If he wins the second round, then $u_2(x_1,x_2) = v - w_2 + x_2$, and $x_2 \leq x_1$. Clearly, the optimal choice for the adversary is to let $x_2 = x_1$. With this choice, $w_2 \geq x_2 = x_1 > 1/2$ and $w_2 + x_2 > 1$, and $v \geq x_1 = x_2 > 1/2 \geq 1 - x_2$, as $x_1 \leq 1$. Moreover, $x_2 - 1 > x_1 - 1 > -x_1$, whence

$$v - 1 + x_2 > v - x_1,$$

and the adversary should bid $x_2 = x_1$. His profit is

$$u_2(x_1,x_1) = v - \min\{v, 1 - x_1\}. \tag{5}$$

The player receives a profit of

$$u_1(x_1,x_1) = v - x_1. \tag{6}$$

(d) $w_2 > 1 + x_1, x_1 \leq v/2$. In this case, the adversary can choose to win either the first, the second or both rounds. If he wins just the first round, his profit will be $u_2(x_1,x_2) = v - x_1$. If he wins the second round, his profit is $u_2(x_1,x_2) = v - 1 + x_1$ (by the argument of the previous case). Finally, if he wins both rounds, his profit is $u_2(x_1,x_2) = v - x_1 + (v-1)_+$, which clearly dominates winning only the first round. Then, if $x_2 = x_1 \leq v/2$,

$$\begin{aligned} v - 1 + x_1 &\leq v - 1 + v - x_1 \\ &= 2v - 1 - x_1, \\ &\leq v - x_1 + (v-1)_+. \end{aligned}$$

So the adversary should bid greater than x_1, and will receive a profit of:

$$u_2(x_1,x_2) = 2v - 1 - x_1. \tag{7}$$

The player receives a profit of

$$u_1(x_1,x_2) = 0. \tag{8}$$

(e) $w_2 > 1 + x_1, x_1 > v/2$ As with the case (4), the adversary can win either or both of the rounds. Also, as before, the profit functions are the same. If $v = \min\{v, 1 - x_2\} = \min\{1, 1 - x_1\}$, then $2v - x_1 - \min\{v,1\} \geq 0 = v - \min\{v, 1 - x_2\}$ and the best adversary strategy, profit, and corresponding player profit is as in Case (4). However, if $1 - x_1 < v$, then

$$\begin{aligned} (v - 1 + x_1)_+ &= v - 1 + x_1, \\ &= v - 1 + 2x_1 - x_1, \\ &> 2v - 1 - x_1. \end{aligned}$$

Thus, the adversary should bid $x_2 = x_1$ which results in a profit of

$$u_2(x_1,x_1) = v - 1 + x_1. \tag{9}$$

The player then receives a profit of

$$u_1(x_1,x_1) = v - x_1. \tag{10}$$

■

Theorem 1 specifies the exact bid that the adversary used as a function of his budget, the player's bid, and the value of the objects, which results in the following Corollary.

Corollary 1 *The adversary gains nothing by using randomization to determine his round one bid.*

Let $s_2 : [0,1] \times [0,\omega] \to [0,\omega]$, denote the best-response function for the adversary as function of the players bid x_1 and his own budget w_2 Given the response function s_2, the expected profit is

$$m_i(x_1) := \mathbb{E}_{W_2}[u_i(x_1, s_2(x_1, W_2))] \tag{11}$$

In Figure 1(a) we plot the player profit as a function of her round 1 bid x_1. The optimal fixed strategy is clearly $x_1 = 1/2$. In Figure 1(b) the optimal fixed strategy for the player appears to be $x_1 = 1/2$, but because of the way in which ties are resolved, the actual best strategy is only realizable in an approximate sense. Note that the discontinuity around $x_1 = 1/2$ is caused by the switch in adversary strategy from *not jamming* (for $x_1 \leq 1/2$) to *jamming* (for $x_1 > 1/2$). This motivates the following weakened definition of dominance for our model:

Definition 2 (Weakly limit-dominant strategy) *Let $m : \mathcal{X} \to \mathbb{R}$ be some given function representing the expected profit of a bidder playing a strategy from $\mathcal{X}$. $\bar{x}$ is a weakly limit-dominant strategy for the player, if, for all $\varepsilon > 0$ there exists a $\delta > 0$ and strategy x such that $|\bar{x} - x| < \delta$, and, for all $y \in \mathcal{X}$,*

$$m(x) + \varepsilon \geq m(y). \tag{12}$$

Note that, in our model, $m \equiv m_1$ and the strategy space is the budget set for player one, i.e., $\mathcal{X} = [0,1]$.

For object values greater or equal to two (i.e., twice the player's budget), and any distribution F that has all its probability mass on the interval $[0,3/2]$, we can calculate either a weakly dominant or a weakly limit-dominant strategy.

Theorem 2 *Given that $v \geq 2$, and $\mathbb{P}$ is any probability measure on $[0,3/2]$, with associated distribution function,m F, $x_1 = 1/2$ is either a weakly dominant strategy or a weakly limit-dominant strategy.*

Proof: To show this theorem, we will show that $x_1 = 1/2$ is weakly dominant on $[0,1/2]$, and weakly limit-dominant on $(1/2,1]$.

We will first show that, for $x \in [0,1/2)$, $m_1(1/2) \geq m_1(x)$. To see this, choose some $x \in [0,1/2)$, and note that, by Theorem 1

$$m_1(1/2) = \int_{[0,1/2]} 2(v-w)\mathrm{d}\mathbb{P}(w) + \int_{(1/2,3/2]} (v+1/2-w)\mathrm{d}\mathbb{P}(w).$$

However,

$$\begin{array}{ll} & \int_{[0,1/2]} 2(v-w)\mathrm{d}\mathbb{P}(w) \\ = & \int_{[0,x]} 2(v-w)\mathrm{d}\mathbb{P}(w) + \int_{[x,1/2]} 2(v-w)\mathrm{d}\mathbb{P}(w), \\ \geq & \int_{[0,x]} 2(v-w)\mathrm{d}\mathbb{P}(w) + \int_{[x,1/2]} (v+x-w)\mathrm{d}\mathbb{P}(w), \quad (a) \end{array}$$

whence $m(1/2) \geq m(x)$. Note that (a) is true since $v-w \geq 1/2 > x$ for all $w \in [0,3/2]$, as $v \geq 2$, whence

$$\begin{aligned} 2(v-w) &= v-w+(v-w), \\ &> v-w+x. \end{aligned}$$

Also note that the inequality in (a) is not strict since $\mathbb{P}$ may have no mass on $[x,3/2]$ for some value $x \in [0,1/2)$.

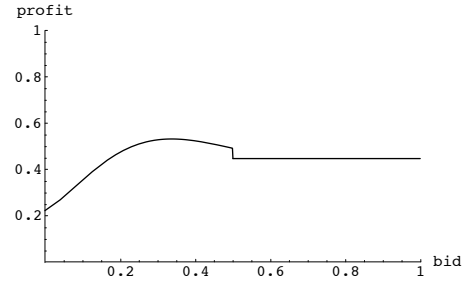
We now show that $x_1 = 1/2$ is weakly limit-dominant on $(1/2,1]$. First, for $\varepsilon \in (0,1/2)$, and $x \in (1/2+\varepsilon,1]$, we claim that $m_1(1/2+\varepsilon) \geq m_1(x)$. To see this, choose some $x \in (1/2+\varepsilon,1]$, and note that

$$\begin{array}{ll} & \int_{(1/2,1/2+\varepsilon]} (v-w)\mathrm{d}\mathbb{P}(w) \\ & + \int_{(1/2+\varepsilon,3/2]} (v-1/2-\varepsilon)\mathrm{d}\mathbb{P}(w) \\ = & \int_{(1/2,1/2+\varepsilon]} (v-w)\mathrm{d}\mathbb{P}(w) \\ & + \int_{(1/2+\varepsilon,x]} (v-1/2-\varepsilon)\mathrm{d}\mathbb{P}(w) + \\ & \int_{(x,3/2]} (v-1/2-\varepsilon)\mathrm{d}\mathbb{P}(w) \\ \geq & \int_{[0,1/2]} 2(v-w)\mathrm{d}\mathbb{P}(w) + \int_{(1/2,1/2+\varepsilon]} (v-w)\mathrm{d}\mathbb{P}(w) \\ & + \int_{(1/2+\varepsilon,x]} (v-w)\mathrm{d}\mathbb{P}(w) + \int_{(x,3/2]} (v-x)\mathrm{d}\mathbb{P}(w) \quad \text{(b)} \end{array}$$

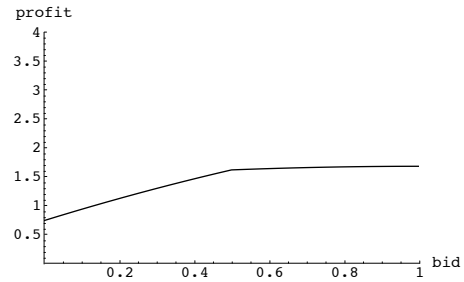
Note that (b) follows since $v-(1/2+\varepsilon) > v-w$ for $w > 1/2+\varepsilon$, and, as before, the inequality used is not strict in case $\mathbb{P}$ has no mass on $(1/2+\varepsilon,3/2]$. However, this inequality implies that $m(1/2+\varepsilon) \geq m_1(x)$ (see [12]).

The rest of the proof is analogous and is proven in [12] for space reasons.

■



(a) $W_2 \sim Beta(2,5), v = 0.5$



(b) $W_2 \sim U[0,2], v = 2$

Figure 2. Cases not covered by Theorem 1

We note that Theorem 2 does not apply to distributions where $\mathbb{P}(W_2 > 3/2) > 0$ or when the value $v < 2x$. The former can be seen in Figure 2(b), and the latter in Figure 2(a). In [12], we provide evidence that a version of Theorem 1 may hold once assumptions about the specific aspects of the distribution are made.

RANDOMIZED STRATEGIES

We now consider the case when the player round 1 bid X_1 is uniformly distributed on the interval $[a,b]$ where $0 \leq a \leq b \leq 1$. In the context of adword auctions, this could correspond to a the player randomizing their bid every appropriate time unit. Note that Lemma 1 continues to hold, so the second round bids are determined by the round 1 bids and the common value v. Let

$$m_2(x_2, X_1) = \mathbb{E}_{X_1}[u_2(X_1, x_2)]. \tag{13}$$

where the expectation is over the random round 1 player bid. When the player round 1 bid X_1 is clear from the context, we will simply write $m_2(x)$.

Theorem 3 *Suppose X_1 is chosen uniformly in $[a,b]$, $1/2 \leq a < b \leq 1$. Then a weakly limit-dominant strategy for the adversary with a budget of w_2 is:*

$$x_2 := \begin{cases} w_2 & w_2 < \max\left\{\frac{b+1}{3}, a\right\}, \\ \max\{a, \frac{b+1}{3}\} & w_2 \in (\max\{a, \frac{b+1}{3}\}, 1+\max\{a, \frac{b+1}{3}\}], \\ w_2 - 1 & w_2 \in [1+a, 1+b), w_2 - 1 \geq \frac{b+1}{3}, \\ w_2 & \textit{otherwise}. \end{cases} \tag{14}$$

Moreover, the strategy is weakly dominant except when $w_2 \in [1+a, 1+b)$ and $w_2 - 1 \geq \frac{b+1}{3}$.

We prove this theorem in [12]. We now give some examples of distributions that illustrate this theorem. Suppose the adversary has a budget that is distributed uniformly on the interval $[1,2]$, and that $v = 2$. The profit function for a player bidding a fixed-point strategy x, is:

$$m_1(x) = \int_1^{1+x} (2-x)dw = 2x - x^2.$$

This concave function is maximized at $x = 1$, and the player therefore bids $x = 1$ with $m(1) = 1$. Now consider the uniform strategy with $a = 1/2$ and $b = 1$. Then, according to Theorem 3, the adversary bids:

$$x_2 = \begin{cases} 2/3 & w_2 \leq 5/3, \\ w_2 - 1 + \varepsilon & \text{otherwise.} \end{cases}$$

For notational convenience, we will assume we are operating at the limit as $\varepsilon \to 0$, and ignore the ε. Then, the player receives:

$$\begin{aligned} & m_1(\Sigma) \\ &= \int_1^2 \int_{1/2}^1 \big[\mathbf{1}\{w \leq 5/3\}(\mathbf{1}\{x < 2/3\}(2-w+x) \\ &\quad + \mathbf{1}\{x \geq 2/3\}(4/3)) \\ &\quad + \mathbf{1}\{w > 5/3\}\mathbf{1}\{x > w-1\}(2-(w-1))\big] 2dxdw, \\ &= 2\big[\int_1^{5/3} \int_{1/2}^{2/3} (2-w+x)dxdw \\ &\quad + \int_1^{5/3} \int_{2/3}^1 (4/3)dxdw + \int_{5/3}^2 \int_{w-1}^1 (3-w)dxdw\big], \\ &= 15/54 + 32/54 + 11/81 = 163/162. \end{aligned}$$

The essential difference between the fixed-point strategy and the uniform strategy is that the adversary is no longer able to jam the player as successfully – any nontrivial bid the adversary makes has a positive probability of winning in the first round, causing the player to jam the adversary.

NON-IDENTICAL OBJECTS

Consider the case when the objects auctioned in each round are no longer identical, and instead have values v_i in rounds $i = 1,2$. Note that, in this case, Lemma 1 still holds. However, the profit functions for the player and adversary are now given by

$$\begin{aligned} u_1(x_1,x_2) &:= \mathbf{1}\{x_1 \geq x_2\}\big(v_1 - x_2 + \mathbf{1}\{1 - x_2 \geq W_2\}(v_2 - W_2)\big) \\ &\quad + \mathbf{1}\{x_1 < x_2\}\mathbf{1}\{W_2 - x_1 \leq 1\}(v_2 - W_2 + x_1) \quad (15) \\ u_2(x_1,x_2) &:= \mathbf{1}\{x_1 < x_2\}\big(v_1 - x_1 + \mathbf{1}\{W_2 - x_1 \geq 1\}(v_2 - 1)\big) \\ &\quad + \mathbf{1}\{x_1 \geq x_2\}\mathbf{1}\{1 - x_2 \leq W_2\}(v_2 - 1 + x_2) \quad (16) \end{aligned}$$

For certain item, adversarial budget and player bid values, the strategy of the adversary will shift from winning the first auction to jamming the first auction. The "critical value" (c.f. [20]) is defined as follows.

$$\bar{c}(v_1, v_2) = \frac{1 + v_1 - v_2}{2}. \quad (17)$$

We will focus on this function defined only along the player's feasible bidding region, i.e.

$$c(v_1, v_2) = \max\big\{\min\{c(v_1, v_2), 1\}, 0\big\} \quad (18)$$

Note that in the case that $v_1 = v_2, c(v_1, v_2) = 1/2$. We can then generalize Theorem 1 to this setting:

Theorem 4 *Given that the player bids x_1 in round 1,a weakly dominant strategy for the adversary is to bid*

$$x_2 := \begin{cases} w_2 & \begin{array}{l} w_2 < x_1, \\ x_1 < w_2 \leq 1 - x_1, \\ \max\{x_1, 1-x_1\} < w_2 \leq 1 + x_1, x_1 \leq c(v_1, v_2), \\ w_2 > 1 + x_1, \text{ and } (x_1 \leq v/2 \text{ or } v_2 \leq 1); \end{array} \\ \hline x_1 & \begin{array}{l} \max\{x_1, 1-x_1\} < w_2 \leq 1 + x_1, x_1 > c(v_1, v_2), \\ w_2 > 1 + x_1, x_1 > v_1/2, \text{ and } v_2 > 1. \end{array} \end{cases} \quad (19)$$

We prove this theorem in [12]. However, there does not exist a weakly dominant fixed bid strategy that is budget-distribution independent.

Theorem 5 *Suppose $v_1 + v_2 > 1$, $c := c(v_1, v_2) \in (0,1)$ and $v_1 > c$. Then there is no budget-distribution independent player fixed strategy that is weakly limit-dominant.*

Proof: Recall that a fixed strategy is simply a constant bid. This theorem states that there is no fixed bid that is weakly limit-dominant across all possible adversary budget distributions. We will examine three cases

(i) $x_1 = c$: Suppose the adversary has a budget that is distributed on $(1+c, 2]$, and note that the adversary can win both rounds if $x_1 = c$. The adversary will do this, as opposed to jamming since the profit he receives from jamming satisfies:

$$\begin{aligned} v_2 - 1 + c &= v_2 - 1 + (v_1 + 1 - v_2)/2 \\ &= v_1 + v_2 - 1 - v_1 + (v_1 + 1 - v_2)/2 \\ &= v_1 + v_2 - 1 - v_1/2 - 1/2 + v_2/2 + 1 - v_2 \\ &< v_1 + v_2 - 1 - c, \end{aligned}$$

which is the profit for the adversary if he wins both rounds. Since $v_1 > c$, the player bid of c is dominated by the bid $x \in (c, v_1)$, as $v_2 = v_1 + 1 - c > 1$, and the adversary cannot win in both rounds. No matter which round the adversary wins, the player receives positive profit.

(ii) $x_1 < c$: Suppose that the adversary has a budget that is $1+x_1$ with probability one. Then, according to Theorem 4 a player bid of $c+\varepsilon$ for $\varepsilon = (v_1 - c)/2$ dominates x_1 since the player will win the first round and results in a profit of $v_1 - c > 0$.

(iii) $x_1 > c$: Suppose that the adversary has a budget of $1+c$ with probability one. Then, according to Theorem 4, a player bid of $\bar{x}_1 \in (c, x_1)$ dominates x_1 as the player is jammed in both cases, but receives $x_1 - \bar{x}_1$ more profit.

■

Bidding the critical value is clearly not a dominant strategy across distributions; however, the player can force the adversary to bid his budget, rather than jamming, by bidding the critical value.

Theorem 6 *For values v_1 and v_2 such that $c := c(v_1, v_2) \in [0,1]$, a player bid of $x_1 = c$ causes the adversary to bid his budget, i.e., he will not jam the player.*

Proof: By Theorem 4, if $w_2 \leq c$, the adversary always bids w_2. So assume that $w_2 > c$. In this case, we simply have to show that $c > v_1/2$ and $v_2 > 1$ cannot both be true. To see that this is so, note that

$$v_1/2 < c \quad \Leftrightarrow \quad c - v_1 = -v_2 + 1 > 0 \quad \Rightarrow \quad v_2 < 1.$$

■

CONCLUSIONS

In this extended abstract, we investigated auction phenomena of *bid jamming* by using asymmetric information techniques. Our model provides evidence that bid jamming is a natural consequence of the multi-round budget-constrained auction setting, where it can be a weakly-dominant strategy. Our model also provides insights into the trade-offs between whether to jam or not, and supports the intuition that bidders should not blindly do one or the other. In a setting where the objects auctioned are identically valued, and a bidder knows that her opponent's budget is at most 150% of her own, our model suggests that she should bid half her own budget in the first round. When the objects auctioned are identically valued, but the opponents budget is not known to be bounded above, our model suggests that even simple randomization is a better strategy than any fixed bid. However, the added complication of non-identically valued objects indicates that more knowledge regarding the opponent's strategy will be required in order to strategically bid.

There are a number of interesting questions that remain to be answered.

- An immediate line of research is to extend the results presented here to the case of more rounds and more objects auctioned each round.
- Further investigation into computing useful bidding strategies seem to require:
  - Determining how well an adversary can approximate multiple bidders, i.e., the invisible hand of the rest of the market.
  - Determining if there is an algorithmic way to apply knowledge of the adversary's budget distribution to determine strategies.

REFERENCES

[1] G. Aggarwal, A. Goel, and R. Motwani. Truthful auctions for pricing search keywords. In *Proceedings of the 2006 ACM Conference on Electronic Commerce*, 2006.

[2] Apex Pacific Dynamic Bid Maximizer, see http://www.apexpacific.com/bidmax.html.

[3] Atlas Search BidManager, see http://www.gotoast.com/products/bidmanager.

[4] C. Borgs, J. Chayes, N. Immorlica, M. Mahdian, and A. Saberi. Multi-unit auctions with budget-constrained bidders. In *Proceedings of The Sixth ACM Conference on Electronic Commerce (EC'05)*, 2005.

[5] F. Brandt, T. Sandholm, and Y. Shoham. Spiteful bidding in sealed-bid auctions. In *Proceedings of the 7th Workshop on Game Theoretic and Decision Theoretic Agents (GTDT)*, 2005.

[6] G. Brown, M. Carlyle, R. Harney, E. Skroch, and K. Wood. How to optimally interdict a belligerent project to develop a nuclear weapon. submitted for publication, 2005.

[7] G. Brown, M. Carlyle, J. Royset, and K. Wood. On the complexity of delaying an adversary's project. In B. Golden, S. Raghavan, and E. Wasil, editors, *The Next Wave in Computing, Optimization and Decision Technologies*, pages 3–17. Springer, New York, NY, 2005.

[8] ClickTracs BidHero, see http://www.clicktracks.com/products/bidhero/index.php.

[9] Digital Point online forum, see http://forums.digitalpoint.com/showthread.php?t=30805.

[10] Winning a search advertising bidding war. *Domains Magazine*, 2006. Online magazine, see http://domainsmagazine.com/domain/Domains_18/Domain_2945.shtml.

[11] B. Edelmann, M. Ostrovsky, and B. Schwartz. Internet advertising and the generalized second price auction. Technical Report 11765, NBER Working Paper, 2005.

[12] G. Iyengar, D. J. Phillips, and C. Stein. Strategies for sequential auctions with budgets. Technical report, CORC, Columbia University, 2005. Working paper.

[13] KeywordMax BidDirector, see http://www.keywordmax.com/bid_director.html.

[14] V. Krishna. *Auction Theory*. Academic Press, San Diego, CA, 2002.

[15] S. Lahaie. An analysis of alternative slot auction designs for sponsored search. In *Proceedings of The Seventh ACM Conference on Electronic Commerce (EC'06)*, 2006.

[16] Quick tips for bid jamming prevention. *MarketerToday.com*, 2005. Online magazine, see http://www.marketertoday.com/archives/2005/10/quick_tips_for.html.

[17] P. R. Milgrom and R. J. Weber. Theory of auctions and competitive bidding. *Econometrica*, 50(6):1089–1122, 1982.

[18] Shmuel S. Oren and Michael H. Rothkopf. Optimal bidding in sequential auctions. *Operations Res.*, 23(6):1080–1090, 1975.

[19] D. Pastemak and K. Lee. Winning the search bidding wars. *Multichannel Merchant*, 2003. Online magazine, see http://multichannelmerchant.com/webchannel/seo/marketing_winning_search_bidding.

[20] C. Pitchik. Budget-constrained sequential auctions with incomplete information. Unpublished Manuscript, 1994.

[21] S. Spencer. Web marketing blog, see http://www.stephanspencer.com/archives/2005/09/06/bid-jamming-and-gap-surfing/.

[22] V. H. Stackelberg. *The Theory of Market Economy*. Oxford University Press, Oxford, 1952.

[23] H. R. Varian. Position auctions. Technical report, UC Berkeley, 2006.

[24] Web1marketing company homepage, see http://www.web1marketing.com/glossary.php?term=Bid+Jamming.

[25] Y. Zhou and R. Lukose. Vindictive bidding in keyword auctions. In *Proceedings of the Second Workshop on Sponsored Search Auctions (SSA)*, 2006.