

# SA305: Linear models and optimization

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UNITED STATES NAVAL ACADEMY

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# Who am I?

A quick work bio:

- 1997: B.A. in Mathematics from Oberlin College
- 1997–2000: Research Analyst at The Lewin Group and PA Consulting
- 2001–2006: M.S., M.Phil., Ph.D. in Operations Research from Columbia University
- 2006–2011: Assistant Professor at the College of William & Mary
- Fall 2011–now: Assistant Professor at USNA

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**Please use email to contact me – your course policies has my email address**

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- Or by appointment

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- **Group work: 10%**
- **Tests: 40%**
  - Two tests as on syllabus, 20% each.
- **Final exam: 30%**
  - Usual USNA policies

# Make up and late policies

- Late homework graded at my discretion
- Exams: You must communicate with me prior to the examination if you will miss an examination or else you will receive a zero on the exam.
- Final examination: any scheduling change must be arranged through the Dean.

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- Talk to me: Ask questions, go to EI, etc.
- Forming a study group is encouraged, but....
  - You need to make sure you can actually do the problems.

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One answer:

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# When in doubt, consult the internet

from Wiki:

*Operations research (also referred to as decision science, or management science) is an interdisciplinary mathematical science that focuses on the effective use of technology by organizations. In contrast, many other science & engineering disciplines focus on technology giving secondary considerations to its use.*

# When in doubt, consult the internet

from Wiki:

*Operational research (OR) encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency. Some of the tools used by operational researchers are statistics, optimization, probability theory, queuing theory, game theory, graph theory, decision analysis, mathematical modeling and simulation. Because of the computational nature of these fields, OR also has strong ties to computer science and analytics. Operational researchers faced with a new problem must determine which of these techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power.*

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from the Institute for Operations Research and Management Science(INFORMS) site:

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At the INFORMS Annual Meeting last year

- Estimated 4500 attendees: academics and industry practitioners
- Over 70 parallel tracks, four sessions per day over four days
- Topics include applied probability, optimization, specific application areas (e.g., healthcare, military, finance, transportation), logistics and planning

# A little history

17th and 18th century: Some mathematical underpinnings

- Expected value, B. Pascal (1654)
- Newton's Method, Newton (1665)
- Bridges of Königsberg, Euler (1736)
- Bayes Rule, Bayes (1763)
- Lagrangian multipliers, Lagrange (1788)
- Least Squares, Gauss (1795)

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- Uses OR to analyze mail delivery, leads to Sir Rowland Hill introducing the Penny Post to Britain
- Key quote from the Babbage Room of the Totnes Museum: *He broke down the service into various elements – the operations involved, the manpower required, the expense of each process – and discovered that the cost of handling mail was greater than the cost of transportation.*

## 19th and early 20th century results

- Solution of inequalities, Fourier (1826)
- Solution of linear inequalities, Gauss (1826)
- Gantt charts, Gantt, Taylor (1900)
- Farkas lemma, Farkas (1902)
- Pareto optimality, Pareto (1906)
- Markov chains, Markov (1907)
- “The Theory of Probabilities and Telephone Conversations”, Erlang (1909)

Formal beginnings:

1936 The term “operational research” is first used in Great Britain to describe experiments studying the most effective use of radar.

1939,1947 Kantorovich (1939) and Dantzig (1947) discover linear programming. Dantzig (1947) describes the simplex method.

1941 Operational Research Section (ORS) was established in Britain directed by P. Blackett.

- Organizes flying maintenance and inspection
- Improvement of antisubmarine operations

1949 Monte Carlo simulation, S. M. Ulam, J. von Neumann

1952 First graduate programs (M.A. and Ph.D.) established at Case Western.

## Historical connections to the Navy:

1942,1945 U.S. Navy begins to use OR formally via the Antisubmarine Warfare Operations Research Group (1942) and the Operations Evaluation Group (OEG, 1945).

1951 Naval Post Graduate School OR program established.

1954 Naval Research Logistics Quarterly established.

1962 Center for Naval Analyses established.

# OR applications

- Airline: scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet,
- Pharmaceutical: R & D management,
- Delivery companies: routing and planning,
- Financial services: credit scoring, marketing, and internal operations,
- Government: deployment of emergency services, scheduling police and fire department, regulation of environmental pollution, air traffic safety, AIDS intervention
- Healthcare: kidney transplant donor matching, departmental staffing and scheduling, radiation therapy

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- 1 Model formulation & building
  - 1 Discussing problem with decision makers
  - 2 Gathering data
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- ③ Interpreting and validating model results
  - ① Are the results meaningful to the decision maker?
  - ② What are the strengths and weaknesses of the model?
  - ③ How can we improve the current model?

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- 1 Model formulation & building
  - 1 Discussing problem with decision makers
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  - 3 **Abstracting into a mathematical model**
- 2 Analysis of the model
  - 1 **Can the model be solved?**
  - 2 **How is uncertainty being taken into account?**
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  - 1 Are the results meaningful to the decision maker?
  - 2 What are the strengths and weaknesses of the model?
  - 3 **How can we improve the current model?**
- 4 Implementation: practice versus theory
  - 1 Are the computations performed efficiently with respect to the decision maker's needs and resources?
  - 2 Are the results in a form that are useful to the decision maker?

# Reading assignment

## Discussion questions for Friday

- Describe the situation:
  - Who is the principal player?
  - What are their objective(s)?
  - What can they change? What are their decisions?
- Describe the methodology:
  - If you understand the model, describe it. If not, at what point are you not sure what's going on?
  - List the methods and models used with definitions.
- Describe the outcomes:
  - What is the benefit of the study?
  - Are their long-term benefits?
  - What are the future directions?