SA305: Linear Models and Optimization
Spring 2012
NPS Format
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The following is the required format for our modeling in SA305. Based on the format used at the Naval Postgraduate School (NPS), we refer to the format as the *NPS Format*. In order to organize our models for both readability and to help improve correctness, the format divides the model into different sections. Each section has a title and requires the following information:

1. Index and set use In this section, describe any indices and/or index sets used. If none are used, simply write "n/a", i.e., "not applicable".
2. Given Data Describe the notation used for all fixed data, if any.
3. Decision Variables Describe and define all decision variables used.
4. Formulation Write out the formulation in this section and make sure to **label the objective and all constraints**.
5. Discussion Describe your objective and constraints here. Any other details about the model should also be described here.

Lemonade Stand. Here is the lemonade formulation written out in NPS format. We are including the extra details discussed in class:

- Mary can buy extra lemons for \$.05 per lemon. She can buy up to 800 extra lemons.
- Mary can whip lemonade with a blender and sell a quart of whipped lemonade for \$8. Whipping lemonade requires an additional 5 minutes.
- Mary must make 10 quarts of lemonade (whipped or regular) and 15 quarts of sorbet.
- Mary must make at least three quarts of lemonade for every quart of sorbet made.

Index and set use: Let $\mathcal{P}$ denote the products that Mary produces and sells. In the current description, we have $\mathcal{P} = \{\text{lemonade, whipped lemonade, sorbet}\}$. We let $\mathcal{R}$ denote the resources used to produce the products Mary makes. Currently, $\mathcal{R} = \{\text{lemons, sugar, time}\}$. In what follows, we use the indices $j \in \mathcal{P}$ and $r \in \mathcal{R}$.

Data: We use s_j to denote the selling price of product j, and a_{jr} to denote the amount of resource r that product j requires to produce one unit. We use b_r to denote the amount of resource r that Mary has on hand. We use ℓ_r to denote limit on the amount of extra resource r that can be bought and c_r to denote the cost of purchasing one unit of resource r.

Decision Variables: Our decision variables are x_j, which denotes the amount of product j is made. We use y_r to denote the amount of resource r that Mary buys.

Formulation:

$$\begin{array}{lll} \max & \sum_{j\in\mathcal{P}} s_j x_j + \sum_{r\in\mathcal{R}} c_r y_r & \text{(a)} \\ \text{s.t.} & \sum_{j\in\mathcal{P}} a_{rj} x_j \le b_r + y_r, \quad \forall r \in \mathcal{R} & \text{(b)} \\ & 0 \le y_r \le \ell_r \quad \forall r \in \mathcal{R} & \text{(c)} \\ & x_j \ge 0 \quad j \in \mathcal{P}. & \text{(d)} \end{array}$$

Discussion: The objective, (a), represents the profit Mary would receive if only sales and the cost of purchasing additional resources is taken into account. Constraints (b) represent the constraint on the resources available for use in making products and constraints (c) represent the nonnegativity and upper bound on additional resources that Mary can purchase. Based on the description given, ℓ_r would be set to zero for all resources except lemons. Constraints (d) are the nonnegativity of the amount of each product Mary makes.